6.055/2.038 desert-island diagnostic

Name:

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These questions are a diagnostic to improve how I teach this course and to give you an overview of what you will learn this semester.

The results will show me where we are at the beginning of the course: What areas and concepts need more explanation throughout the term? What can be emphasized less? What topics are most confusing to you as a class?

I do not expect you to solve many of the questions now! Rather, my goal is that by the end of the course you can solve every question. Most or all of the questions will appear on the problem sets, so (that) you can compare your understanding then with your understanding now.

Spend up to 3 hours on the questions, and do them all in one sitting. Do not spend too much time on any one question, and answer all questions as best you can.

Answer all questions. Educated guessing is strongly encouraged! See the particular instructions at the start of each section for how to describe any uncertainty about the answer.

For each question, jot down a one- or two-sentence explanation of how you chose your answer.

Due Wednesday at 10am most likely online. I'll post the details on the course website and the announcement list.

Please no collaboration, calculators, computers, smart phones, or other outside sources of information. Instead, imagine that you are alone on a desert island – except for the table of useful constants provided on the next page.

The correctness of your answers has no effect on your course grade. Do the best you can with your knowledge now, and enjoy the problems!

—*Sanjoy*

Useful numbers for the backs of envelopes

π	pi	3	
G	Newton's constant	$7 \cdot 10^{-11}$	$\mathrm{kg^{-1}\,m^3\,s^{-1}}$
c	speed of light	$3 \cdot 10^{8}$	$\mathrm{m\,s^{-1}}$
k_{B}	Boltzmann's constant	10^{-4}	$\mathrm{eV\,K^{-1}}$
e	electron charge	$1.6 \cdot 10^{-19}$	C
σ	Stefan–Boltzmann constant	$6 \cdot 10^{-8}$	$\mathrm{W\,m^{-2}\,K^{-4}}$
m_{sun}	Solar mass	$2 \cdot 10^{30}$	kg
R_{earth}	Earth radius	$6 \cdot 10^{6}$	m
$\theta_{\mathrm{moon/sun}}$	angular diameter	10^{-2}	
ρ_{air}	air density	1	$\mathrm{kg\,m^{-3}}$
ρ_{rock}	rock density	5	$\mathrm{g\,cm^{-3}}$
$\hbar c$		200	eV nm
$L_{\mathrm{vap}}^{\mathrm{water}}$	heat of vaporization	2	$\mathrm{MJ\,kg^{-1}}$
γ_{water}	surface tension of water	10^{-1}	$\mathrm{N\,m^{-1}}$
a_0	Bohr radius	0.5	Å
a	typical interatomic spacing	3	Å
N_{A}	Avogadro's number	$6 \cdot 10^{23}$	
$\mathcal{E}_{\mathrm{fat}}$	combustion energy density	9	$\mathrm{kcal\,g^{-1}}$
E_{bond}	typical bond energy	4	eV
$\dfrac{e^2/4\pi\epsilon_0}{\hbar c}$	fine-structure constant α	10^{-2}	
p_0	air pressure	10^{5}	Pa
ν_{air}	kinematic viscosity of air	$1.5 \cdot 10^{-5}$	$\mathrm{m^2\,s^{-1}}$
ν_{water}	… of water	10^{-6}	$\mathrm{m^2\,s^{-1}}$
day		10^{5}	s
year		$\pi \cdot 10^{7}$	s
F	solar constant	1.3	$\mathrm{kW\,m^{-2}}$
L_{sun}	solar luminosity (radiated power)	$4 \cdot 10^{26}$	W
AU	distance to sun	$1.5 \cdot 10^{11}$	m
P_{basal}	human basal metabolic rate	100	W
K_{air}	thermal conductivity of air	$2 \cdot 10^{-2}$	$\mathrm{W\,m^{-1}\,K^{-1}}$
K	… of non-metallic solids/liquids	1	$\mathrm{W\,m^{-1}\,K^{-1}}$
K_{metal}	… of metals	10^{2}	$\mathrm{W\,m^{-1}\,K^{-1}}$
$c_{\mathrm{p}}^{\mathrm{air}}$	specific heat of air	1	$\mathrm{J\,g^{-1}\,K^{-1}}$
c_{p}	… of solids/liquids	25	$\mathrm{J\,mole^{-1}\,K^{-1}}$

Multiple-choice questions

For each of the following multiple-choice questions, you have 10 tokens to distribute among all the answers. Divide the tokens among the choices to represent your relative confidence in each answer. If you are sure of your answer, place all 10 tokens on that one answer. If you're equally split between two answers, give 5 to each; etc. Ensure that the tokens add up to 10 for each question (leave the box blank for a choice to which you give 0 tokens).

Here is an example question and answer showing the tokens in action.

Example problem. Turbulence research
What is the future of research on turbulence?

[3] Within 20 years, a large breakthrough will happen.

[7] It will forever remain beyond human comprehension.

[ ] There is nothing more we need to learn.

Problem 1 Xylophones
If a 10-cm long slat of a xylophone rings with middle C ($f \sim 260\,\mathrm{Hz}$), how long is the slat that rings with low C ($f \sim 130\,\mathrm{Hz}$)? (The slats of a xylophone all have the same thickness.)

[ ] 5 cm

[ ] 7 cm

[ ] 14 cm

[ ] 20 cm

Problem 2 Boiling water up high
Roughly what is the boiling point of water on top of Mount Everest (altitude 10 km)?

[ ] 70 °C

[ ] 85 °C

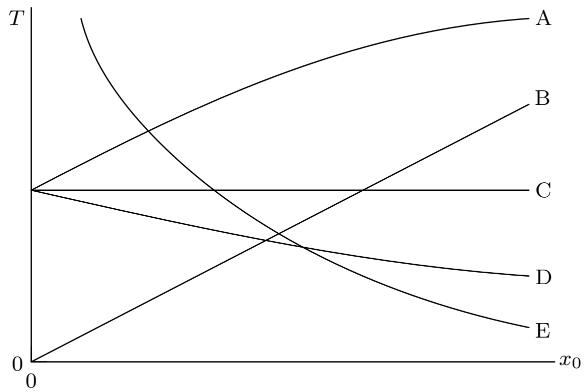
[ ] 100 °C

[ ] 115 °C

[ ] 130 °C

Problem 3 Non-Hooke's law spring

Imagine a mass connected to a spring with force law $F = Cx^3$ (instead of the usual Hooke's law behavior $F = kx$) and therefore potential energy $V \sim Cx^4$ (where C is a constant). Which curve shows how the system's oscillation period T depends on the amplitude x_0?



☐ Curve A

☐ Curve B

☐ Curve C

☐ Curve D

☐ Curve E

Problem 4 Rate of growth

Define the sequence f_n by the recurrence

$$f_n = f_{n-1} + f_{n-2},$$

where $f_0 = f_1 = 2$.

Which value is closest to f_{100}?

☐ 1.2^{100}

☐ 1.4^{100}

☐ 1.6^{100}

☐ 1.8^{100}

☐ 2.0^{100}

Problem 5 Power radiated by an accelerating charge

If the velocity and acceleration of a (nonrelativistic) electric charge are doubled, how does the power radiated by the charge change?

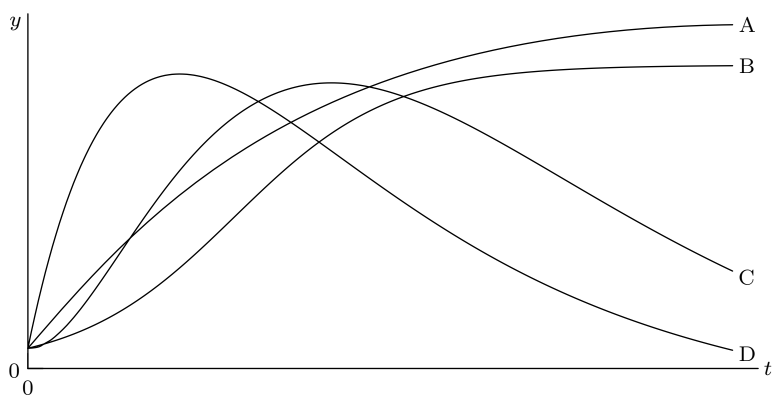
- [ ] The power increases by a factor of 16.
- [ ] The power increases by a factor of 8.
- [ ] The power increases by a factor of 4.
- [ ] The power increases by a factor of 2.
- [ ] The power increases by a factor of $\sqrt{2}$.

Problem 6 Differential-equation solution

Which sketch shows a solution of the differential equation

$$\frac{dy}{dt} = Ay(M - y),$$

where A and M are positive constants?



- [ ] Curve A
- [ ] Curve B
- [ ] Curve C
- [ ] Curve D

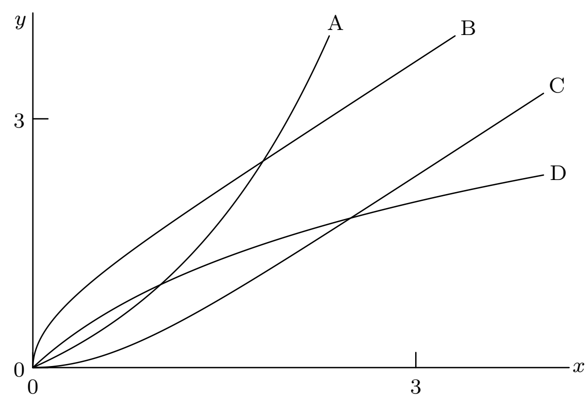
Problem 7 Speed of sound

The speed of sound in oxygen (molecular mass 32) at standard temperature and pressure is $316\,\mathrm{m\,s^{-1}}$. What is the speed of sound for hydrogen (molecular mass 2)?

- [ ] $78\,\mathrm{m\,s^{-1}}$
- [ ] $157\,\mathrm{m\,s^{-1}}$
- [ ] $316\,\mathrm{m\,s^{-1}}$
- [ ] $637\,\mathrm{m\,s^{-1}}$
- [ ] $1284\,\mathrm{m\,s^{-1}}$

Problem 8 Sketch

Which graph is $\ln\cosh x$ (where $\cosh x \equiv (e^x + e^{-x})/2$)?



- [ ] Curve A
- [ ] Curve B
- [ ] Curve C
- [ ] Curve D

Problem 9 Blue skies

Which reason is part of the explanation for why the sky looks blue?

- [ ] The power radiated by an oscillating charge decreases strongly with frequency.
- [ ] The power radiated by an oscillating charge increases strongly with frequency.
- [ ] Our eyes are more sensitive to blue than to red light.
- [ ] The sun radiates more energy in the short-wavelength (blue) end of the visible spectrum than in the long-wavelength (red) end of the spectrum.
- [ ] The sun radiates more energy in the long-wavelength (red) end of the visible spectrum than in the short-wavelength (blue) end of the spectrum.

Problem 10 Golf-ball dimples

Why do golf balls have dimples?

- [ ] The dimples make the main airflow around the ball become turbulent.
- [ ] The dimples stabilize the flight.
- [ ] The dimples are there by tradition but have no physical justification.
- [ ] The dimples make the airflow turbulent in the thin boundary layer adjacent to the ball.

Numerical-range answers

For each of the following numerical-answer questions, give your best estimate of the exponent (rounding to within 0.5) and of your uncertainty in the exponent (rounding to within 0.5). The more uncertain you are of the answer, the larger the uncertainty in the exponent.

Choose your exponent and uncertainty so that the numerical range is such that you'd be somewhat surprised – but not too surprised – if the true value lay outside the range. To make that standard quantitative, choose your range so that you expect to have a 2/3 probability of being correct.

As an example, if I had to estimate the mass of the earth but could not find a simple method and just had to guess, my uncertainty would large. My answer might be $10^{21\pm5}$ kg. In contrast, after making a divide-and-conquer estimate for the pit spacing on a CD, my uncertainty would be smaller. My answer might be $10^{-6\pm0.5}$ m, to indicate that $0.3\,\mu\text{m} \ldots 3\,\mu\text{m}$ is my plausible range for the spacing. (An exponent of 0.5, i.e. $10^{0.5}$, is roughly a factor of 3.)

Here is an example problem to show how to enter an exponent and uncertainty.

Example problem. *CD pit spacing*

How far apart are the data pits on a CD or CDROM?

$$10^{\boxed{-6.0}\pm\boxed{0.5}}\ \text{m}$$

Problem 11 Flea

What is roughly the mass of a 1-mm-long flea?

$$10^{\boxed{}\pm\boxed{}}\ \text{kg}$$

Problem 12 Pool temperature

A large outdoor swimming pool in the Arizona desert has a time constant of 4 days for exchanging heat with the air. Roughly how large are the peak-to-peak fluctuations of the water temperature caused by 30 °F (peak-to-peak) night–day fluctuations in the air temperature?

$$10^{\boxed{}\pm\boxed{}}\ {}^\circ\text{F}$$

Problem 13 Alpha Centauri A

On a clear Southern-hemisphere night, roughly how many visible-light photons per second make it into your eye from Alpha Centauri A (a sun-like star roughly 4 light-years away)?

$$10^{\boxed{}\pm\boxed{}}\ \text{photons/second}$$

Problem 14 Babies

Roughly how many babies are born in the United States every year?

$10^{\square} \pm \square$ babies/year

Problem 15 Boiling away mercury

The surface tension of mercury (a liquid at room temperature) is roughly $0.5\,\mathrm{N\,m^{-1}}$. Roughly how much energy is required to boil away $1\,\mathrm{m^3}$ of mercury?

$10^{\square} \pm \square$ J

Problem 16 Waves on a swimming pool

Roughly what is the minimum speed of waves on the surface of a swimming pool?

$10^{\square} \pm \square$ $\mathrm{m\,s^{-1}}$

Problem 17 Irradiation

In an experiment I learnt about at a Caltech biology seminar in the 1990s, researchers repeatedly irradiated a population of bacteria in order to generate mutations. In each round of radiation, 5% of the bacteria got mutated. After 140 rounds, roughly what fraction of bacteria remained unmutated?

$10^{\square} \pm \square$

(For fun the seminar speaker gave the audience 5 s to make a guess, hardly enough time to even find a calculator.)

Problem 18 Cold day

You stand outside on a calm (i.e. not very windy) but cold winter day wearing only a thin T-shirt and equally thin pants. Roughly at what rate does your body lose heat?

$10^{\square} \pm \square$ W

Problem 19 Candy bar

How many flights of stairs can you climb thanks to the energy from eating a candy bar?

$10^{\square} \pm \square$ flights

Problem 20 Local black hole

What is roughly the largest radius the earth could have, with its current mass, and be a black hole (i.e. light cannot escape from its surface)?

$10^{\square \pm \square}$ m

Problem 21 9V battery

Roughly how much energy is stored in a typical (disposable) 9V battery?

$10^{\square \pm \square}$ J

Problem 22 747

What is roughly the fuel efficiency of a (mostly full) 747 jumbo jet?

$10^{\square \pm \square}$ passenger–miles per gallon

Problem 23 Raindrop speed

What is the terminal velocity of a typical raindrop?

$10^{\square \pm \square}$ $\mathrm{m\,s^{-1}}$

Problem 24 Wire

Roughly what is the number density of free (conduction) electrons in a copper wire?

$10^{\square \pm \square}$ $\mathrm{m^{-3}}$

Problem 25 Drag on a cyclist

Roughly what is the force due to air drag on a bicyclist traveling at 25 mph?

$10^{\square \pm \square}$ N

Problem 26 Should you be worried?

Assume that 1 in 10^4 bridges in the United States are in danger of collapse unless repaired soon, and that a new test for bridge integrity has been devised. This test is 90% accurate: It always detects an unsafe bridge (no false negatives); and 10% of the time it says that a safe bridge is unsafe (10% false positives).

You learn that a nearby bridge, on which you often walk, failed the test (the test said it was unsafe). What are the odds that the bridge is safe?

$$10^{\boxed{} \pm \boxed{}}$$

Note: If p_{safe} is the probability that the bridge is safe, then the corresponding odds are defined by

$$\text{odds} \equiv \frac{p_{\text{safe}}}{1 - p_{\text{safe}}}.$$

(Odds, unlike probabilities, range from 0 to ∞ and are thus more suitable for describing in the form $10^{a \pm b}$.)

And one final question to help me calibrate myself.

Problem 27 Length of the diagnostic

How long did you spend taking this desert-island diagnostic? (If possible, give a time accurate to within 0.5 hours.)

$\boxed{}$ hours