

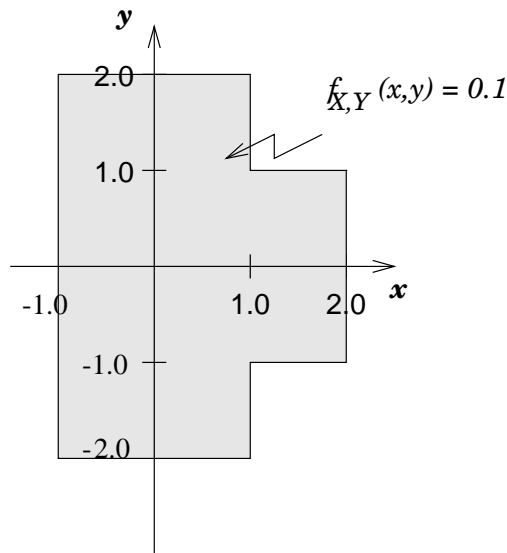
Tutorial 5
October 9/10, 2008

1. One of two wheels of fortune, A and B , is selected by the flip of a fair coin, and the wheel chosen is spun once to determine the numerical value of random variable Z . Random variable X , the reading obtained with wheel A , and random variable Y , the reading obtained with wheel B , are described by the PDFs

$$f_X(x) = \begin{cases} 1, & 0 < x \leq 1; \\ 0, & \text{otherwise,} \end{cases} \quad \text{and} \quad f_Y(y) = \begin{cases} 3, & 0 < y \leq \frac{1}{3}; \\ 0, & \text{otherwise.} \end{cases}$$

- (a) If we are told the numerical value of Z was *less than* $\frac{1}{4}$, what is the conditional probability that wheel A was the one selected?
- (b) If we are told the numerical value of Z was *exactly* $\frac{1}{4}$, what is the conditional probability that wheel A was the one selected?
2. Let X have a uniform distribution in the unit interval $[0, 1]$, and let Y have an exponential distribution with parameter $\lambda = 2$. Assume that X and Y are independent. Let $Z = X + Y$.
- (a) Find $\mathbf{P}(Y \geq X)$.
- (b) Find the conditional PDF of Z given that $Y = y$.

3. Random variables X and Y have the joint PDF shown below:



- (a) Prepare neat, fully labeled sketches of $f_X(x)$, $f_Y(y)$, $f_{Y|X}(y | x)$ and $f_{X|Y}(x | y)$.
- (b) Are X and Y independent?
- (c) Find $f_{X,Y|A}(x, y)$, where the event A corresponds to points (x, y) within the unit circle centered at the origin.
- (d) Find $\mathbf{E}[X | Y = y]$ and $\text{var}(X | Y = y)$.