MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Spring 2008)

Recitation 13
April 3, 2008

1. Define X as the height in meters of a randomly selected Canadian, where the selection probability is equal for each Canadian, and denote $\mathbf{E}[X]$ by h. Bo is interested in estimating h. Because he is sure that no Canadian is taller than 3 meters, Bo decides to use 1.5 meters as a conservative (large) value for the standard deviation of X. To estimate h, Bo averages the heights of n Canadians that he selects at random; he denotes this quantity by H.

   (a) In terms of h and Bo's 1.5 meter bound for the standard deviation of X, determine the expectation and standard deviation for H.

   (b) Help Bo by calculating a minimum value of n (with $n > 0$) such that the standard deviation of Bo's estimator, H, will be less than 0.01 meters.

   (c) Say Bo would like to be 99% sure that his estimate is within 5 centimeters of the true average height of Canadians. Using the Chebyshev inequality, calculate the minimum value of n that will make Bo happy.

   (d) If we agree that no Canadians are taller than three meters, why is it correct to use 1.5 meters as an upper bound on the standard deviation for X, the height of any Canadian selected at random?

2. Random variable X is uniformly distributed between -1.0 and 1.0. Let $X_1, X_2, \ldots$, be independent identically distributed random variables with the same distribution as X. Determine which, if any, of the following sequences (all with $i = 1, 2, \ldots$) are convergent in probability. Give reasons for your answers. Include the limits if they exist.

   (a) X_i

   (b) $Y_i = \dfrac{X_i}{i}$

   (c) $Z_i = (X_i)^i$

   (d) $T_i = X_1 + X_2 + \ldots + X_i$

   (e) $U_i = \dfrac{X_1 + X_2 + \ldots + X_i}{i}$

   (f) $V_i = X_1 \cdot X_2 \cdot \ldots \cdot X_i$

   (g) $W_i = \max(X_1, \ldots, X_i)$