**Tutorial 2**
**September 18/19, 2008**

1. Problem 1.58, page 69 in the text.

   We draw the top 7 cards from a well-shuffled standard 52-card deck. Find the probability that

   (a) The cards include exactly 3 aces.

   (b) The cards include exactly 2 kings.

   (c) (**Optional**) The probability that the 7 cards include exactly 3 aces, or exactly 2 kings, or both.

2. Consider the random variable $X$ with PMF

$$p_X(x) = \begin{cases} \frac{x^2}{a} & \text{if } x = -3, -2, -1, 0, 1, 2, 3, \\ 0 & \text{otherwise.} \end{cases}$$

   (a) Find $a$ and $\mathbf{E}[X]$.

   (b) What is the PMF of the random variable $Z = (X - \mathbf{E}[X])^2$ ?

   (c) Using part (b), compute the variance of $X$.

   (d) Compute the variance of $X$ using the formula $\sum_x (x - \mathbf{E}[X])^2 p_X(x)$.

3. **Lottery.** Suppose you choose any $r$ of the first $n$ positive integers, and a lottery similarly chooses a random subset $L$ of the same size. What is the probability that:

   (a) the numbers in $L$ are drawn in increasing order?

   (b) the numbers in $L$ are the same as those chosen by you?

   (c) exactly $k$ of the numbers in $L$ match the numbers chosen by you?

   (d) (**Optional**) $L$ includes no consecutive integers?

   (e) (**Optional**) $L$ includes exactly one pair of consecutive integers?