Tutorial 9
November 6/7, 2008

1. A busy professor likes taking each Saturday to sail to his cottage on a nearby island off the coast. The professor is an avid fisherman, and enjoys fishing off of his boat on the way to and from the island, as long as the weather is good. Unfortunately, the weather is good on the way to or from the island with probability p, independently of what the weather was on any past trip (so the weather could be nice on the way to the island, but poor on the way back). Now, if the weather is nice, the professor will take one of his n fishing rods for the trip, but if the weather is bad, he will not bring a fishing rod with him. We want to find the probability that on a given leg of the trip to or from the island the weather will be nice, but the professor will not fish because all his fishing rods are at his other home.

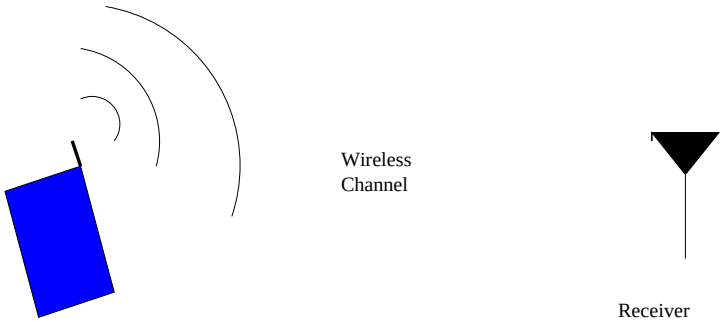
   (a) Formulate a minimum state Markov chain which allows you to find the required probability. Explain why your formulation is a legitimate Markov chain.

   (b) Find the steady state probabilities in the above chain.

   (c) What is the probability that on a given trip, the professor sails with clement weather but without a fishing rod?

   (d) If the professor owns 4 fishing rods, find p such that the time he wants to, but cannot fish is maximized.

2. A PDA/phone transmits one packet per time slot over a wireless connection. Each packet is not received correctly with probability p. Whenever the receiver discovers it has received an error five consecutive times, it sends a 6.041-guaranteed-error-free packet back to the transmitter notifying it of the problem (through the magic of 6.041/6.431 hand-wavy-unrealistic-question-physics). When the transmitter receives this packet, the transmitter enters a "timeout" state in order to avoid wasting power when the link quality is poor. During such a timeout, the mobile terminal performs an independent Bernoulli trial with success probability q in every slot. When a success occurs, the mobile terminal starts transmitting in the next slot as though no packets had been in error.



   (a) Construct a discrete-time Markov chain for this system, which includes
      i. defining an appropriate state space and
      ii. drawing the transition probability graph.

   (b) Solve for the steady-state probabilities in terms of parameters p and q.