

Tutorial 8
October 30/31, 2008

1. Let X be a random variable with $\mathbf{E}[X] = 0$ and $\text{var}(X) = 1$. The Chebyshev inequality tells us, for example, that

$$\mathbf{P}(|X| \geq 2) \leq 0.25.$$

Is there a random variable Y with $\mathbf{E}[Y] = 0$ and $\text{var}(Y) = 1$ such that

$$\mathbf{P}(|Y| \geq 2) = 0.25,$$

or in other words, achieves the bound? If so, find such a Y . If not, explain why not.

2. We are laying out 25 plastic planks lengthwise, trying to make a path of about 1000 meters. The plastic planks are made in molds, and any variation in the lengths of the planks is due entirely to variation between different molds. The length in meters, X , of any particular mold used for making planks is independent of the length of all other molds. X is uniformly distributed between $40 - \sqrt{3}$ and $40 + \sqrt{3}$ meters. X has an expected value of 40 meters and a standard deviation of 1 meter. What is the probability that the resulting path will be within 1000 ± 7.5 meters if we use 25 planks ...
- (a) ... all made from the same mold?
(b) ... each made from a different mold?

Explain the difference between the answers to parts (a) and (b).

3. Problem 6.3, page 326 in the text.

A computer system carries out tasks submitted by two users. Time is divided into slots. A slot can be idle, with probability $p_I = 1/6$, and busy with probability $p_B = 5/6$. During a busy slot, there is probability $p_{1|B} = 2/5$ (respectively, $p_{2|B} = 3/5$) that a task from user 1 (respectively, 2) is executed. We assume that events related to different slots are independent.

- (a) Find the probability that a task from user 1 is executed for the first time during the 4th slot.
(b) Given that exactly 5 out of the first 10 slots were idle, find the probability that the 6th idle slot is 12.
(c) Find the expected number of slots up to and including the 5th task from user 1.
(d) Find the expected number of busy slots up to and including the 5th task from user 1.
(e) Find the PMF, mean, and variance of the number of tasks from user 2 until the time of the 5th task from user 1.