

Recitation 12¹
October 16, 2008

1. Consider n independent tosses of a k -sided fair die. Let X_i be the number of tosses that result in i . What is the distribution of X_i ? Compute the covariance of X_1 and X_2 . Give an interpretation of the result. Compute the correlation coefficient of X_1 and X_2 . Give an interpretation of the result.
2. Suppose that X_1 and X_2 are independent exponential random variables with parameter λ . Let random variables Y and Z be defined according to:

$$Y_1 = X_1 + X_2, \quad Y_2 = X_1/X_2.$$

Are Y_1 and Y_2 independent?

3. Romeo and Juliet have a date at a given time, and each, independently, will be late by an amount of time (measured in minutes) that is exponentially distributed with parameter $\lambda = 2/\text{min}$. If Romeo arrives before Juliet, he will wait for her for 20 minutes. However, if Juliet arrives before Romeo, she will wait for him for 1 minute only! What is the probability that they will meet? Derive your answer by direct integration of the joint PDF of their arrival times. Verify your answer by computing the PDF of the difference of their arrival times using the convolution formula.

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