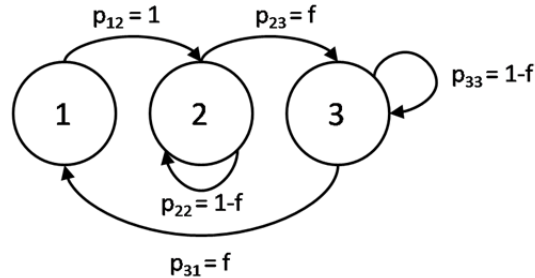
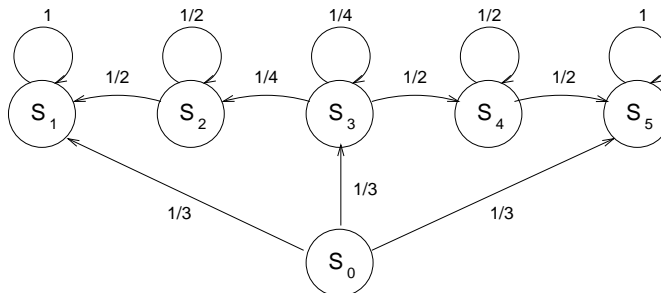


Recitation 17
November 4, 2008

1. Consider the three-state Markov chain in the figure.



- (a) If $0 < f < 1$, list the recurrent and transient states.
 - (b) For what value of f does state 1 become a transient state?
 - (c) For what value of f , if any, does this Markov chain become periodic?
2. There are n fish in a lake, some of which are green and the rest blue. Each day, Helen catches 1 fish. She is equally likely to catch any one of the n fish in the lake. She throws every fish she catches back into the lake, but when she catches a green fish, she paints it blue before throwing it back in. Helen is so artistically inclined, that the blue painted fish are indistinguishable from the blue fish. Let G_i denote the event that there are i green fish left in the lake.
- (a) Show how to model this fishing exercise as a Markov chain, where $\{G_i\}$ are the states. Explain why your model satisfies the Markov property.
 - (b) Find the transition probabilities $\{p_{ij}\}$.
 - (c) List the transient and the recurrent states.
3. Consider the following Markov chain:



Given that the above process is in state S_0 just before the first trial, determine by inspection the probability that:

- (a) The process enters S_2 for the first time as the result of the k th trial.
- (b) The process never enters S_4 .
- (c) The process enters S_2 and then leaves S_2 on the next trial.
- (d) The process enters S_1 for the first time on the third trial.
- (e) The process is in state S_3 immediately after the n th trial.