Massachusetts Institute of Technology

Department of Electrical Engineering & Computer Science

6.041/6.431: Probabilistic Systems Analysis

(Fall 2008)

Recitation 21
November 25, 2008

1. (See problem 10 pg. 447.) A police radar always overestimates the speed of incoming cars by an amount that is uniformly distributed between 0 and 5 miles/hour. Assume that car speeds are uniformly distributed between 55 and 75 miles/hour. What is the (LMS) Least Mean Squares estimate of a car's speed based on the radar's measurement? You might find it helpful to plot the region where the joint distribution of the speed and radar measurement is nonzero, and the LMS estimate.

2. One metric for the merit of an investment analyst is his or her ability to correctly recommend which of two stocks will yield a greater return over the next year, given two stocks chosen at random from the New York Stock Exchange. Let the metric of merit A be the probability that such a recommendation turns out to be correct, and assume that A has the uniform density

$$f_A(a) = 1, 0 \leq a \leq 1.$$

You give your advisor three different pairs of stocks (chosen at random from the New York Stock Exchange) and ask her to advise you which one from each pair will perform best over the next year. One year later K of her predictions turn out to be accurate, and $3 - K$ of her recommendations underperform, $0 \leq K \leq 3$. Assume that the outcomes of all her recommendations are conditionally independent, given $A = a$.

   (a) Suppose we are given the value taken by A and asked to find the most probable value for K. Is this an estimation problem or a hypothesis testing problem?

   (b) Suppose we are given the value taken by K and asked to find the conditional mean of A. Is this an estimation problem or a hypothesis testing problem?

   (c) Find the LMS (Least Mean Squares) estimate for A, given that $K = 2$.

   (d) Now suppose that the distribution of A is not uniform, but instead

$$f_A(a) = \begin{cases} 6a(1-a) & 0 \leq a \leq 1. \\ 0 & \textit{otherwise}. \end{cases}$$

   Will the LMS estimate of A, given that $K = 2$, be greater than or less than the answer you found in part (d)? First see if you can answer this without doing any calculations, and explain your reasoning. Then carry out the calculations to find the exact value for the LMS of A, given that $K = 2$ (and hopefully this will confirm your intuitive answer that required no calculations).

3. Let Θ be a random variable which is distributed in the interval $[1, 3]$. The distribution of Θ is piecewise linear and symmetric around 2. That is:

$$f_\Theta(\theta) = \begin{cases} \theta - 1 & \text{if} \quad \theta \in [1, 2] \\ 3 - \theta & \text{if} \quad \theta \in [2, 3] \end{cases}$$

   (a) What is the Least Mean Squares estimate of θ in the absence of any observation?

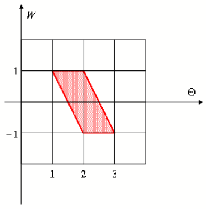

Figure 1: The joint distribution of Θ and W is uniform over the shaded area.

(b) Suppose now, that we can observe the value of Θ through noisy measurements. The observed value is $X = \Theta + W$, where W is not independent of Θ. The joint distribution of Θ and W is uniform over the shaded region shown in Figure 1 and is zero everywhere else.

Given an observation $X = x$, what is the LMS estimate of Θ? Plot the region where the joint pdf of Θ and X is non-zero, and on the same graph sketch the LMS estimate. Give an interpretation of your results.