

**Tutorial 7**<sup>1</sup>  
**October 23/24, 2008**

1. Using a fair three-sided die (construct one, if you dare), we will decide how many times to spin a fair wheel of fortune. The wheel of fortune is calibrated infinitely finely and has numbers between 0 and 1. The die has the numbers 1, 2 and 3 on its faces. Whichever number results from our throw of the die, we will spin the wheel of fortune that many times and add the results to obtain random variable  $Y$ .
  - (a) Determine the expected value of  $Y$ .
  - (b) Determine the variance of  $Y$ .
2. Joe wishes to estimate the true fraction  $f$  of smokers in a large population without asking each and every person. He plans to select  $n$  people at random and then employ the estimator  $F = S/n$ , where  $S$  denotes the number of people in a size- $n$  sample who are smokers. Joe would like to sample the minimum number of people, but also guarantee an upper bound  $p$  on the probability that the estimator  $F$  differs from the true value  $f$  by a value greater than or equal to  $d$  i.e., for a given accuracy  $d$  and given confidence  $p$ , Joe wishes to select the minimum  $n$  such that

$$\mathbf{P}(|F - f| \geq d) \leq p \quad .$$

For  $p = 0.05$  and a particular value of  $d$ , Joe uses the Chebyshev inequality to conclude that  $n$  must be at least 50,000. Determine the new minimum value for  $n$  if:

- (a) the value of  $d$  is reduced to half of its original value.
  - (b) the probability  $p$  is reduced to half of its original value, or  $p = 0.025$ .
3. Let  $X_1, X_2, \dots$  be independent random variables, uniformly distributed in  $[0, 1]$ . Let

$$Z_n = \min\{X_1, X_2, \dots, X_n\}.$$

Determine whether or not the sequence  $\{Z_n\}$  is convergent in probability to a real value  $c$  (if it is, what is  $c$ ?).

---

<sup>1</sup>Published October 20, 2008