

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Electrical Engineering & Computer Science
6.041/6.431: Probabilistic Systems Analysis
(Fall 2008)

Tutorial 1
September 11/12, 2008

1. Imagine a drunk tightrope walker, who manages to keep his balance, but takes a step forward with probability p and takes a step back with probability $(1 - p)$.
 - (a) What is the probability that after two steps the tightrope walker will be at the same place on the rope?
 - (b) What is the probability that after three steps, the tightrope walker will be one step forward from where he began?
 - (c) Given that after three steps he has managed to move ahead one step, what is the probability that the first step he took was a step forward?
 2. **Knives and Forks:** In the kitchen in your apartment, you put all your 10 forks in the left drawer and all 10 knives in the right drawer. Your roommate, who does not agree with your organizational approach, comes in, takes two forks from the left drawer and tosses them into the right drawer. She then takes at random an item (knife or fork) from the right drawer and tosses it in the left drawer.

After this exchange, you come in and randomly pick up an item from a randomly chosen drawer. Given you have picked up a knife, what is the probability that you have opened the left drawer?
 3. Three people each roll a fair n -sided die once. Let A_{ij} be the event that person i and person j roll the same face. Show that the events A_{12} , A_{13} , and A_{23} are pairwise independent but are not independent.
 4. **(Galton's Paradox)** You flip three fair coins.

Your friend argues that at least two of the outcomes are alike, and there is an even chance that the third is a head or a tail. Therefore $P(\text{"all alike"}) = 1/2$. Do you agree?
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