Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2008)

Recitation 19
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1. Sasha just got a new puppy when they moved into a new home. As it turns out, the puppy is a domestic terrier, and destroys a random amount X square meters of the front lawn every day. X is uniformly distributed over the interval $[0, \theta]$. At the end of each day, an annoyed staff of gardeners fix the damage wrought by the puppy. Sasha, a budding Bayesian statistician, is more fascinated than horrified with the puppy's behavior, and chooses to model the destruction of the front lawn. She treats the parameter θ as an unknown value of a random variable Θ, uniformly distributed between zero and one square meter.

   (a) On one day, Sasha observes that the puppy destroyed x m^2. How should Sasha use this information to update the distribution of Θ?

   (b) Let X_1, ..., X_n be the amounts of lawn destroyed by the puppy on n consecutive days. Given $\Theta = \theta$, X_1, ..., X_n are independent and uniformly distributed in the interval $[0, \theta]$. Sasha keeps careful measurements over the n days, and records $X_1 = x_1$, ..., $X_n = x_n$. How should Sasha use this information to update the distribution of Θ?

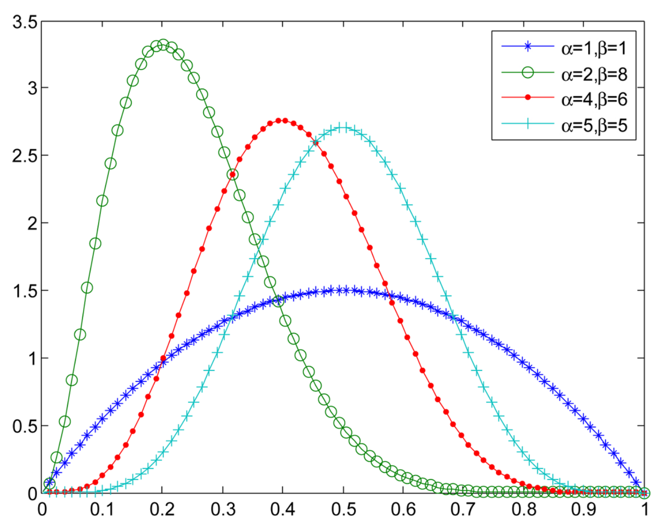
2. We wish to estimate the probability of heads of a biased coin, which we denote θ. We model θ as the value of a random variable Θ with known prior PDF f_Θ. We consider n independent tosses and let X be the number of heads observed.

   (a) Find the posterior PDF of Θ in terms of its prior f_Θ.

   (b) Assume that the prior PDF is of the form

   $$f_\Theta(\theta) = \begin{cases} \frac{\theta^\alpha (1-\theta)^\beta}{B(\alpha,\beta)} & \text{if } 0 < \theta < 1 \\ 0 & \text{otherwise} \end{cases},$$

   where $\alpha \geq 1$ and $\beta \geq 1$. This type of distribution is part of the family of *Beta* distributions. $B(\alpha, \beta)$ is a normalizing constant, known as the Beta function. The graph below illustrates example Beta distributions:



   Find the shape of the posterior PDF of Θ and compare it to the prior.