Recitation 15
October 28, 2008

1. Random variable X is uniformly distributed between -1.0 and 1.0. Let $X_1, X_2, \ldots$, be independent identically distributed random variables with the same distribution as X. Determine which, if any, of the following sequences (all with $i = 1, 2, \ldots$) are convergent in probability. Give reasons for your answers. Include the limits if they exist.

   (a) X_i

   (b) $Y_i = \dfrac{X_i}{i}$

   (c) $Z_i = (X_i)^i$

   (d) $T_i = X_1 + X_2 + \ldots + X_i$

   (e) $U_i = \dfrac{X_1 + X_2 + \ldots + X_i}{i}$

   (f) $V_i = X_1 \cdot X_2 \cdot \ldots \cdot X_i$

   (g) $W_i = \max(X_1, \ldots, X_i)$

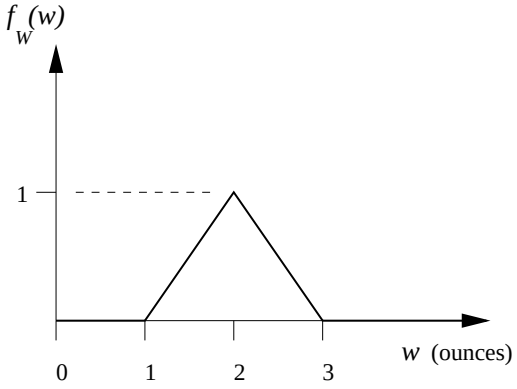
2. You have two nearly identical coins in your pocket, a weighted coin that comes up heads with probability 0.55, along with a fair coin. With no obvious way to tell the two apart in appearance, you take out one coin and decide to flip it 1000 times. If it comes up heads more than 525 times, you'll presume that you've been flipping the biased coin. Assume that in reality you have the fair coin.

   (a) By using the de Moivre - Laplace normal approximation to the binomial, approximate the probability that you will presume it is the biased coin.

   (b) Find an upper bound on the probability that you will presume it is the biased coin by using the Markov inequality.

   (c) Find an upper bound on the probability that you will presume it is the biased coin by using the Chebyshev inequality.

3. The weight of a Pernotti Parabolic Pretzel, W, is a continuous random variable described by the probability density function



$$f_W(w) = \begin{cases} 0 & w \le 1 \\ w-1 & 1 \le w \le 2 \\ 3-w & 2 \le w \le 3 \\ 0 & 3 \le w \end{cases}$$

(a) What is the probability that 102 pretzels weigh more than 200 ounces?

(b) What is the smallest integer (the pretzels are not only inedible, they are also unbreakable) n for which the total weight of n pretzels will exceed 200 ounces with probability 0.990?