Massachusetts Institute of Technology
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2008)

Recitation 24
December 9, 2008

1. Let's consider binary hypothesis testing. Under the null hypothesis (H_0), random variable X is uniformly distributed over the interval $[0, 1]$. Under the alternative hypothesis (H_1), the PDF of X is given by

$$f_X(x; H_1) = \begin{cases} 2x & \text{if } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

We are interested in figuring out whether X is uniformly distributed or if the PDF of X is a linear ramp as described above.

   (a) Construct the likelihood ratio test (LRT) for a critical value ξ.

   (b) Find the probability of false rejection , $\alpha(\xi)$, of H_0. (This is called P_{FA} in Monday's lecture.)

   (c) Find the probability of false acceptance, $\beta(\xi)$, of H_0. (This is called P_M in Monday's lecture.)

   (d) Plot the two errors. Let $\alpha(\xi)$ be on the X axis and $\beta(\xi)$ on the Y axis. Mark the points on the plot that correspond to $\xi = 0, 2/3, 1, 4/3, 2$.

   (e) Suppose we want to limit the probability of false rejection to 1/4. Find the decision rule that minimizes the probability of false acceptance. Calculate the probability of false acceptance achieved by this rule.

   (f) Suppose we want to limit the probability of false acceptance to 1/4. Find the decision rule that minimizes the probability of false rejection. Calculate the probability of false rejection achieved by this rule.

   (g) Bob is a true Bayesian. Instead of using LRT, he puts a prior on each hypothesis and performs Bayesian hypothesis test to determine which distribution generated X. He lets p be the probability that the alternative hypothesis (H_1) is true. Help Bob construct the MAP decision rule. Compare to the LRT rule in (a).

   (h) Suppose x=0.9. What is the p-**value** of the claim that H_1 is true based on this measurement. (See page 499.)

2. (Problem 23, see page 519.)

The number of phone calls received by a ticket agency on any one day is Poisson distributed. On an ordinary day, the expected value of the number of calls is λ_0, and on a day where there is a popular show in town, the expected value of the number of calls is λ_1, with $\lambda_1 > \lambda_0$. Describe the LRT for deciding whether there is a popular show in town based on the number number of calls received.

   (a) Assume a given probability of false rejection, and find an expression for the critical value ξ.

   (b) Given $\lambda_0 = 7$, $\lambda_1 = 9$ and the probability of false alarm (i.e, false rejection), $P_{FA} = 0.25$, calculate the lowest probability of false acceptance achieved by the LRT rule.