Recitation 11
October 9, 2008

1. Let X be a continuous random variable with the uniform distribution on $[-1, 1]$. Find the PDF of $\sqrt{|X|}$ and the PDF of $-\ln|X|$.

2. (Problem 3.34, page 198 of the text.) A defective coin minting machine produces coins whose probability of heads is a random variable X with PDF

$$f_X(x) = \begin{cases} xe^x, & x \in [0, 1], \\ 0, & \text{otherwise.} \end{cases}$$

A coin produced by this machine is selected and tossed repeatedly, with successive tosses assumed independent.

(a) Find the probability that a coin toss results in heads.

(b) Given that a coin toss resulted in heads, find the conditional PDF of X.

(c) Given that the first coin toss resulted in heads, find the conditional probability of heads on the next toss.

3. Many natural phenomena involving counts (emitted or absorbed photons, decaying particles, etc.) are modeled well with Poisson random variables. Suppose that for each minute that a Geiger counter is exposed to some source of radiation, the number of "beeps" N has the Poisson PMF with parameter $\lambda > 0$:

$$p_N(n) = e^{-\lambda} \frac{\lambda^n}{n!}, \qquad \text{for } n = 0,\ 1,\ 2,\ \ldots \text{ (and zero otherwise).}$$

The numbers of beeps in disjoint minutes are independent.

(a) Find the probability of observing 1, 3, 3, and 2 beeps in four disjoint minutes. (That is, the probability of observing 1 beep in the first minute, 2 beeps in the second minute, and so on.) The answer will depend on λ.

(b) As a way of estimating λ from the observations in part (a), find the value of λ that maximizes the probability found in part (a). This estimate of λ is called the *maximum likelihood* estimate. We will return to this topic in Chapter 9.