**Tutorial 3** [1]
**September 25/26, 2008**

1. Consider an experiment in which a fair four-sided die (with faces labeled 0, 1, 2, 3) is thrown once to determine how many times a fair coin is to be flipped. In the sample space of this experiment, random variables $N$ and $K$ are defined by

   - $N$ = the result of the die roll
   - $K$ = the total number of heads resulting from the coin flips

   Determine and sketch each of the following functions for all values of their arguments:

   (a) $p_N(n)$
   (b) $p_{K|N}(k \mid 2)$
   (c) $p_{N|K}(n \mid 2)$
   (d) $p_K(k)$
   (e) Determine the conditional PMF for random variable $N$, given that the experimental value of $K$ is an odd number.
   (f) Compute the expected value and variance for the distribution you computed in part (b).

2. The newest invention of the 6.041/6.431 staff is a three-sided die. On any roll of this die, the outcome $x$ is:

$$p_X(x) = \begin{cases} \frac{1}{2}, & \text{if } x = 1 \\ \frac{1}{4}, & \text{if } x = 2 \\ \frac{1}{4}, & \text{if } x = 3 \\ 0 & \text{otherwise.} \end{cases}$$

   Consider a sequence of six independent rolls of this die, and let $x_i$ be the random variable corresponding to the $i$th roll.

   (a) What is the probability that exactly three of the rolls have an outcome equal to 3?
   (b) What is the probability that the first roll is 1, given that exactly two of the six rolls had an outcome of 1?
   (c) We are told that exactly three of the rolls resulted in 1 and exactly three resulted in 2. Given this information, what is the probability that the sequence 121212 resulted?
   (d) Conditioned on the event that at least one roll resulted in 3, find the conditional PMF of the number of 3's.

3. A company is interviewing potential employees. Suppose that each potential employee is either qualified or unqualified. The company tries to determine this by asking 20 true or false questions. A candidate gets a $C$ for each correct answer, and an $I$ for each incorrect answer. A qualified candidate has probability $p$ of answering a question correctly, while an unqualified candidate has a probability $p$ of answering incorrectly, with questions independent of each other. If the company considers anyone with at least 15 $C$'s qualified, and everyone else unqualified, what is the probability that the 20 questions will correctly categorize the candidate? Is there enough information to answer this question?

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