6.055J/2.038J (Spring 2010)

Homework 4

Submit your answers and explanations online by **10pm on Wednesday, 10 Mar 2010**.

**Open universe:** Collaboration, notes, and other sources of information are encouraged. However, avoid looking up answers to the problem, or to subproblems, until you solve the problem or have tried hard. This policy helps you learn the most from the problems.

Homework is graded with a light touch: P (made a decent effort), D (made an indecent effort), or F (did not make an effort).

**Problem 1  Bandwidth**
To keep your divide-and-conquer muscles strong, here is an exercise from lecture: Estimate the bandwidth of a 747 crossing the Atlantic filled with CDROMs.

\[
10 \pm \quad \text{bits/s} \quad \text{or} \quad 10 \quad \ldots \quad \text{bits/s}
\]

**Problem 2  Gravity versus radius**
Assume that planets are uniform spheres. How does \( g \), the gravitational acceleration at the surface, depend on the planet’s radius \( R \)? In other words, what is the exponent \( n \) in

\[
g \propto R^n?
\]

\[
\pm \quad \text{or} \quad \ldots
\]

**Problem 3  Gravity on the moon**
The radius of the moon is one-fourth the radius of the earth. Use the result of Problem 2 to predict the ratio \( g_{\text{moon}}/g_{\text{earth}} \). In reality, \( g_{\text{moon}}/g_{\text{earth}} \) is roughly one-sixth. How might you explain any discrepancy between the predicted and actual ratio?

**Problem 4  Minimum power**
In the readings we estimated the flight speed that minimizes energy consumption. Call that speed \( v_E \). We could also have estimated \( v_P \), the speed that minimizes power consumption. What is the ratio \( v_P/v_E \)?

\[
\pm \quad \text{or} \quad \ldots
\]
Problem 5  Highway vs city driving
Here is a measure of the importance of drag for a car moving at speed $v$ for a distance $d$:

$$\frac{E_{\text{drag}}}{E_{\text{kinetic}}} \sim \frac{\rho v^2 A d}{m_{\text{car}} v^2}.$$  

This ratio is equivalent to the ratio

$$\frac{\text{mass of the air displaced}}{\text{mass of the car}}$$

and to the ratio

$$\frac{\rho_{\text{air}}}{\rho_{\text{car}}} \times \frac{d}{l_{\text{car}}},$$

where $\rho_{\text{car}}$ is the density of the car (its mass divided by its volume) and $l_{\text{car}}$ is the length of the car.

Make estimates for a typical car and find the distance $d$ at which the ratio becomes significant (say, roughly 1).

$$10 \pm \boxed{} \text{ m or } 10 \boxed{} \cdots \boxed{} \text{ m}$$

To include in the explanation box: How does the distance compare with the distance between exits on the highway and between stop signs or stoplights on city streets? What therefore are the main mechanisms of energy loss in city and in highway driving?

Problem 6  Mountains
Here are the heights of the tallest mountains on Mars and Earth.

<table>
<thead>
<tr>
<th></th>
<th>Mars</th>
<th>27 km (Mount Olympus)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earth</td>
<td>9 km</td>
<td>(Mount Everest)</td>
</tr>
</tbody>
</table>

Predict the height of the tallest mountain on Venus.

$$10 \pm \boxed{} \text{ km or } 10 \boxed{} \cdots \boxed{} \text{ km}$$

To include in the explanation box: Then check your prediction in a table of astronomical data (or online).

Problem 7  Raindrop speed
Use the drag-force results from the readings to estimate the terminal speed of a typical raindrop (diameter of about 0.5 cm).

$$10 \pm \boxed{} \text{ m s}^{-1} \text{ or } 10 \boxed{} \cdots \boxed{} \text{ m s}^{-1}$$

To include in the explanation box: How could you check this result?
Problem 8 Cruising speed versus air density
For geometrically similar animals (same shape and composition but different size), how does the minimum-energy speed \( v \) depend on air density \( \rho \)? In other words, what is the exponent \( \beta \) in \( v \propto \rho^\beta \)?

\[ \pm \boxed{\text{or}} \boxed{\ldots} \]

Problem 9 Cruising speed versus mass
For geometrically similar animals (same shape and composition but different size), how does the minimum-energy speed \( v \) depend on mass \( M \)? In other words, what is the exponent \( \beta \) in \( v \propto M^\beta \)?

\[ \pm \boxed{\text{or}} \boxed{\ldots} \]

Problem 10 Speed of a bar-tailed godwit
Use the results of Problem 8 and Problem 9 to write the ratio \( v_{747}/v_{\text{godwit}} \) as a product of dimensionless factors, where \( v_{747} \) is the minimum-energy speed of a 747, and \( v_{\text{godwit}} \) is the minimum-energy speed of a bar-tailed godwit (i.e. its cruising speed). By estimating the dimensionless factors and their product, estimate the cruising speed of a bar-tailed godwit. [Useful information: \( m_{\text{godwit}} \sim 0.4 \, \text{kg}; v_{747} \sim 600 \, \text{mph}.]

\[ 10 \boxed{\pm} \boxed{\ldots} \text{m s}^{-1} \boxed{\text{or}} \boxed{\ldots} \text{m s}^{-1} \]

To include in the explanation box: Compare your result with the speed of the record-setting bar-tailed godwit, which made its 11,570 km journey in 8.5 days.