Massachusetts Institute of Technology

Department of Electrical Engineering & Computer Science

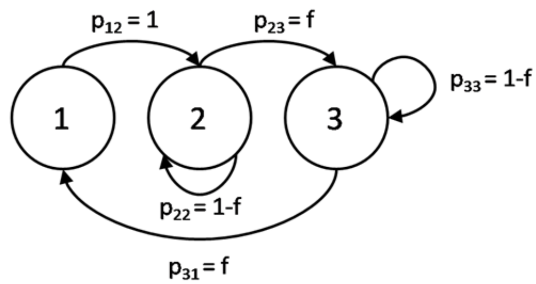
6.041/6.431: Probabilistic Systems Analysis

(Fall 2008)

Recitation 18

November 6, 2008

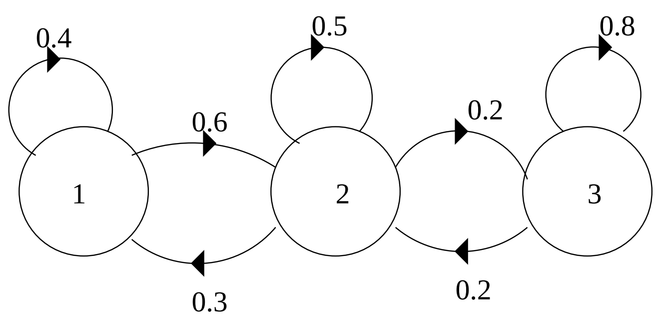
1. Consider the three-state Markov chain in the figure.



   (a) For what value of f, if any, does this Markov chain have a steady state distribution? If it exists, what are the steady state probabilities?

2. The parking garage at MIT has installed a card operated gate, which, unfortunately, is vulnerable to absent-minded faculty and staff. In particular, each day a car crashes into the gate with probability p, in which case a new gate must be installed. Also a gate that has survived for m days must be replaced as a matter of periodic maintenance. What is the steady-state expected frequency of gate replacements?

3. Problem 7.13 in textbook. Consider the Markov chain below. Let us refer to a transition that results in a state with a higher (respectively, lower) index as a birth (respectively, death). Calculate the following quantities, assuming that when we start observing the chain, it is already in steady-state.



   (a) For each state i, the probability that the current state is i.

   (b) The probability that the first transition we observe is a birth.

   (c) The probability that the first change of state we observe is a birth.

   (d) The conditional probability that the process was in state 2 before the first transition that we observe, given that this transition was a birth.

   (e) The conditional probability that the process was in state 2 before the first change of state that we observe, given that this change of state was a birth.

(f) The conditional probability that the first observed transition is a birth given that it resulted in a change of state.

(g) The conditional probability that the first observed transition leads to state 2, given that it resulted in a change of state.