

Tutorial 4
October 2/3, 2008

1. Consider the continuous random variable X with the following PDF:

$$f_X(x) = \begin{cases} 1/8 & \text{if } 1 \leq x < 2 \text{ or } 3 \leq x < 4 \\ 3/4 & \text{if } 2 \leq x < 3 \\ 0 & \text{otherwise.} \end{cases}$$

Let X_i denote the experimental value of random variable X on trial i . All X_i s are independent of each other. Find the expected number of trials before a value greater than $7/2$ is observed. Your answer should be a number.

2. An ATM is replenished with cash every day at 12 p.m. The amount withdrawn (in hundreds of thousands of dollars) in the 24 hours between refills behaves like a random variable with probability density function:

$$f_Z(z) = \begin{cases} 5(1-z)^4 & 0 < z < 1 \\ 0 & \text{otherwise} \end{cases}$$

- (a) Find the expectation and the variance of the amount of money withdrawn.
(b) How much money should the bank deposit in the ATM so it can be 99% confident that the supply will last the 24 hours?
3. Let X be a normal random variable $X \sim N(\mu, \sigma^2)$.

- (a) Compute the probabilities

$$\mathbf{P}(X \leq \mu + \sigma)$$

$$\mathbf{P}(X \leq \mu - \sigma)$$

$$\mathbf{P}(X \leq \mu + 2\sigma)$$

- (b) Compute the probabilities

$$\mathbf{P}(\mu - \sigma \leq X \leq \mu + \sigma)$$

$$\mathbf{P}(\mu - 2\sigma \leq X \leq \mu + 2\sigma)$$