Tutorial 6 [1]
October 16/17, 2008

1. Let continuous random variables X, Y and Z be independent and identically distributed according to the uniform distribution in the unit interval $[0, 1]$.

   (a) Consider two new random variables defined by $V = XY$ and $W = Z^2$. Derive the joint PDF $f_{V,W}(v, w)$.

   (b) Show that $\mathbf{P}(XY < Z^2) = \frac{5}{9}$.

2. Alice and Bob flip bias coins independently. Alice's coin comes up heads with probability 1/4, while Bob's coin comes up head with probability 3/4. Each stop as soon as they get a head; that is, Alice stops when she gets a head while Bob stops when he gets a head. What is the PMF of the total amount of flips until both stop? (That is, what is the PMF of the combined total amount of flips for both Alice and Bob until they stop?)

3. Consider four random variables, W, X, Y and Z, with

$$\mathbf{E}[W] = \mathbf{E}[X] = \mathbf{E}[Y] = \mathbf{E}[Z] = 0,$$
$$\text{var}(W) = \text{var}(X) = \text{var}(Y) = \text{var}(Z) = 1,$$

   and assume that W, X, Y and Z are pairwise uncorrelated. Find the correlation coefficients $\rho(R, S)$ and $\rho(R, T)$, where $R = W + X$, $S = X + Y$, and $T = Y + Z$.

4. Suppose that X and Y are random variables with the same variance. Show that $X - Y$ and $X + Y$ are uncorrelated. What can be said about dependence or independence of $X + Y$ and $X - Y$?

[1] Compiled October 14, 2008