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How does this relate to divide and conquer other than checking solutions we found that way?

I'm not sure either. I think the point of this article is to show that checking your answer with different methods is best though. Also, these examples were confusing to me, so I might be missing something.

It relates to divide and conquer because the redundant solutions use divide and conquer, but also to address an inherent flaw in the divide and conquer method. When we break down a problem in a particular way, we may inherently bias our answer (we pretended the CD was a square). Without checking using a different route, our confidence in our estimate is limited.

I am amazed by how you can come up with such methods to calculate the spacing? How do you know that you can utilize the diffraction principles etc, I would have never thought of that? Will I be able to relate problems as such to the different principles I have learned from different classes by the end of the semester?

I agree, I think the first and hardest step is actually realizing that one may actually have knowledge that is applicable to help solve an approximation problem. I would never have thought to think about the CD player to predict spacing.

Is there some sort of method that you can default to if you do not know an equation? Just multiply them maybe?

I don't think there will ever be a default to fall back on, but as others said I'm realizing that I do have knowledge to help me estimate. However I feel like to apply knowledge such as wave diffraction one would need to conduct a small test. Easy for a CD, hard for a 747.

This experiment was done in 8.02 and something similar was done in 5.112 – makes it easier for me to visualize.

Why does the laser wavelength need to be smaller than the pit spacing?

The wavelength is tuned to the distance of where the laser is emitted and the bottom of the pit on the CD so that when the light hits a pit, destructive interference occurs. If the wavelength was larger than the pit, it would be impossible for the reflected light to be knocked out of phase by half a wavelength.

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Does "divide and conquer" as used in this paragraph refer to an umbrella approach to answering any question? As in, divide the way one approaches a given problem and solve it in different ways in order to check your estimation? I agree with previous comments, this paragraph is a bit nebulous.

I found what might be a bug with the NB script. When I already have one of the files loaded, and I click the "Files" tab and select a document, the script becomes unresponsive and crashes Firefox. It works fine when I go to the NB in the beginning with no particular document loaded though. Also, NB doesn't work in Safari at all.

Another bug: When typing a comment, if you try to click to another part of the comment to edit something (say a spelling error) the drop down just reappears there and I cannot make the edit. This is only the case for notes on the right side; not globals.

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I just learned a new word! Are there lots of terms used like this in the text?

I did the homework before reading this. Now your solutions make a lot of sense when you say few.

I think if you were to publish a new edition to this book, you should mention that since we are dealing with order of magnitude estimation, 10ˆ.5 is about 3 is a justified reason to claim few=3. When you explained this in class, it really helped me feel comfortable with applying that approximation.

I agree with this. Maybe mentioning that the exponents are related to keeping the estimate in a logarithmic scale would be helpful.

This is one instance where an example is amazing. I would prefer to have it written in equation form, outside the body of the paragraph, to show exactly how a calculation would resolve into fewˆsome power * 10ˆsome power. This particular example may not be best in this case.

Why do they say to calculate (few)ˆ3 = 30, if they change their answer to 50?

I think it's done just cuz 50 is an easier number to work with than 30.

He explains in the next line that because each number is slightly greater than 3, the few technique is a bit of an underestimate so he bumps it up to 50.

how do you know to guess 50, is this a gut feeling problem again? I know it would be greater than 30, but I don't really know whether to guess 40 or 50.

I think the general theme is that it doesn't really matter whether you guess 40 or 50 – they're pretty close to each other, and we're only aiming for an estimate anyway. One way to guess though, is that 3.2ˆ2 10, and 3.6 is roughly 10% greater than 3.2 (actually 12.5%). So 3.6*4.4*3.2 3.2ˆ2*1.1*4.4 3.2ˆ2*4.8 10 * 5.

Regarding to the notes visibility button: It would be nice to have an option to filter out notes by person or to just look at one's notes.

This comment is exactly what I was going to post. A little bit back I tried to look at my last post and was wondering how to do that. It seems there is no way... is there?

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I'm not sure how you could fix this, but when I started reading section 1.2, I assumed that we would be presented with other divide and conquer methods, like we saw in section 1.1. However, this section is about ways to ensure the accuracy of a divide and conquer approximation. This section just seemed out of place since only the one example of the CD has been given so far.

Read Section 1.2 and do the memo by Sunday at 10pm. Have fun!

If it can make the reading easier, check out this **untested pre-alpha preview :)** of what will evolve into the next NB interface: http://nb.csail.mit.edu:8080/?t=p23

I tried using the new interface, but it wouldn't link the comments with their location on the page. I would either click a comment box on the page and it wouldn't identify a comment, or I'd click a comment and it wouldn't identify the text it referenced. Did anyone else have this issue? — Update: after a third try, it worked. Not sure why.

No issues for me, sometimes it takes a few clicks to be linked to the comment

This example is pretty crazy. After reading the explanations I begin to understand the example and methods better, but for an introductory divide and conquer example, I feel the oil import question is a little better to start with.

It's great that this section is here, but many people were concerned about this when reading section 1.1. Perhaps you should make a note in that section that their concerns would be addressed in a future reading.

I agree. I think it would also be helpful to explicitly indicate that the capacity example will be supplemented by further approaches in section 1.2, so that as we read section 1.1 we can anticipate checking these assumptions through redundancy.

I disagree with these two comments and think it's fine the way it is. To us, it seems like it's a good idea to do that because there's been a break of a few day since we visited that material - however, to someone reading this book in physical form or as a whole entire PDF, it would be kind of redundant since they're only a page apart. Just flip back a page if you need a refresher.

I feel like you can never trust the estimate to make any important decisions. Then... I am confusing myself? Why do we make estimates anyways.

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Since we're working with Nˆa+/-b, how large is "embarrassingly" large? Or does that not apply in this case?

I think being off by more than an order or two of magnitude is when it begins to become embarrassingly far off, but I suppose this number gets larger as the numbers we're using get larger.

I agree with that comment. I think it's important to step back from a problem and ask yourself "does this answer make sense?" If you're getting a number that's physically impossible/unlikely (like a speed faster than the speed of light), I think it might be time to look at your assumptions/steps again to see if you can do better. This ties into the whole "gut" feeling that we talked about in class.

i've never done this, but i suppose it makes sense. i don't see how it's much different than just adding them again though.

displaying the list of numbers first would clarify this

Agreed - if you put 12, 15, and 18 in a list, and "rearrange" them to show the two different additions, the redundancy would be more clear.

I don't think I've ever used this method of checking my mental math, but it does make more sense than checking by doing it in the same order again. My way could allow me to make the same mistake twice in a row using the same thought process, similar to how you could type your e-mail incorrectly twice in a row as we discussed in class.

I've never heard of anybody doing this. Is this common? If I was going to check it, I would just add them again in the same order, or add them more slowly/carefully.l

It's better to do this way I think because that way you don't make the same easy mistake twice.

I agree. I often subtract some numbers from the sum and see if i can get the original numbers

I didn't know it was common either and did the same as you, adding them again in the same order or more carefully. It does make a lot more sense to go backwards. I can definitely remember some "mindless redundancy" mistakes way back when...

It's not clear to me what you're doing here

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I've actually never thought of doing this, but I like the idea of it. I'll probably use this method in the future.

I agree, I have never thought of doing this before, but I think it could have really helped me in the past. I'll definitely remember this one the next time I have to check a sum.

Another good method for double checking sums/differences is the method of "casting out nines." I learned this as a child, and I found it useful because it didn't involve doing the problem again using the same method (adding the numbers forwards and backwards to me is the same method).

what is "casting out nines"?

The idea is that you add up the digits of what you're adding, and make every instance of "9" a "0,"then you add what is left (which is a single digit number). This answer should be the same as your original answer (assuming you add up the digits and "cast out" the 9s there as well).

For example, 193 + 324 = 517. I know this is correct. But if I didn't, I could use casting out nines to check. 1+9+3=4 (I made the 9 a 0). 3+2+4=9=0. So when we add up the digits of our answer, they should equal 4 (since 4+0=4). Alas, 5+1+7 = 13, and 1+3 =4. Therefore, our answer is most likely right. Of course, we could have made some weird error that this method wouldn't catch, but the method is usually correct.

The Wikipedia article on the method is decent–you should read it if you are unclear.

That's pretty useful. It might be beneficial to share it with everyone. It's at least a fun litle trick.

I've actually never checked my answer using a different method. This could be useful.

what exactly is "casting out nines"?

So I am having some troubles with this site now that I didn't have before; I cannot check all the comments. I want to see all the comments on an area, but can only see the largest one. Additionally, I am having global comment trouble. I cannot make or reply to global comments.

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I agree with one of the global comments; how would you go about estimating something you know nothing about? Could you just say, "Well, I can't see the pits, so it's probably somewhat smaller than what I can make out visually... maybe a few nms?"

This goes with a lot of the problems on the pre-test; What are we supposed to do about estimation when we have no general knowledge on a subject?

I think a lot of us are adapting to this. You do need to start somewhere and I'm hoping to have a better feeling for sizes of things and more useful knowledge coming out of this class.

two totals agree, as they do here, each is probably correct: The chance is low that both additions contain an error of exactly the same amount.

Redundancy, it seems, reduces errors. Mindless redundancy, however, offers little protection. As an example, if we repeatedly add the numbers from top to bottom, we are likely to repeat our mistakes from the first attempt. Similarly, reading your rough drafts several times usually means repeatedly overlooking the same spelling, grammar, or logic faults. Instead, put the draft in a drawer for a week, then look at it, or ask a colleague or friend – in both cases, use fresh eyes.

This robustness heuristic was in the Laser Interferometric Gravitational Observatory (LIGO), an extremely sensitive system to detect gravitational waves. It contains one detector in Washington and a second in Louisiana. The LIGO fact sheet explains the redundancy:

> Local phenomena such as micro-earthquakes, acoustic noise, and laser fluctuations can cause a disturbance at one site, simulating a gravitational wave event, but such disturbances are unlikely to happen simultaneously at widely separated sites.

Robustness, in short, comes from *intelligent* redundancy.

This principle helps us make reliable, robust estimates. Not only should we use several methods, we should make the methods different from one another; for example, make the methods use unrelated knowledge and information. This approach is another use of divide and conquer (which may explain why the approach belongs in this chapter): The hard problem of making a robust estimate becomes several simpler subproblems – one per estimation method.

So, to supplement the divide-and-conquer estimate for the pit spacing (Section 1.1), here are two intelligently redundant methods:

1. An optics method is based on turning over a CD to enjoy and explain the brilliant, shimmering colors. The colors are caused by how the pits diffract different wavelengths of light. (Diffraction is beautifully explained in Feynman's *QED* [12].) For a pristine example of diffraction, find a red-light laser pointer, the kind often used for presentations. When you shine it onto the back of a CD, you'll see several red dots on the wall. These dots are separated by the diffraction angle. This angle, we learn from optics, depends on the wavelength (or color): It is λ/D, where λ is the wavelength and D is the pit spacing. Since light

Comments on page 2

This is not always the case, especially if a computer is doing the calculations and spanning several orders of magnitude with small numbers first/larger laterr. We were learning about this in 2.086 (numerical computation) last week... something to do with the number of decimal places the program stores when doing calculations

What would be the possibility of both additions containing an error?

are there certain cases or pitfalls where this may not be true?

But doesn't redundancy only reduce careless mistakes? If we keep doing the math over and over, but our assumptions are wrong, our final estimation will still be off. Or does "intelligent redundancy" refer to coming back after we've gained new knowledge?

> It does seem like there needs to be a distinction between redundancy to reduce arithmetic mistakes and redundancy to challenge assumptions through multiple approaches.
>
> > I had the same thought; its addressed in the next sentence
> >
> > > But I wouldn't call checking for arithmetic mistakes "mindless redundancy". For example, repeating the addition from the bottom up was a good way to check the addition, however it did not guard against wrongly assumed numbers to begin with. I realize the example is meant to be representative of the process as a whole, but I think it would still be useful to point out that there seem to be two types of helpful redundancy: reducing arithmetic errors through repetition, and checking assumptions through separate approaches.
> > >
> > > > I think there's a difference between mathematical redundancy (checking your work) and statistical redundancy (doing multiple tests to confirm results). I think this paragraph is trying to explain how to use statistical redundancy (different "directions" of adding) to reduce mathematical errors.
>
> In the next few sentences, this concern is actually explained. It's natural for the mind to become accustomed to what's written and automatically assuming what we see is what we had in mind, and as a result, it's often difficult to catch one's mistakes without taking a step back and using 'fresh eyes'.
>
> Down two paragraphs it explains intelligent redundancy as making the methods different from one another.

What level of redundancy is ideal?

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Looks like we agree. (See my previous note.)

answered my comment/question above on pitfalls

I wonder if there are multiple methods of corrected errors in rough drafts by yourself without the time delayed by resting for fresh eyes, or asking others to read

I feel like this comparison might be a bit of a stretch. I can understand why your brain will assume the same answer to a calculation you did a few seconds earlier, but when you proofread a paper, it is usually very obvious when you make mistakes.

My experience is very different. Often I still find typos in papers I've spent years working on; and I usually find them when I have put down the paper for several months and then come back to it.

Agreed, for me I just have a problem spending too much time focused on paper/math problem. So the time away allows my mind to see the paper/math problem in a new light.

There are also those cases where you make one mistake so often that you're own brain has a hard time picking it out. This is especially common with poor spelling.

i agree, sometimes just the difference of looking at it on paper instead of a computer screen is enough.

This is a very common proofreading technique, as is reading drafts aloud. Sometimes if you talk your way through a problem aloud, careless mistakes become even more obvious.

Agreed. Both taking a break and reading drafts aloud can help. In 9.00, I learned that when you concentrate for too long on one thing, you are more likely to look over or make mistakes. However, switching your focus to another subject may not have the same effect because it activates a different part of your brain.

The switch to active imperative voice from the passive of the previous sentence was strange to me. "Putting the draft away for a week or asking someone else to read it, like adding numbers in reverse, provides a fresh look at the problem."

The analogy was helpful.

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Or we can change the font and the size and we will be able to see the text in a "new" pictorial way, the brain will probably catch new characters.

Also a good strategy when debugging code. In 6.005 Professor Rinard suggested going surfing before coming back to the problem to have fresh eyes or a new perspective.

which robustness heuristic?

I am also confused as to which robustness heuristic being referred to,

The heuristic refers to mindful redundancy. I think it's just unclear- Ihad to read it a few times to understand that too.

You seem to be missing a verb. "This robustness heuristic was -used- at LIGO," otherwise it makes it sound like a physical object.

How does the system take advantage of this robustness heuristic?

I was curious as to what a gravitational wave was. The most succinct description I found was that bodies (stars, etc.) leak energy in the form of gravitational waves: Here's the Wikipedia entry: http://en.wikipedia.org/wiki/Gravitational_wave

I think NASA has a pretty good page on gravitational waves, describing them as "ripples in space-time." Here's the link: http://imagine.gsfc.nasa.gov/docs/features/topics/gwaves/gwaves

Wasn't sure if this was WA or DC so I looked it up. Its in Hanford, WA

not sure what the point of this is or what it mean.

what kind of acoustic noise? this seems like it would be everything...

I think that the acoustic noise does mean the background noise but also any other kind of noise fluctuation that would be site-specific and could be mistaken for a gravitational wave event, but not the kind of gravitational waves that they are looking for - they want ones that are global, not local.

This makes the class example of entering one's email twice seem insignificant in comparison. Having both does help clear up the idea, though.

After re-reading, I understand, but could be clearer

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I think I understand this metaphor to mean that it's unlikely to make the same mistake over a variety of methods? However, it's a bit unclear, and I think the metaphor would benefit from being more explicit.

Oops...seems my comment didn't highlight the intended section - I was talking about the LIGO fact sheet example on page 2.

Yeah I agree. This example is a little cloudy.

The example made sense to me. Whether using different types of data (capacity, optics, hardware) or using the same type of data from discrete locations (Washington, Louisiana) the estimations are unlikely to agree unless they are reasonable. If the numbers are greatly different you know one of them must be inaccurate.

I think that it's just a little over kill to offer the LIGO example. There are already 2 'common' examples and adding a third that's less relatable to most people muddies the water.

Are there any tools we can use for approximating the level of redundancy that would be declared "intelligent?"

what was intelligent about this example? merely the fact that they were located far apart?

The intelligent use of redundancy in this case is the assumption that false gravitational wave events can be found (and ignored I suppose) because they will not occur in both, far-apart locations - however, the events that they want to find will occur at both locations

it seems that "intelligent" is a keyword in this section meaning "reasonable" or "intended by design". It is also repeated below as "intelligently redundant methods." However, I do not see "intelligent" being defined (although it might be somewhere), but it would help if it were defined.

I completely agree, but wouldn't it make sense to err on the side of being too redundant than not enough?

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Is there anything we can do about the fact that the forces of nature are seemingly always against us? We assume that freak accidents are very very unlikely to happen, and yet they do...

> This is a good point but if the system is constantly detecting gravitational waves then it should calculate a virtually accurate average measurement. This average would also make it easier to detect the microearthquakes because they would deviate from this number.

can the methods use completely disjoint knowledge and information though?

This seems like a very helpful and reliable process... however, for some problems it's hard enough just to find one way of getting an answer, let alone two or three different ways.

> I agree; for the time being, we're having considerable trouble coming up with one trustworthy method, but perhaps over the course of 6.055 we'll be able to learn how to come up with a few of them relatively quickly.
>
> > Well, in the previous CD example we found, for instance, the storage capacity of a CD a few different ways. The entire problem may have one logical method of solving, but if we can tackle each constituent estimate from a few angles, they become more precise and robust.

But what if we do not have enough knowledge to try another way, and we can only run the numbers in one direction?

this reminds me of the diagram prof mahajan drew in class about gaining an understand about an area by exploring two unrelated examples

This is interesting...From lecture, I thought of divide and conquer as drawing from related sources that will influence your estimation. However using unrelated knowledge makes it easier for me to grasp. Initially, I was concerned about using divide and conquer because I assume I wouldn't know about every topic to make a estimate on it. ie. suppose I didn't know the population of the world...I figured that would hinder me in making a estimation of how many toothbrushs are used yearly (disclaimer: this is a random example)

> How would you go about doing this, without knowing the population. I guess you could think about how much $$ a toothbrush company makes and the production/profit cost of a toothbrush, and how many companies there are...

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It seems this is more of an "attack from multiple fronts" argument a than a "divide & conquer" one. I think the phrase divide and conquer gets misused quite a bit.

What about it is divide and conquer? It seems more like a "multiply and conquer" Since we are actually making more problems of the same length, rather than more smaller problems.

I think the "divide and conquer" similarity is that you want to divide it up different ways and see if you get the same answer.

Leave comments in the margins instead of overlaying over everyone else! It's very convenient to be able to pinpoint a commented section and click on it so the right comment bar centers in on the relevant comment...

Perhaps an improvement to NB that could be looked into would be prohibiting comments covering the same area or something. Although I think you should be able to comment on an entire paragraph and also a section of the paragraph with separate comments, this paragraph highlights just how bad it gets when overlap isn't prevented.

but then that defeats the purpose of being able to "pinpoint the area of confusion." but i agree, 75 people writing on one page is ridiculous.

Is there some other better reason for putting it in this chapter? This parenthetical seems unnecessary.

I agree with the comment above that this parenthetical insertion is unnecessary. In general, we would like to trust the author's judgement about the contents of a chapter.

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This explanation of "divide and conquer" doesn't make sense to me. Hearing the name of that tactic makes me think that we are going to break up a problem into several pieces and conquer each small piece and then put them together to make an estimate. This idea of "divide and conquer" as a method of looking at several ways to solve one problem seems like its harder than just making one estimation because you have to find several ways to look at it.

I agree, I also thought this was a strange definition of divide and conquer. What seemed strange to me was that the CD example was just explained as a divide and conquer problem, but now we're being introduced to this heirarchy of divide and conquer problems. First, there is the CD problem, and finding an approximation for that is a divide and conquer problem. But now there is also this larger divide and conquer problem of obtaining a reliable approximation for the CD problem. Couldn't you infinitely come up with more divide and conquer problems which seek to refine the answer to another?

the idea of finding redundant methods is a _supplement_ to the divide and conquer tactic. it's a way to 'sanity check,' if you will.

Ok, so we have been using "divide and conquer" by splitting up each method of estimation into smaller problems, and then combining these smaller estimation problems in order to produce a "reliable" estimate. Are you also defining the use different methods of the same problem as a "dividing and conquering" method? It seems that we a dividing a checking here, not dividing in order to overcome the problem.

I think this is supposed to be a supplement to the divide and conquer method.

just because you split them up, does that guarantee that the error will be less?

It just means if there is error, then it is likely to be different in each place; if the answers don't match (for some reasonable definition of match), then you have at least one problem.

it is the same idea as adding numbers in different orders.

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I like this explanation.

I think this confuses me more actually. What makes more sense to me is that you have divide & conquer to take a single estimation method and break it down into simpler subproblems. And then, outside of divide & conquer, you make your estimate more robust by repeating this exercise from a different approach. What the text implies (and what I don't understand) is how this robustness relates to divide & conquer, since it's not really dividing the problem so much as coming at it from a different approach.

I would have to agree with this. The explanation kind of muddies the concept of divide and conquer. I understood it to refer to the process of breaking a large problem into reasonable bits and then solving from there. The way this explains making the estimate more robust makes it seems like we are validating each component of the original estimate rather than the final solution. The examples, however take the problem as a whole, divide it differently and then conquer it differently.

I think this is a very valuable skill to be applied to engineering.

Would it be possible to include examples that are less technical but still get the concept and lesson across? It is sometimes difficult to follow technical-heavy examples such as this CDROM example (and the bandwidth 747 example in class).

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Is this suggesting that we could have used any of the following methods (or any of the ones from the past reading) and we would come up with the same estimate or is it rather saying that in order to confirm/ disprove our past estimate that we should check it again with one of these? It was hard enough for me personally to think of way of looking at that problem- I'm really not sure I could look at it in several different ways unless prompted to (i.e. "Consider this now as an optics problem")

I agree, it would have been hard for me also to come up with another method of approximating this value. I wouldn't have been able to come up with either of the 2 following methods. Is there anything we can do if we aren't familiar enough with the subject material of the problem to come up with multiple methods?

I don't think that we should approach this as an 'exhaustive' list of methods - these are two methods that CAN be used, but don't necessarily have to be. Personally, i doubt I would have come up with either of them either, but that doesn't mean other methods that we may or may not come up with aren't applicable. My concern comes is at what point the error becomes too great, even with some redudancy precautions.

I would like to reword "intelligently" with "reasonable" in the document's sentence.

"optics-based"?

He uses 'based' later in the sentence.

I made a comment below for the analogous section of the second method. It is awkward to say "an optics method..." because too much information is being condensed into only 3 words. Instead, you might say: "A redundancy method that makes use of knowledge about optics is..." I proposed a similar phrase for the beginning of the second method. Another thing that my suggestion accomplishes is that it acknoledges that each method requires certain knowledge. Many students (including me) are complaining that these methods are very sophisticated and not very intuitive. By emphasizing that each method makes use of specific knowledge, you are telling the reader, "if you know A, you might do B as an option."

This method is pretty intimidating, I don't think I could do this.

me neither.

I don't know. These are equations that we learned in 8.02 and 3.091, the only part that really takes an intuitive leap is the measurement of the 0.5 rad.

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but if the pits are in a grid pattern, how do they look so smooth and connected rather than just a mix of a bunch of other colors

How interesting that the pits tend to refract into rainbow patterns so consistently.

Would it be possible for you to post the notes for the entire chapter so that we can look at the references?

This paragraph has two separate points: one, that you can use basic optics to get the pit spacing, and two, that the same principle is why CD's are pretty. The first is the main point, and would benefit from a bit more elaboration (perhaps a diagram). The second is an interesting side note, and should probably be a different paragraph.

A diagram of what this looks like would have been helpful here

> I agree. Either a diagram or an in-class demonstration. Or both!
>
> > I also agree. It would be helpful if you provided a link to the demonstration if drawing it in the readings isn't possible.
> >
> > > I don't really understand how someone could think this up, and I agree that a demonstration would greatly help visualize this experiment.

so if were one were to use a green laser, the angle of the diffraction would be different, but would the pattern also change?

Random, but it would be really cool to test this using a green laser and a red laser, and see how the diffraction pattern shifts

> I agree! It would also help us see how accurate our estimations are.

I don't understand how the D is automatically assumed to be the Pit spacing, what if the reflection pattern is cause by some different aspect of the CD such as the thickness, the coating, or the angle of the sides of the pits or anything along those lines.

rad

Not very understanding of these derivations. I'm not very acquainted with optics

contains a spectrum of colors, each color diffracts by its own angle. Tilting the disc changes the mix of spots – of colors – that reach your eye, creating the shimmering colors.

Their brilliance hints that the diffraction angles are significant – meaning that they are comparable to 1 rad. To estimate the angle more precisely, and lacking a laser pointer, I took a CD to a sunny spot and noted what appeared on the nearest wall. There was a sunny circle, the reflected image of the CD, surrounded by a diffracted rainbow. Relative to the reflected image, the rainbow appeared at an angle of roughly $30°$ or $0.5\,\text{rad}$. This data along with the diffraction relation $\theta \sim \lambda/D$ implies that the pit spacing is approximately 2λ. Since visible-light wavelengths range from $0.35\,\mu\text{m}$ to $0.7\,\mu\text{m}$ – let's call it $0.5\,\mu\text{m}$ – I estimate the pit spacing to be $1\,\mu\text{m}$.

2. A hardware method is based on how a CD player or a CDROM drive reads data. It scans the disc with a tiny laser that emits – I seem to remember – near-infrared radiation. The *infrared* means that the radiation's wavelength is longer than the wavelength of red light; the *near* indicates that its wavelength is close to the wavelength of red light. Therefore, *near infrared* means that the wavelength is only slightly longer than the wavelength of red light. For the laser to read the pits, its wavelength should be smaller than the pit spacing or size. Since red light has a wavelength of roughly $700\,\text{nm}$, I'll guess that the laser has a wavelength of $800\,\text{nm}$ or $1000\,\text{nm}$ and that the pit spacing is slightly larger – $1\,\mu\text{m}$. (The actual wavelength is $780\,\text{nm}$.)

Three significantly different methods give comparable estimates: $1.4\,\mu\text{m}$ (capacity), $1\,\mu\text{m}$ (optics), and $1\,\mu\text{m}$ (hardware). Therefore, we have probably not committed a blunder in any method. To make that argument concrete, imagine that the true spacing is $0.1\,\mu\text{m}$. Then three *independent* methods all contain an error of a factor of 10 – and each time producing an overestimate. Such a coincidence is not common. Although any method can contain errors – the world is infinite but our abilities are finite – the errors would not often agree in sign (being an over- or underestimate) and magnitude.

The lesson – that intelligent redundancy produces robustness – seems plausible now, I hope. But the proof of the pudding is in the eating: What is the true pit spacing? It depends whether you mean the radial or the transverse spacing. The data pits lie on a tremendously long spiral

But if your shining a red laser pointer, how can you get different colors out? The laser emits red at its wavelength and then refracts by the relation you mentioned. A red laser doesn't really have a spectrum of colors.

You are right, lasers should not emit a broad range of wavelengths, if any at all. Also, the sentence is not well written because it contains a false cause-effect relationship. That "lights contains a spectrum of colors" does not cause colors to diffract by their own angles.

Does this mean there would have been a different angle of diffraction if you had used green or blue instead of red in class today? I assume it would because it would change the lambda/D relationship, but it's hard for me to grasp an intuition for why that's true.

Different colors have different wavelengths (e.g. the visible spectrum), changing the values in the equation.

Are you talking about just shining white light onto the disc, or is this still referring to the red laser?

meaning that the colors look pretty? or they make interesting shapes?

I don't understand this leap.

Me too–is it because distinct red dots show up when a laser pointed is shined onto the CD? or is it because each color diffracts by its own angle? or some other reason?

I think the point is because there ARE so many colors, this implies that the colors are significant. Were the back of the CD uniform in color than we would not think to examine it.

I think poster #3's explanation is correct, but it would be great to have this more explicitly/thoroughly stated.

I don't know if brilliance is really the right word to use here either, unless it's defined explicitly to mean something like "the wide variation in color".

Agreed; I thought brilliance meant "brightness" here, and didn't realize it meant the variety of color until reading comments.

So simply the separation of colors is enough to conclude that the angles are significant?

also, what are we defining as significant? i probably wouldn't have come to the conclusion of 1 rad in difference.

contains a spectrum of colors, each color diffracts by its own angle. Tilting the disc changes the mix of spots – of colors – that reach your eye, creating the shimmering colors.

Their brilliance hints that the diffraction angles are significant – meaning that they are comparable to 1 rad. To estimate the angle more precisely, and lacking a laser pointer, I took a CD to a sunny spot and noted what appeared on the nearest wall: There was a sunny circle, the reflected image of the CD, surrounded by a diffracted rainbow. Relative to the reflected image, the rainbow appeared at an angle of roughly 30° or 0.5 rad. This data along with the diffraction relation $\theta \sim \lambda/D$ implies that the pit spacing is approximately 2λ. Since visible-light wavelengths range from 0.35 µm to 0.7 µm – let's call it 0.5 µm – I estimate the pit spacing to be 1 µm.

2. A hardware method is based on how a CD player or a CDROM drive reads data. It scans the disc with a tiny laser that emits – I seem to remember – near-infrared radiation. The *infrared* means that the radiation's wavelength is longer than the wavelength of red light; the *near* indicates that its wavelength is close to the wavelength of red light. Therefore, *near infrared* means that the wavelength is only slightly longer than the wavelength of red light. For the laser to read the pits, its wavelength should be smaller than the pit spacing or size. Since red light has a wavelength of roughly 700 nm, I'll guess that the laser has a wavelength of 800 nm or 1000 nm and that the pit spacing is slightly larger – 1 µm. (The actual wavelength is 780 nm.)

Three significantly different methods give comparable estimates: 1.4 µm (capacity), 1 µm (optics), and 1 µm (hardware). Therefore, we have probably not committed a blunder in any method. To make that argument concrete, imagine that the true spacing is 0.1 µm. Then three *independent* methods all contain an error of a factor of 10 – and each time producing an overestimate. Such a coincidence is not common. Although any method can contain errors – the world is infinite but our abilities are finite – the errors would not often agree in sign (being an over- or underestimate) and magnitude.

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Significant meaning that the definition of significant angles is 1 rad or that the angles are truly unique and then there was some math involved in order to arrive a the 1 rad?

wait how did we get to 1 rad?

I believe it has to do with powers of 10. A 'few' radians would be 3, or pi. This would correspond to an angle of 180deg, which wouldn't make sense for this reflection example. Thus, because we see such a large variation in colors, angles are on the order of 1 radian (60 deg).

I would also have preferred that this was more explicitly explained. I'm not sure I understand where 1 rad comes from..?

I agree that the rainbow effect means the diffraction angles are significant, however I don't follow the leap to 1 rad. I'm not entirely sure what is comparable to 1 rad.

Are the results of this test affected by the sunlight?

I just tried this with a CD and didn't see the "surrounding rainbow" (either that or I didn't know what to look for). If the point of this estimation is that anyone could do this, then I think either a diagram or a better explanation of what to look for would be helpful.

I've seen this experiment/calculation done in a freshman seminar and 6.007 - I feel like the presentation of the mathematics in those classes helped to understand what's being talked about. Can that be included in a future revision?

I had to read this part 3 times to figure out what it was trying to say...I think that images would really help me out here.

How do you compare this angle if they are reflected on the same surface?

I don't understand how they get that angle- perhaps a diagram?

an angle relative to what?

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this section goes through approximately a class and a half of 8.02 in like a paragraph. it's very hard for me to follow and i'm still a little hazy.

So...the reflected light from the CD onto the wall made a CD-shaped bright spot, and then a rainbow (I'm assuming it stretched above or below the bright spot) that reached around 30 degrees, which is equivalent to 0.5 rad. Plugging this into theta yields 1/2 wavelength/pit space->pit space 2*wavelength. Given that the average value of visible light (which is what is seen in the rainbow on the wall) is about 0.5 micrometers, he plugged that into the equation pit space 2*0.5 to yield a spacing of around 1 micron

A figure or diagram would definitely help with the explanation of this.

Also, when solving problems, it is hard enough to come up with one method to solve them. Without having the proper knowledge to solve the problem in more than one way (like with the knowledge of diffraction in this case) how can we be redundant?

Your explanation was unclear, and this explains it much better.

I think it would be a lot easier to follow this paragraph if you've taken 8.03, however a diagram would definitely be appropriate here. I'm still not really sure what he means by "the rainbow appeared at an angle of roughly 30 degrees". In relation to what? Pictures Please!

I feel like some of these more complicated examples defeat their purpose in teaching about estimation since we're so caught up in trying to understand the physics a lot of us haven't learned that we sort of miss the point. Maybe using a simpler example that people can more easily conceptualize would be better for getting the estimation technique across?

I am also having trouble visualizing what is going on here. A diagram would definitely be useful in defining your angles.

maybe a modification to NB is a way in which figures and diagrams and references can be supported. Basically, if a person needs more clarification, he can click on a symbol besides the statement which will then open a diagram, figure, reference off to the side or in a pop up window.

I agree with this, although once you look past the complexities, it becomes apparent what point you are trying to make - you arrived at the same answer via a different method. While understanding the method more fully would make the example more easily repeatable for the reader, just a vague understanding gets the point across.

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I also agree that pictures would be make things much easier to visualize. It also seems like some of the things we're estimating require knowledge that we probably haven't learned, and it'd be nice to develop some sort of intuitive feel of how to estimate these things with real-world comparisons rather than drawing equations from physics courses we've never taken.

agreed with all of the above. it's nice to space out all this info so its easier to think about. also pictures.

I agree with Anon#5 a thousand times.

I would agree with this comment. I found myself stuggling trying to remain focused on the point that you were trying to get across about redundancy, but kept getting distracted by understanding the details of the problem

I understand how these checks work, but they still rely on some assumptions about CDs that, unless you knew in advance, you'd have no way of verifying yet.

I don't get where the 2(lambda) came from

Is it saying D=2lambda because that would make sense

I have been feeling generally confused during these kinds of trains of thought, and I think I understand why. The specific figures make logical sense what I read them, but I doubt I could recall them on my own, especially the ones from the previous section about the pits in a CD. Can there be some elaboration on a systematic approach to a problem if you aren't sure of numbers to start with, or a way to figure out which numbers to use in certain circumstances? I realize this basically the heart of approximation and thus a pretty loaded question, but most of these examples have figures ready at hand, and it would be nice to see a problem that is approached with the same initial shock of knowing nothing (similar to the feeling I am faced with almost every problem on the pre-test).

As for the hardware method, I feel that it requires knowing/looking up information which makes the inquiry redundant. For example, if I know that the CD players use red light, I can easily find out the spacing of the pits.

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I feel like in terms of understanding, this example works a lot better than the previous one since wavelength is something we can all conceptualize a bit better. (maybe it'd help to see this example first so by the time we get to the next one, we at least understand the approach?)

This wording is awkward because a lot of information is being compressed into only 3 words. When I saw "hardware" I thought "hammer and wood". Instead of abbreviating a concept you might say: "A different method that relies on knowledge about how a CD works as a computer hardware device is..."

Although I don't feel like this is common knowledge either, you have to remember that if we were to estimate something in real life, it would most likely be in a field related to what we are working on. In that case something like the wavelength of infrared radiation would be common knowledge

this sounds like a transcript of a lecture you're giving. since this is a book, i don't think "i seem to remember" is necessary.

Actually, I think he's emphasizing a situation where we might not have any tools or lecture notes and only vaguely remember some facts. The point of this class is to show us how we can mentally arrive at a guess on the spot. The way he is writing here seems to be literally simulating what might go on in our own heads, if we were doing the calculation the way he is trying to teach us to do.

I don't think the fact that the laser is near-infrared would be common knowledge, though it is entirely possible that many MIT students have a rough idea of the wavelength o red light - this brings up the question - what should be considered "common knowledge"?

As much knowledge as you can common! And the way to do that is to practice. One reason for not doing 'regular' grading in this class is that the base of knowledge each person has is so different, and my goal is not to grade people on their common knowledge but to encourage people to expand that knowledge.

yeah it does sound weird

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initially I didn't understand what this section was talking about and I missed the "near" in the expression so I was very confused by the following sentences describing the terms. When I read it a second time however, it made a lot more sense and I think the hardware method is a really cool way to estimate the pit spacing. The confusion may have come from the fact that the boxes from people's previous comments make the text more difficult to follow because they are distracting.

I believe Blu-ray players work just like this except the laser is blue (smaller wavelength), thus it the disk can hold way more space

Correct: "While a standard DVD uses a 650 nanometer red laser, Blu-ray uses a shorter wavelength, a 405 nm blue-violet laser, and allows for almost ten times more data storage than a DVD." - http://en.wikipedia.org/wiki/Bluray

(Also note that the DVD has a shorter wavelength than the CD, as noted later in the paragraph).

Is there something preventing us from, therefore, making a system employing UV light and thus cramming even more onto a disk? What I mean is that I'm wondering whether it's a theoretical or purely technological problem.

I would not have know the details of how a CDROM drive reads data, but I like this section a lot otherwise. It is much simpler to follow than some of the more technical details in the previous section.

and my journal of numbers increases

I agree with the other comments about this section. Although I can understand how you got to your answer by reading this method, I lack the necessary "givens" to make these approximations.

How does the laser relay information back to the hardware, does it refract in varying directions and that signals the information?

Does the accuracy change as the ratio of the laser wavelength to the pit spacing changes?

How much smaller, or does that not matter? Is there some sort of equivalent to sampling and aliasing here, such that a much larger wavelength would not have worked?

Curious about this as well. Is it really wavelength or frequency at play here?

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Three significantly different methods give comparable estimates: 1.4 µm (capacity), 1 µm (optics), and 1 µm (hardware). Therefore, we have probably not committed a blunder in any method. To make that argument concrete, imagine that the true spacing is 0.1 µm. Then three *independent* methods all contain an error of a factor of 10 – and each time producing an overestimate. Such a coincidence is not common. Although any method can contain errors – the world is infinite but our abilities are finite – the errors would not often agree in sign (being an over- or underestimate) and magnitude.

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Is this considered lumping?

If the laser wavelength and the pit are similar sizes, Why doesn't the laser hit in between the pits often? I would think having a laser much smaller than the size of the pit would make it easier to be sure which pit the laser is looking into.

Why is there the uncertainty between 800- 1000 nm? I understand that it has to be greater than the red light wavelength but I don't get why the reading just says "800 nm or 1000 nm". I think it would read better as "between 800 to 1000 nm" as that denotes a range as opposed to 2 distinct values.

Why does the laser wavelength have to be shorter than the pit spacing? Also, you seem to be assuming that the laser and pit spacing are optimized for each other. Technically, couldn't the pit spacing be any distance less than the wavelength?

which was the design limiting factor? the lasers readily available for reading or the spacing of the pits?

1um isn't slightly larger than 1000nm, do you mean on the larger end of that range of 800-1000nm?

I think he meant the wavelength of laser is smaller than the size of the pit.

Right, but how does he make a jump from 1000nm (which is 1um) directly to 1um? If we assume the near IR scanner is at 1um, then this breaks apart. It seems very very rough

It comes from the assumption that the wavelength will actually be smaller than 1000nm while the pit spacing is slightly larger, but for the sake of simple math we'll just approximate them both to be 1um with the knowledge that the theory would actually work out in the end (that is, that even though we're approximating both to be the same length, we know they are actually slightly different).

It does seem very rough, and this method almost seems as if it's "tailored" to get the 1 um answer that all the other methods have arrived at. It's a little too convenient...,

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The change in units here from nano to micro and back is a little confusing for someone who doesn't have a scale of them.

you should probably know that if you're reading this however

where did this number come from?

The 1.4 came from the previous section.

Isn't 1.4 significantly different than 1? Does this discount the first method?

Remember that we are judging accuracy by orders of magnitude. All guesses are based on values with around 1 significant figure. Based on significant figures, 1.4 is equivalent to 1.0 in this case. Maybe this fact should be noted since the beginning of the chapter and we should be reminded of it often.

Now having read the whole document, I feel like the addition part at the beginning wasn't quite the same, because the methods were too similar. I mean, you aren't simply suggesting here that we do our multiplication in a different order, you are suggesting entirely different methods. For the addition, shouldn't we have used entirely different methods? Like doing one in our head and counting out one on our fingers?

I sort of agree. There are several more drastically different methods for addition that can be used. One good one I like for adding N numbers is estimating an average value (call it A), and then keeping a running total of how much each number differs from that average (D). The sum is then N*A+D.

I think what this paragraph is getting at is that we come to similar answers with different methods, basically proving that our estimated answer is pretty much correct. The different methods are just a way to check your answer.

That's pretty cool.

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After we reach these three results, assuming they differ, would averaging them out give us an even more accurate result? (My assumption would be that they aren't all under or over estimates)

i'm not sure either because one could also just say they all seem like 1micron and keep the 0.4 as our tolerance, right? or would it make more sense to say 1.1+/-0.3 or 1.2+/-0.2?

In chapter 7 we'll talk about how to combine estimates. But the basic idea is (usually) to use the geometric mean. So, if your two estimates are a and b, and they are both equally reliable, then your best estimate is sqrt(a*b). If one estimate is much more reliable than the other, then the best estimate will be closer to that value than to the other one (a weighted geometric mean).

How valuable can this be? How many types of problems really have enough information to be able to solve in a multitude of ways?

This phrase gets me to thinking. For this example, the estimates were all very similar in value. If the different estimates were not so similar, could the disparity between answers be factored in the uncertainty?

I don't think the answers should have to do with the uncertainty, but you may be able to adjust the uncertainty if you have underestimated it. For example, if I said that I had a 0.1 micrometer error after my 1.0 micrometer estimation, and then I got 1.4 micrometers the other way, I could probably assume I miscalculated the error.

How could such a consistent error have happened?

There could be some implicit assumption common to all three methods (lack of independence).

It didn't, I think. This example error is hypothetical.

I think this example is a little redundant. You have already mentioned in the intro to the section that the purpose of this exercise is to increase robustness and provide a "check" that your answer is probably close to correct. I think that it is obvious enough to the reader that being a factor of ten off consistently is unlikely on this scale.

contains a spectrum of colors, each color diffracts by its own angle. Tilting the disc changes the mix of spots – of colors – that reach your eye, creating the shimmering colors.

Their brilliance hints that the diffraction angles are significant – meaning that they are comparable to 1 rad. To estimate the angle more precisely, and lacking a laser pointer, I took a CD to a sunny spot and noted what appeared on the nearest wall: There was a sunny circle, the reflected image of the CD, surrounded by a diffracted rainbow. Relative to the reflected image, the rainbow appeared at an angle of roughly 30° or 0.5 rad. This data along with the diffraction relation $\theta \sim \lambda/D$ implies that the pit spacing is approximately 2λ. Since visible-light wavelengths range from 0.35 µm to 0.7 µm – let's call it 0.5 µm – I estimate the pit spacing to be 1 µm.

2. A hardware method is based on how a CD player or a CDROM drive reads data. It scans the disc with a tiny laser that emits – I seem to remember – near-infrared radiation. The *infrared* means that the radiation's wavelength is longer than the wavelength of red light; the *near* indicates that its wavelength is close to the wavelength of red light. Therefore, *near infrared* means that the wavelength is only slightly longer than the wavelength of red light. For the laser to read the pits, its wavelength should be smaller than the pit spacing or size. Since red light has a wavelength of roughly 700 nm, I'll guess that the laser has a wavelength of 800 nm or 1000 nm and that the pit spacing is slightly larger – 1 µm. (The actual wavelength is 780 nm.)

Three significantly different methods give comparable estimates: 1.4 µm (capacity), 1 µm (optics), and 1 µm (hardware). Therefore, we have probably not committed a blunder in any method. To make that argument concrete, imagine that the true spacing is 0.1 µm. Then three *independent* methods all contain an error of a factor of 10 – and each time producing an overestimate. Such a coincidence is not common. Although any method can contain errors – the world is infinite but our abilities are finite – the errors would not often agree in sign (being an over- or underestimate) and magnitude.

The lesson – that intelligent redundancy produces robustness – seems plausible now, I hope. But the proof of the pudding is in the eating: What is the true pit spacing? It depends whether you mean the radial or the transverse spacing. The data pits lie on a tremendously long spiral

Why is the error a factor of 10?

It means that the numbers are all in the same order of magnitude.

Does this mean that we can allow room for error as long as the results are consistent?

perhaps a discussion of systematic vs. random errors would be good here. In this hypothetical example, we just made a systematic error. In conclusion, redundancy cannot provide valid error-bars for systematic errors. Systematic errors might often be due to similarities between the redundant methodologies...

Is there a way to try to catch yourself, or check yourself, to ensure you haven't make some systematic error? Is it just a gut feeling?

The best way is to use methods that are very different. Then the systematic errors in each are likely to be different in type, and are not likely to point only in one direction.

So if the errors weren't coincidentally about the same, where would we go from there? Which estimates do we take as the more accurate if we didn't know the actual value? We wouldn't be able to keep doing this–given common knowledge, there aren't too many more ways to estimate this right?

I'm not sure knowing that CDs are read by near IR is common knowledge! And I have no idea where one would go next...maybe back to the basics? But this assumes you know the answer, so you'd have to find where the fudge factors went wrong

I think we'd at first have to look over all of our estimates and make sure we didn't make any foolish mistakes. And if they looked robust, look for a 3rd or 4th method of getting the estimate, perhaps by consulting with peers for ideas. For example, we could try another method that is not divide and conquer at all. Possibly some sort of dimensional analysis method.

Or one could evaluate all the assumptions made and see if one could have possibly have been made in error.

welcome to the dirty world of engineering, where you often need to know what your answer will be before actually obtaining it.

Yeah, what if the errors are not similar at all. Why give an example, and then say, "this example generally does not apply" without discussing an example that does?

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With this sort of problem I would actually think these sort of errors might happen more often than this author seems to imply, most guesses are of a power of ten in one direction, and it is reasonable to assume that the guesses would be close to 1 order of magnitude off assuming the guesser is using some sort of logic, and so the chance that these errors might be similar should not be discounted. In the case of three I would guess it would be like flipping a coin three times, and saying either three heads or three tails means the errors line up, and in this case that would be 25%. In a case of 2 independent guesses, it would be 50/50. Obviously the magnitude that the guess is off would have a chance of being more than an order of magnitude so these % are a bit high, but I think it is reasonable for the guesses to all be wrong based on probability.

I would love for you to elaborate on this in class.

Does this mean we should repeat our calculation using multiple methods every time?

Would we then average the result and add an appropriate +/- tolerance, or make a gut judgment? Also, how far must a result vary from the others before we throw it out as bad data?

The second question is also unclear to me. Because the numbers here are so small, the different between 1.4 and 1 microns seems like a big enough difference to me to discount the first estimate.

I disagree that it is a big difference... they are the same order of magnitude which I think is more what we are after in this problem than an exact number.

I think we'd just do a quick mental average of the methods and forgo error bars, as error bars from averaging are negligable when compared to the errors from the approximations made.

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It won't let me comment on the last page

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Thats a funny expression. Where does that come from? Sounds like a British expression.

Heh, yeah it is a weird one. Never heard that before. I do like though how the text repeats the main idea multiple times, without being annoyingly 'redundant'.

the phrase is "the proof is in the pudding".... its just adapted here i guess

"the proof is in the pudding" reminds me of the slogan of a Yale all-female a capella group I met last semester, hahaha.

How long? I estimated 10km.

I agree – The rings are 1um apart and they cover about 3cm radially, so there should be about 3e4 rings. Average radius of a ring is about 4cm, so average circumference is about 20cm. That give us about 6e3 meters, or close to 10km total.

Where are these numbers from?

The 1um spacing is from the previous reading, and the 3cm radial coverage and average radius are from the ruler and CDs in my desk drawer.

I was the first poster–I used the lattice approach and thought of it as a 10 cm square of pits 1 um apart. One row across would be 10 cm = 10ˆ-1 m long, and there are 10 cm/1um = (10ˆ-1 m)/(10ˆ-6) m = 10ˆ5 rows, so the total length = 10ˆ5 rows * 10ˆ-1 m/row = 10ˆ4 m = 10 km. The first hit in Google for length of track on cd says 6 km, so we're in the right order of magnitude.

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Given that these are different (I didn't know that) couldn't we have taken it into account when we did the first estimation by making our square have unequal numbers of pits between column and row?

Yes, you could have. But doing so would make the problem harder and defeat the purpose of getting a simple "guess" at the pit-spacing. I think so long as we get something within the order of magnitude, it is "good enough". Introducing complications at the beginning will slow us down. This isn't to say that you shouldn't consider the fact that they are spaced differently transversely versus longitudinally, but that you should arrive at a simple answer first, then modify it using additional knowledge. It's like making a sculpture: you make big cuts to get the shape approximately right, then you start using smaller tools to carve out and edit the chunk into a more detailed/accurate representation.

I really like your sculpting analogy!

I agree that for a good guess, it is not necessary to actually calculate row and column spacing of the pits. However, it would be good to be aware of this difference since the beginning. In this case we were very fortunate that the spacings were very close. However, if they were far apart (eg. factor of 100), then we would be looking for a single number that has no physical significance at all.

I understand what this looks like, but a picture or figure would be a cool addition to the words.

What does this spiral look like? I'd understood it as concentric circles and not a single line curled into a spiral.

track whose ‘rings’ lie 1.6 µm apart. Along the track, the pits lie 0.9 µm apart. So, the spacing is between 0.9 and 1.6 µm; if you want just one value, let it be the midpoint, 1.3 µm. We made a tasty pudding!

Problem 1.3 Robust addition

The text offered addition as an example of intelligent redundancy: We often verify an addition by by redoing the sum from bottom to top. Analyze this practice using simple probability models. Is it indeed an example of intelligent redundancy?

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Think of and describe a few real-life examples of intelligent redundancy.

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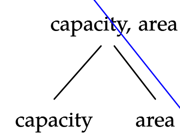
Tasty though the estimation pudding may be, its recipe is long and detailed. It is hard to follow – even for its author. Although I wrote the analysis, I cannot quickly recall all its pieces; rather, I must remind myself of the pieces by looking over the text. As I do, I am reminded that sentences, paragraphs, and pages do not compactly represent a divide-and-conquer estimate.

Linear, sequential information does not match the estimate’s structure. Its structure is hierarchical – with answers constructed from solving smaller problems, which might be constructed from even solving still smaller problems – and its most compact representation is as a tree.



As an example, let’s construct the tree representing the elaborate divide-and-conquer estimate for a CDROM’s pit spacing (Section 1.1). The tree’s root is ‘capacity, area’, a two-word tag reminding us of the method underlying the estimate. The estimate dissolves into finding two quantities – the capacity and area – so the tree’s root sprouts two branches.

Of the two new leaves, the area is easy to estimate without explicitly subdividing into smaller problems, so the ‘area’ node remains a leaf. To estimate the capacity – rather, to estimate the capacity reliably – we used intelligent redundancy: (1) looking on a CDROM box; and (2) estimating how many bits are required to represent the music that fits on an audio

Why are there different spacings for the different axes?

I feel like compared to the previous estimations, this answer is either very easy/ brief or not explained. Should I know what is meant by ’rings’ when we’ve only been talking about ’pits’?

he’s making the assumption that you can visualize the cd: "the data pits lie on a tremendously long spiral track..." the spiral forms "rings" around the cd, not individual rings, but still rings

These two values measure the pit spacing in two different ways, "radially" and "linearly." Was there a specific way that the different estimates done before tried to measure the distance? Like maybe the estimates are more accurate than explained because they are measuring the distance one way versus another?

I think that these measurements are actually given (by manufacturers) data points that we are comparing ’our’ estimates to.

ummm... ok. is this helpful to our understanding?

While I really enjoy the casual tone of this text (it makes it much more enjoyable to read), I think this sort of statement is a little over the line, and not as useful as a conclusion that pointed out how some of our calculations overestimated, and others underestimated (or some other conclusion). That’s just my humble opinion, though. - Edit: I’m not sure if the tag came through, but this was referring to "We made a tasty pudding!"

We’ve defined intelligent redundancy, but what is unintelligent redundancy? Is it just when we repeat what we’ve done before or are there ways to be redundant without repeating your previous methods?

Great line.

I don’t know if it’s just me or if NB is broken, but none of the links on page 10 are working for me. The other 3 pages are working fine, just not this one.

I’m having trouble. Does anyone know an example using probability?

typo....also, why are no note boxes showing up on this page?

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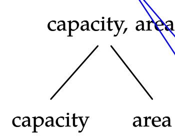

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Of the two new leaves, the area is easy to estimate without explicitly subdividing into smaller problems, so the 'area' node remains a leaf. To estimate the capacity – rather, to estimate the capacity reliably – we used intelligent redundancy: (1) looking on a CDROM box; and (2) estimating how many bits are required to represent the music that fits on an audio

I wasn't sure how to analyze the robust addition practice in terms of probability models, could someone please clarify/answer this?

But, I do agree that robust addition is a good example of intelligent redundancy, whether it involves going from top to bottom, bottom to top, or grouping similar terms (like 3s and 7s, 4s and 6s, etc.).

I think it has to do with the probability of missing the same thing twice...if you increase the number of times and directions you look at the same thing, the likelihood of missing the same value decreases. Also, it gives more data sets to average over and find mistakes. I don't know much about probability models though.

In the previous version of the chapter, there was a section here on making probability models to determine the accuracy of an estimate. But I recently moved it to the new chapter on probabilistic reasoning. The question about probability models belongs there too. Thanks for pointing out that it is out of place.

Typo. Is it helpful for us to point these out?

Yes, thank you. The reading memos are a form redundancy. I've been staring at all these sentences for so long that I do not see the mistakes (whether typographical, mathematical, or conceptual). But all your fresh eyes find so problems so quickly. Sigh!

The probability of adding wrong going down is at least a little different than going up.

P(down) intersect P(down) = P(down)

P(down) intersect P(up) < P(down)

Such as coin tosses and picking marbles out of a jar? If you need enough repetitions, then yes I think it's intelligently redundant – right? But is it intelligently redundant with only a small number of repetitions?

what do you mean here by simple probability models?

It is smart because repeating an arithmetic mistake with different numbers is way more improbable than repeating the same mistake with the same numbers.

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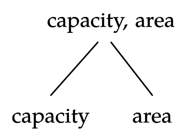

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Of the two new leaves, the area is easy to estimate without explicitly subdividing into smaller problems, so the ‘area’ node remains a leaf. To estimate the capacity – rather, to estimate the capacity reliably – we used intelligent redundancy: (1) looking on a CDROM box; and (2) estimating how many bits are required to represent the music that fits on an audio

I feel like adding backwards and forwards and in whatever order isn’t really intelligent redundancy–you’re not changing methods or gaining any new knowledge...

True, but it is a way of trying to make sure you don’t accidentally skip the same number twice

I also feel like it is very different. to add 12, 15 and 18 forwards requires you to (looking just at ones column) add 2+5, then 7+8, whereas adding in reverse requires 8+5, then 3+2. These are all very different additions. You might get the same result, but how you arrive at it is very different in the two cases, which is the point of intelligent redundancy.

I do not know what you mean by probabilistic methods here. Maybe it means, if you have a 10% chance of messing up an addition problem, you have a 1% chance of messing it up twice and an even smaller chance of messing it up to get the same number twice. ??

When proofreading my papers, I usually don’t have time to set them aside for a few weeks and, when the paper is 20 pages long, it can be hard to find someone else to read it for you. So i usually read it in my head once, and then again out loud. When I read them out loud I sometimes notice things I didn’t originally. "Hey...that sounds weird..."

I think this is one way, I’ve used intelligent redundancy before. When first learning to play the violin, intonation was something that my instructor worked on with me. Say I was learning where to place my finger to play an E in the first position on the A string. I would play it once, compare to the E the instructor played on the piano, compare it to the sound of the open E string, and use a calibrated tuner to help me adjust. In the end I would be able to play that E note in tune.

I still don’t think I’m totally clear on "intelligent redundancy"–is it just making more estimations using different methods? I feel like this isn’t really possible most cases becuase we don’t always know so much about what we’re estimating.

Yes, I think that you’re on the right track. He is saying that by using intelligent redundancy, we can check our cross-check our answers to make sure they are relatively consistent. I agree with you that this isn’t really possible in most cases given what we know yet, but perhaps we will have a better intuition at the end of the class.

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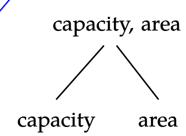

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I thought the example about entering your email address twice when signing up for websites was really effective in class. It might be worth to add that as an example at the beginning of the section in order to provide a simple easy-to-understand example.

measure twice cut once?

Peer editing, grading itself..

In 6.02, we were taught that electronics read voltages as 1's and 0's (binary code). However, in real life, changing the output voltage from zero to one is not instant, it requires many samples because the transition is slow. The redundancy of the samples facilitate the voltage to reach its value

So basically we can try to increase the robustness of our estimates by going top down and then bottom up. Nice