Recitation 23 - Solutions
December 4th, 2008

1. c. We estimate the variance σ_2 with the unbiased estimator $\hat{S}_n^2$ defined in question 1 of recitation 22.. The variance σ^2/n of the mean estimator $\hat{\Theta}_n$ can be estimated by $\hat{S}_n^2/n$. Since we are interested in the 95 % confidence interval we set $\alpha = 0.05$. For n=4, we find from the t-distribution table the value of z, for which $1 - \Psi_3(z) = 0.025$, is 3.182. Therefore the 95% confidence interval is given by,

$$[\hat{\Theta}_n - 1.591\hat{S}_n, \hat{\Theta}_n + 1.591\hat{S}_n].$$

For $n = 16$, we find from the t-distribution table the value of z, for which $1-\Psi_{15}(z) = 0.025$, is 2.131. Therefore the 95% confidence interval is given by,

$$[\hat{\Theta}_n - 0.533\hat{S}_n, \hat{\Theta}_n + 0.533\hat{S}_n].$$

d. The first method yields a narrower confidence interval and is therefore more optimistic. As n increases the difference between the confidence intervals decreases.

2. (See solutions on pg. 515-516)