4.3.2 Power limits

Power production might also limit the jump height. In the preceding analysis, energy is the limiting reagent: The jump height is determined by the energy that an animal can store in its muscles. However, even if the animal can store enough energy to reach that height, the muscles might not be able to deliver the energy rapidly enough. This section presents a simple model for the limit due to limited power generation.

Once again we'd like to find out how power P scales (varies) with the size l Power is energy per time, so the power required to jump to a height h is

$$P \sim \frac{\text{energy required to jump to height } h}{\text{time over which the energy is delivered}}.$$

The energy required is $E \sim mgh$. The mass is $m \propto l^3$. The gravitational acceleration is independent of l. And, in the energy-limited model, the height h is independent of l. Therefore $E \propto l^3$.

The delivery time is how long the animal is in contact with the ground, because only during contact can the ground exert a force on the animal. So, the animal crouches, extends upward, and finally leaves the ground. The contact time is the time during which the animal extends upward. Time is length over speed, so

$$t_{\text{delivery}} \sim \frac{\text{extension distance}}{\text{extension speed}}.$$

The extension distance is roughly the animal's size l. The extension speed is roughly the takeoff velocity. In the energy-limited model, the takeoff velocity is the same for all animals:

$$v_{\text{takeoff}} \propto h^{1/2} \propto l^0.$$

So

$$t_{\text{delivery}} \propto l.$$

The power required is $P \propto l^3/l = l^2$.

That proportionality is for the power itself, but a more interesting scaling is for the specific power: the power per mass. It is

Interesting...is this primarily energy from glucose or carbohydrates or possible energy that can be stored in fats? I can imagine energy from fats will take longer to break down.

The homework is very useful in understanding the 2nd problem.

Is this the animal's size in height only or its size in weight, height etc?

I think size here usually refers to the length dimension. Weight scales as the cube of that, since W mg volume L^3. It's that difference in scaling that provides the interesting findings since things scale differently relative to L and m.

It is so weird that small animals need more power and have higher drag. It seems quite the opposite, when you compare the speed of a fly to say a human.

They don't need higher absolute power, just higher power/mass ratio. And they don't have higher absolute drag, just higher drag energy/kinetic energy.

Is this because the small animals need by energy? Could you explain in more detail how you made these connections?

Is this true for all animals? Or specific for fleas? The height proportion is confusing me...seems wrong to estimate jump height as 1m.

I think this was a very clear and descriptive memo that made it easy to understand. I am starting to feel more comfortable understanding the calculations and examples that are given.

4.3.2 Power limits

Power production might also limit the jump height. In the preceding analysis, energy is the limiting reagent: The jump height is determined by the energy that an animal can store in its muscles. However, even if the animal can store enough energy to reach that height, the muscles might not be able to deliver the energy rapidly enough. This section presents a simple model for the limit due to limited power generation.

Once again we'd like to find out how power P scales (varies) with the size l Power is energy per time, so the power required to jump to a height h is

$$P \sim \frac{\text{energy required to jump to height } h}{\text{time over which the energy is delivered}}.$$

The energy required is $E \sim mgh$. The mass is $m \propto l^3$. The gravitational acceleration is independent of l. And, in the energy-limited model, the height h is independent of l. Therefore $E \propto l^3$.

The delivery time is how long the animal is in contact with the ground, because only during contact can the ground exert a force on the animal. So, the animal crouches, extends upward, and finally leaves the ground. The contact time is the time during which the animal extends upward. Time is length over speed, so

$$t_{\text{delivery}} \sim \frac{\text{extension distance}}{\text{extension speed}}.$$

The extension distance is roughly the animal's size l. The extension speed is roughly the takeoff velocity. In the energy-limited model, the takeoff velocity is the same for all animals:

$$v_{\text{takeoff}} \propto h^{1/2} \propto l^0.$$

So

$$t_{\text{delivery}} \propto l.$$

The power required is $P \propto l^3/l = l^2$.

That proportionality is for the power itself, but a more interesting scaling is for the specific power: the power per mass. It is

So, I am still lost with this whole proportionality thing; can we get a table of proportionality, or a breakdown of exactly what goes into it? I'm still stuck thinking in units, and that's not helping me at all. I see t prop length, and m proportional to l^2. Is everything proportional to some sort of l?

Not everything is proportional to L, for instance, we found that jumping height isn't, since the L's cancel. You're right that units can get confusing if you try to think of all of them at once. Force is proportional to mass, but it's also proportional to acceleration. Acceleration is proportional to distance, and also inversely to time squared. Instead of including all the units at once (kg-m/s^2), you can think of them affecting force independently (mass^1, distance^1, time^-2) or in whatever combination you want.

This section reads very well. The transitions between subsections and topics helps me to anticipate the information I'm about to read and helps me to understand it when I read it.

4.3.2 Power limits

Power production might also limit the jump height. In the preceding analysis, energy is the limiting reagent: The jump height is determined by the energy that an animal can store in its muscles. However, even if the animal can store enough energy to reach that height, the muscles might not be able to deliver the energy rapidly enough. This section presents a simple model for the limit due to limited power generation.

Once again we'd like to find out how power P scales (varies) with the size l Power is energy per time, so the power required to jump to a height h is

$$P \sim \frac{\text{energy required to jump to height } h}{\text{time over which the energy is delivered}}.$$

The energy required is $E \sim mgh$. The mass is $m \propto l^3$. The gravitational acceleration is independent of l. And, in the energy-limited model, the height h is independent of l. Therefore $E \propto l^3$.

The delivery time is how long the animal is in contact with the ground, because only during contact can the ground exert a force on the animal. So, the animal crouches, extends upward, and finally leaves the ground. The contact time is the time during which the animal extends upward. Time is length over speed, so

$$t_{\text{delivery}} \sim \frac{\text{extension distance}}{\text{extension speed}}.$$

The extension distance is roughly the animal's size l. The extension speed is roughly the takeoff velocity. In the energy-limited model, the takeoff velocity is the same for all animals:

$$v_{\text{takeoff}} \propto h^{1/2} \propto l^0.$$

So

$$t_{\text{delivery}} \propto l.$$

The power required is $P \propto l^3/l = l^2$.

That proportionality is for the power itself, but a more interesting scaling is for the specific power: the power per mass. It is

Comments on page 1

Read about power limits and fleas (two subsections) for the Thursday memo.

I feel like this section just jumps into the math much more quickly than the other sections. In addition, I feel like the math is not being explained as much.

4.3.2 Power limits

Power production might al~~so limit the jump height. In the preceding~~ analysis, energy is the limiting reagent: The jump height is determined by the energy that an animal can store in its muscles. However, even if the animal can store enough energy to reach that height, the muscles might not be able to deliver the energy rapidly enough. This section presents a simple model for the limit due to limited power generation.

Once again we'd like to find out how power P scales (varies) with the size l Power is energy per time, so the power required to jump to a height h is

$$P \sim \frac{\text{energy required to jump to height } h}{\text{time over which the energy is delivered}}.$$

The energy required is $E \sim mgh$. The mass is $m \propto l^3$. The gravitational acceleration is independent of l. And, in the energy-limited model, the height h is independent of l. Therefore $E \propto l^3$.

The delivery time is how long the animal is in contact with the ground, because only during contact can the ground exert a force on the animal. So, the animal crouches, extends upward, and finally leaves the ground. The contact time is the time during which the animal extends upward. Time is length over speed, so

$$t_{\text{delivery}} \sim \frac{\text{extension distance}}{\text{extension speed}}.$$

The extension distance is roughly the animal's size l. The extension speed is roughly the takeoff velocity. In the energy-limited model, the takeoff velocity is the same for all animals:

$$v_{\text{takeoff}} \propto h^{1/2} \propto l^0.$$

So

$$t_{\text{delivery}} \propto l.$$

The power required is $P \propto l^3/l = l^2$.

That proportionality is for the power itself, but a more interesting scaling is for the specific power: the power per mass. It is

I thought about this in the last reading but didn't comment about it. Athletes with strong legs are not the ones who jump high, athletes who are "explosive" do. In sports, this means being able to provide a lot of force in a short amount of time, which in physics is obviously power. To summarize, I expect power to be a much better estimator of jump height than energy.

Slow twitch versus fast twitch

great way of saying this!

I don't know about slow twitch versus fast twitch... Baseball is the best example of a fast twitch sport, fielders stand around for minutes between sprints and hitters do the same. I would guess they have the worst verticals of all athletes.

That's because baseball doesn't require you to jump, but I bet they all have a bank robbers first step.

Also, I would strongly disagree with the verticals comment. Think about, for instance, Derek Jeter, or some similarly athletic player. Infielders especially need to jump high for line drives, something greatly effected by vertical jump.

I think that's debateable per se. For instance, slow twich muscles could still be stronger than fast twitch in two relatively similar individuals depending on training and composition. For instance, sprinters often do weight lifting on legs for training to gain speed, but I know some of us long distance runners are actually much faster out of the blocks.

That's unusual, in my experience. I'd say a well trained sprinter should beat out a well trained long distance runner any day. Although, do you mean faster to leave the blocks or going faster upon leaving them (or shortly thereafter)? The first is largely a function of reaction time, whereas the second is more a function of fast vs slow twitch and muscle strength.

Slow twitch muscle fibers aren't "stronger" than fast twitch muscle fibers and vice versa. The slow/fast twitch simply means how quickly individual muscle fibers are recruited to contract when a signal from the brain is given. Having more fast twitch results in recruiting more fibers at once (power) as opposed to slow twitch which recruit fibers over a longer period of time.

I think it was said earlier but these fiber affect power and not strength. You can lift weights and strengthen both muscle fibers....

4.3.2 Power limits

Power production might also limit the jump height. In the preceding analysis, energy is the limiting reagent: The jump height is determined by the energy that an animal can store in its muscles. However, even if the animal can store enough energy to reach that height, the muscles might not be able to deliver the energy rapidly enough. This section presents a simple model for the limit due to limited power generation.

Once again we'd like to find out how power P scales (varies) with the size l Power is energy per time, so the power required to jump to a height h is

$$P \sim \frac{\text{energy required to jump to height } h}{\text{time over which the energy is delivered}}.$$

The energy required is $E \sim mgh$. The mass is $m \propto l^3$. The gravitational acceleration is independent of l. And, in the energy-limited model, the height h is independent of l. Therefore $E \propto l^3$.

The delivery time is how long the animal is in contact with the ground, because only during contact can the ground exert a force on the animal. So, the animal crouches, extends upward, and finally leaves the ground. The contact time is the time during which the animal extends upward. Time is length over speed, so

$$t_{\text{delivery}} \sim \frac{\text{extension distance}}{\text{extension speed}}.$$

The extension distance is roughly the animal's size l. The extension speed is roughly the takeoff velocity. In the energy-limited model, the takeoff velocity is the same for all animals:

$$v_{\text{takeoff}} \propto h^{1/2} \propto l^0.$$

So

$$t_{\text{delivery}} \propto l.$$

The power required is $P \propto l^3/l = l^2$.

That proportionality is for the power itself, but a more interesting scaling is for the specific power: the power per mass. It is

I think this answers my question from last time about why some people can jump higher than others

This is probably why personal trainers recommend rapid repetitions over slower ones.

It depends on what you're doing, fast twitch muscle training involves quick repetitions which we need for explosive motions but if you are trying to build strength then heavy slow repetition is desired. For this example fast twitch muscles will get you more height but if you are trying to squat then you wouldn't want to train the same way.

This is something I never expected to learn in an estimation class!

I'm unsure of what you would not have expected. The reason it has to be rapid is to increase the power. Did you not know how to calculate power?

Kinda random but related to delivery speed: Read an interesting article somewhere about sprinters tending to have shorter moment arms for the achilles tendon. Though this decreased the moment that could be applied, it extended the duration of the "flexxing." Apparently muscles are better at applying force over time and this outweighed the negatives, from a reduced moment arm.

I think it would be nice if you wrote out "jump height limit" here to clarify what limit you are talking about.

yeah i wasn't sure what "limit" we are talking about here. Energy limits a lot of things that we can do.

Did you mean to have both words here? Could you just pick one?

I'd prefer 'varies'

4.3.2 Power limits

Power production might also limit the jump height. In the preceding analysis, energy is the limiting reagent: The jump height is determined by the energy that an animal can store in its muscles. However, even if the animal can store enough energy to reach that height, the muscles might not be able to deliver the energy rapidly enough. This section presents a simple model for the limit due to limited power generation.

Once again we'd like to find out how power P scales (varies) with the size l Power is energy per time, so the power required to jump to a height h is

$$P \sim \frac{\text{energy required to jump to height } h}{\text{time over which the energy is delivered}}.$$

The energy required is $E \sim mgh$. The mass is $m \propto l^3$. The gravitational acceleration is independent of l. And, in the energy-limited model, the height h is independent of l. Therefore $E \propto l^3$.

The delivery time is how long the animal is in contact with the ground, because only during contact can the ground exert a force on the animal. So, the animal crouches, extends upward, and finally leaves the ground. The contact time is the time during which the animal extends upward. Time is length over speed, so

$$t_{\text{delivery}} \sim \frac{\text{extension distance}}{\text{extension speed}}.$$

The extension distance is roughly the animal's size l. The extension speed is roughly the takeoff velocity. In the energy-limited model, the takeoff velocity is the same for all animals:

$$v_{\text{takeoff}} \propto h^{1/2} \propto l^0.$$

So

$$t_{\text{delivery}} \propto l.$$

The power required is $P \propto l^3/l = l^2$.

That proportionality is for the power itself, but a more interesting scaling is for the specific power: the power per mass. It is

Is the variable, l, the length/size of the muscle or animal?

I assume animal simply based on previous examples, but it isn't very clear.

Yeah it's not explicitly said here. It could be muscle size or animal height, and should be defined.

How big a difference would it make?

we've always used it as a characteristic length (or length scale) of the animal.

I also assumed it was size of the animal, because the previous paragraph doesn't mention anything on the size of the actual muscles, only how much energy the muscles can store.

@5:05 ... it would to the actual approximation, just to the general understanding of the process

just missing a "."

Where? I'm not sure what you're talking about.

between the "l" and the capitolized "Power"

I agree.

What about if the animal is designed to maximize the distance of its jump, can we consider that as well? I feel like maybe a grasshopper or something would jump far instead of high.

I was wondering about that as well... Is there any correlation between how high animals can jump and how far they can jump? If we assume everything is a cube, then that factor doesn't change in any direction.

I'd imagine there's a correlation between jump height and standing broad jump. However, if you take into account a run up like in the long jump, for example, speed becomes a major factor.

I'm sure there's a correlation. Since jumping forward and high involve a lot of the same muscles, it would be intuitive that they go together.

I'm sure there's a correlation. Since jumping forward and high involve a lot of the same muscles, it would be intuitive that they go together.

Is this the time before or during the jump?

During, i.e. while the muscles are delivering the power to propel the animal upwards.

4.3.2 Power limits

Power production might also limit the jump height. In the preceding analysis, energy is the limiting reagent: The jump height is determined by the energy that an animal can store in its muscles. However, even if the animal can store enough energy to reach that height, the muscles might not be able to deliver the energy rapidly enough. This section presents a simple model for the limit due to limited power generation.

Once again we'd like to find out how power P scales (varies) with the size l Power is energy per time, so the power required to jump to a height h is

$$P \sim \frac{\text{energy required to jump to height } h}{\text{time over which the energy is delivered}}.$$

The energy required is $E \sim mgh$. The mass is $m \propto l^3$. The gravitational acceleration is independent of l. And, in the energy-limited model, the height h is independent of l. Therefore $E \propto l^3$.

The delivery time is how long the animal is in contact with the ground, because only during contact can the ground exert a force on the animal. So, the animal crouches, extends upward, and finally leaves the ground. The contact time is the time during which the animal extends upward. Time is length over speed, so

$$t_{\text{delivery}} \sim \frac{\text{extension distance}}{\text{extension speed}}.$$

The extension distance is roughly the animal's size l. The extension speed is roughly the takeoff velocity. In the energy-limited model, the takeoff velocity is the same for all animals:

$$v_{\text{takeoff}} \propto h^{1/2} \propto l^0.$$

So

$$t_{\text{delivery}} \propto l.$$

The power required is $P \propto l^3/l = l^2$.

That proportionality is for the power itself, but a more interesting scaling is for the specific power: the power per mass. It is

Trying to apply this proportion to humans... m (2m)^3 seems a lot bigger than a human's actual volume...

True, but maybe it gets close enough since we would just round it to few or 10^1 anyway...

Why is m l^3? Shouldn't we account for density? I'm assuming that p = 1 which is why m l^3, but could someone please explain this to me?

Density for all living creatures is usually bout water weight

...and volume is dependent on meters squared.

Er, no, volume has units of meters cubed. Density is the reason why it's only proportional to, and not equal to. We're not really assuming rho is 1, we're just ignoring it entirely, assuming it's pretty much the same for all animals.

Thanks, that makes sense now: rho is ignored since it's the same for all animals.

In that case, it might be helpful if that was explicitly stated in the notes. I know it may seem like overkill, but I would have thought it beneficial.

To clarify the above comment, m is proportional to l^3 partly because we are omitting a constant density, but also because a person or animal's volume isn't necessarily equal to l^3. l is just a characteristic length that we can use to compare the sizes of different animals.

it's talking about proportionality. mass is proportional with size. density is an unchanging factor.

Shouldn't we be calculating "l" as say the length of a person bent and folded into a cubical shape, like he did in the class example?

Yeah, that's what I would guess. I think we used the value of .5m in class to represent l for a human. It probably evens out the same as if we were to calculate l*w*h but in a more simple way.

4.3.2 Power limits

Power production might also limit the jump height. In the preceding analysis, energy is the limiting reagent: The jump height is determined by the energy that an animal can store in its muscles. However, even if the animal can store enough energy to reach that height, the muscles might not be able to deliver the energy rapidly enough. This section presents a simple model for the limit due to limited power generation.

Once again we'd like to find out how power P scales (varies) with the size l Power is energy per time, so the power required to jump to a height h is

$$P \sim \frac{\text{energy required to jump to height } h}{\text{time over which the energy is delivered}}.$$

The energy required is $E \sim mgh$. The mass is $m \propto l^3$. The gravitational acceleration is independent of l. And, in the energy-limited model, the height h is independent of l. Therefore $E \propto l^3$.

The delivery time is how long the animal is in contact with the ground, because only during contact can the ground exert a force on the animal. So, the animal crouches, extends upward, and finally leaves the ground. The contact time is the time during which the animal extends upward. Time is length over speed, so

$$t_{\text{delivery}} \sim \frac{\text{extension distance}}{\text{extension speed}}.$$

The extension distance is roughly the animal's size l. The extension speed is roughly the takeoff velocity. In the energy-limited model, the takeoff velocity is the same for all animals:

$$v_{\text{takeoff}} \propto h^{1/2} \propto l^0.$$

So

$$t_{\text{delivery}} \propto l.$$

The power required is $P \propto l^3/l = l^2$.

That proportionality is for the power itself, but a more interesting scaling is for the specific power: the power per mass. It is

why a squiggle? isn't this a law of physics?

I was wondering that too. But if I had to guess, it would be because the practice of estimation has so ingrained in us a mentality to go for proportions and relations rather than exact equations, that even when faced with a known equation, the habit of using squiggles still carries over.

I thought that just means we don't have to worry about any constants.

There are a lot of factors and terms that we are neglecting. Drag, muscle efficiency, energy spent on balancing, etc.. But roughly speaking, E goes like mgh.

I get that we don't have to worry about them when we are trying to find out proportionality, but if we later use the results to plug in numbers, do we add them back?

This section is about proportionality, if you haven't noticed, all physics related equations are simplified to a proportionality.

how is h independent of l, I thought l was length I feel like they would be connected

It would be really helpful to have a table of all the proportionalities. I keep forgetting the ones from the previous sections.

I feel like having a large table of proportionalities would kind of defeat the purpose. The ideas is to gain experience with proportional reasoning so that we can apply it to many situations, not just those situations which have already been covered.

Well if the table of proportionalities would kill the purpose, can we at least get a table of equations?

I agree with not having a table. I feel like once you take a moment to really think about it, something clicks and you understand it. Rho is same for all animals so the relationship exists in this case. However I find that I need to see it once - it's not natural yet but practice will only help so I wont to be forced to think about it.

how would you even estimate this time for each type of animal?

even when it's straightening its legs and pushing against the ground itself?

4.3.2 Power limits

Power production might also limit the jump height. In the preceding analysis, energy is the limiting reagent: The jump height is determined by the energy that an animal can store in its muscles. However, even if the animal can store enough energy to reach that height, the muscles might not be able to deliver the energy rapidly enough. This section presents a simple model for the limit due to limited power generation.

Once again we'd like to find out how power P scales (varies) with the size l Power is energy per time, so the power required to jump to a height h is

$$P \sim \frac{\text{energy required to jump to height } h}{\text{time over which the energy is delivered}}.$$

The energy required is $E \sim mgh$. The mass is $m \propto l^3$. The gravitational acceleration is independent of l. And, in the energy-limited model, the height h is independent of l. Therefore $E \propto l^3$.

The delivery time is how long the animal is in contact with the ground, because only during contact can the ground exert a force on the animal. So, the animal crouches, extends upward, and finally leaves the ground. The contact time is the time during which the animal extends upward. Time is length over speed, so

$$t_{\text{delivery}} \sim \frac{\text{extension distance}}{\text{extension speed}}.$$

The extension distance is roughly the animal's size l. The extension speed is roughly the takeoff velocity. In the energy-limited model, the takeoff velocity is the same for all animals:

$$v_{\text{takeoff}} \propto h^{1/2} \propto l^0.$$

So

$$t_{\text{delivery}} \propto l.$$

The power required is $P \propto l^3/l = l^2$.

That proportionality is for the power itself, but a more interesting scaling is for the specific power: the power per mass. It is

A previous comment suggested that the time over which the energy is delivered occurs during the jump, not before. However, this makes it sound that the energy comes from before the jump until takeoff, which is the case?

Are we assuming that the same force is being exerted on the animal whenever the animal is on the ground? It would seem to me that this force is related to the surface area of the animal that is touching the ground. So, as the animal gets closer to leaving the ground and only it's toes, for instance, are touching the ground, less force is exerted on the animal.

I'm pretty sure that we can assume that the force applied is constant throughout the whole jumping process.

This is really interesting. I have always wonder how you can do measurements like these.

From 2.671 go forth, the constant force approximation for jumping is not far off from what's actually happening. For those curious, this is pretty easy to measure using a force plate.

So is delivery time the same thing as contact time here?

Yes, I believe that is what this is saying.

Would it be better to say distance here since that's the more typical variable and the following equation uses "extension distance" anyways?

i don't really understand this value. what are we defining as "extension" here. The crouching down and then back up again?

Yes. I believe the "extension length" is the difference between crouched height and fully extended height, which is the distance over which the animal exerts force on the ground.

This explains my earlier question about delivery time.

I like this method of finding the "lag time" between telling your muscles to jump and actually leaving the ground.

4.3.2 Power limits

Power production might also limit the jump height. In the preceding analysis, energy is the limiting reagent: The jump height is determined by the energy that an animal can store in its muscles. However, even if the animal can store enough energy to reach that height, the muscles might not be able to deliver the energy rapidly enough. This section presents a simple model for the limit due to limited power generation.

Once again we'd like to find out how power P scales (varies) with the size l Power is energy per time, so the power required to jump to a height h is

$$P \sim \frac{\text{energy required to jump to height } h}{\text{time over which the energy is delivered}}.$$

The energy required is $E \sim mgh$. The mass is $m \propto l^3$. The gravitational acceleration is independent of l. And, in the energy-limited model, the height h is independent of l. Therefore $E \propto l^3$.

The delivery time is how long the animal is in contact with the ground, because only during contact can the ground exert a force on the animal. So, the animal crouches, extends upward, and finally leaves the ground. The contact time is the time during which the animal extends upward. Time is length over speed, so

$$t_{\text{delivery}} \sim \frac{\text{extension distance}}{\text{extension speed}}.$$

The extension distance is roughly the animal's size l. The extension speed is roughly the takeoff velocity. In the energy-limited model, the takeoff velocity is the same for all animals:

$$v_{\text{takeoff}} \propto h^{1/2} \propto l^0.$$

So

$$t_{\text{delivery}} \propto l.$$

The power required is $P \propto l^3/l = l^2$.

That proportionality is for the power itself, but a more interesting scaling is for the specific power: the power per mass. It is

How is the extension distance roughly the animal's size? If extension is simply motion before leaving the ground, it would have to be more like half the animals size.

I agree, maybe it's since we're just estimating and we know it's not going to extend any MORE than its own size.

Movement through the distance is not constant, but goes through a lot of acceleration. Are we trying to take an average speed here?

Yes, I believe we are taking an average speed just to simplify the calculation.

So we are allowed to make this approximation because the extension distance is proportional to the animal's size? Therefore, we can throw out any constant factor like extension distance is 1/2*L?

I feel like this is a little funny because there is certainly a constant factor being ignored here. In the end, should be same order of magnitude...

Ahh, the magic of the "proportional to" sign.

Yeah, in the last few readings we've ignored the constant terms. this is because we're interested in proportionality and not concrete values.

I agree with you guys, it reads a little funny but still makes sense. Maybe it would be a bit clearer if it notes the extension distance is a function of the animals length, and then use that proportionality?

It seems to make sense: think of a cat jumping from a crouch. Right before they leave the ground they're fully extended with only their back legs on the ground, so they're at their full height. Then the extension hieght is that full height minus their crouched height... and you can assume that more most animals their crouched height is some fraction of their full height and assume that that fraction is the same for most animals... So the proportionality works out. I think.

But how does that work for humans? I could believe that extension length is maybe half of our body size. This is counter intuitive if you picture someone jumping. Ah approximation.

However, if you consider the l to be 0.5 m like mentioned earlier when we were talking about human volume then it is not that difficult to imagine extending you legs from a folded stance about 0.5m.

4.3.2 Power limits

Power production might also limit the jump height. In the preceding analysis, energy is the limiting reagent: The jump height is determined by the energy that an animal can store in its muscles. However, even if the animal can store enough energy to reach that height, the muscles might not be able to deliver the energy rapidly enough. This section presents a simple model for the limit due to limited power generation.

Once again we'd like to find out how power P scales (varies) with the size l Power is energy per time, so the power required to jump to a height h is

$$P \sim \frac{\text{energy required to jump to height } h}{\text{time over which the energy is delivered}}.$$

The energy required is $E \sim mgh$. The mass is $m \propto l^3$. The gravitational acceleration is independent of l. And, in the energy-limited model, the height h is independent of l. Therefore $E \propto l^3$.

The delivery time is how long the animal is in contact with the ground, because only during contact can the ground exert a force on the animal. So, the animal crouches, extends upward, and finally leaves the ground. The contact time is the time during which the animal extends upward. Time is length over speed, so

$$t_{\text{delivery}} \sim \frac{\text{extension distance}}{\text{extension speed}}.$$

The extension distance is roughly the animal's size l. The extension speed is roughly the takeoff velocity. In the energy-limited model, the takeoff velocity is the same for all animals:

$$v_{\text{takeoff}} \propto h^{1/2} \propto l^0.$$

So

$$t_{\text{delivery}} \propto l.$$

The power required is $P \propto l^3/l = l^2$.

That proportionality is for the power itself, but a more interesting scaling is for the specific power: the power per mass. It is

I wouldn't have thought this was so similar at first, but when I think about it now - it does make some sense (to first order of course), but I'm trying to think about what factors go into this.

I would have thought this was a bit faster than the extension speed, just because I feel animals undergo an acceleration while extending, and not quite a constant velocity.

Yeah, but you push off the ground when you jump. Are we assuming that that pushing contributes to the acceleration in the extension or the jump itself?

Animals do undergo an acceleration when extending, but my thought is that it the acceleration happens in very little distance, so the average speed might be comparable to the takeoff velocity. What do you think?

What model is this referring to?

why can we make this assumption? it seems like takeoff velocity would vary a lot among different animals

Yeah I also feel like this could affect the delivery time.

4.3.2 Power limits

Power production might also limit the jump height. In the preceding analysis, energy is the limiting reagent: The jump height is determined by the energy that an animal can store in its muscles. However, even if the animal can store enough energy to reach that height, the muscles might not be able to deliver the energy rapidly enough. This section presents a simple model for the limit due to limited power generation.

Once again we'd like to find out how power P scales (varies) with the size l Power is energy per time, so the power required to jump to a height h is

$$P \sim \frac{\text{energy required to jump to height } h}{\text{time over which the energy is delivered}}.$$

The energy required is $E \sim mgh$. The mass is $m \propto l^3$. The gravitational acceleration is independent of l. And, in the energy-limited model, the height h is independent of l. Therefore $E \propto l^3$.

The delivery time is how long the animal is in contact with the ground, because only during contact can the ground exert a force on the animal. So, the animal crouches, extends upward, and finally leaves the ground. The contact time is the time during which the animal extends upward. Time is length over speed, so

$$t_{delivery} \sim \frac{\text{extension distance}}{\text{extension speed}}.$$

The extension distance is roughly the animal's size l. The extension speed is roughly the takeoff velocity. In the energy-limited model, the takeoff velocity is the same for all animals:

$$v_{takeoff} \propto h^{1/2} \propto l^0.$$

So

$$t_{delivery} \propto l.$$

The power required is $P \propto l^3/l = l^2$.

That proportionality is for the power itself, but a more interesting scaling is for the specific power: the power per mass. It is

Where does this come from? Is this from an earlier reading? If not, I think it needs more explanation because I'm confused as to why this is.

I agree. I don't think it is from a previous reading. And its not a clearly intuitive relation that we can just assume.

A mention of the conservation chapter and kinetic/potential energy conservation would be nice here, since it's a concept that comes up a lot, like in the roller coaster design example.

I too am a bit confused where this came from exactly

I think this was mentioned briefly before, but a refresher would be nice. I believe it comes from setting the kinetic takeoff energy to the potential energy of the jump height: v^2=gh. That v_takeoff is the same for all animals follows from our conclusion that all animals jump to the same height.

can you conclude that the takeoff velocities are the same from the fact that they jump to the same height?

Generally, I would say yes. If we set KE=PE, mv^2=mgh, cancel the m's and v is proportional to h^1/2. This is the equation he has here, and this is where it comes from. It makes sense since all animals have the same acceleration to contend with, they must start at similar velocities to get to similar heights.

great explanation, perhaps a little refresher here would still be good though.

great explanation, perhaps a little refresher here would still be good though.

great explanation, perhaps a little refresher here would still be good though.

great explanation, perhaps a little refresher here would still be good though.

great explanation, perhaps a little refresher here would still be good though.

great explanation, perhaps a little refresher here would still be good though.

Great comment. This helped me understand.

confusing derivation

4.3.2 Power limits

Power production might also limit the jump height. In the preceding analysis, energy is the limiting reagent: The jump height is determined by the energy that an animal can store in its muscles. However, even if the animal can store enough energy to reach that height, the muscles might not be able to deliver the energy rapidly enough. This section presents a simple model for the limit due to limited power generation.

Once again we'd like to find out how power P scales (varies) with the size l Power is energy per time, so the power required to jump to a height h is

$$P \sim \frac{\text{energy required to jump to height } h}{\text{time over which the energy is delivered}}.$$

The energy required is $E \sim mgh$. The mass is $m \propto l^3$. The gravitational acceleration is independent of l. And, in the energy-limited model, the height h is independent of l. Therefore $E \propto l^3$.

The delivery time is how long the animal is in contact with the ground, because only during contact can the ground exert a force on the animal. So, the animal crouches, extends upward, and finally leaves the ground. The contact time is the time during which the animal extends upward. Time is length over speed, so

$$t_{\text{delivery}} \sim \frac{\text{extension distance}}{\text{extension speed}}.$$

The extension distance is roughly the animal's size l. The extension speed is roughly the takeoff velocity. In the energy-limited model, the takeoff velocity is the same for all animals:

$$v_{\text{takeoff}} \propto h^{1/2} \propto l^0.$$

So

$$t_{\text{delivery}} \propto l.$$

The power required is $P \propto l^3/l = l^2$.

That proportionality is for the power itself, but a more interesting scaling is for the specific power: the power per mass. It is

I'm not sure I quite understand the 0 exponent...am I missing something obvious?

Me too... Doesn't l^0 = 1? Also what is h? The length extension? I fee like the variables here weren't defined properly.

The 0 is hard to read in this font at the magnification I normally use.

Shouldn't you just magnify it more then?

This is only a valid complaint, if when you print the file on actual paper (where you don't have the luxury of magnifying), it is still too small to read.

What is h again?

ohh yeah, it's the height

I like how you take time to spell this out. Walking through it really helps.

I understand that this comes from the above equation - it might be helpful to make this more explicit (as in, directly plugging in the proportions) to make this easier to follow.

It took me a few minutes to see that relationship. I agree that a quick step-by-step showing the plug-in values would help.

I think it definitely makes sense intuitively, smaller animals will take less time to extend in comparison to larger animals.

It's kind of ironic thinking about how something like power is only dependent on length. This method is pretty cool.

4.3.2 Power limits

Power production might also limit the jump height. In the preceding analysis, energy is the limiting reagent: The jump height is determined by the energy that an animal can store in its muscles. However, even if the animal can store enough energy to reach that height, the muscles might not be able to deliver the energy rapidly enough. This section presents a simple model for the limit due to limited power generation.

Once again we'd like to find out how power P scales (varies) with the size l Power is energy per time, so the power required to jump to a height h is

$$P \sim \frac{\text{energy required to jump to height } h}{\text{time over which the energy is delivered}}.$$

The energy required is $E \sim mgh$. The mass is $m \propto l^3$. The gravitational acceleration is independent of l. And, in the energy-limited model, the height h is independent of l. Therefore $E \propto l^3$.

The delivery time is how long the animal is in contact with the ground, because only during contact can the ground exert a force on the animal. So, the animal crouches, extends upward, and finally leaves the ground. The contact time is the time during which the animal extends upward. Time is length over speed, so

$$t_{\text{delivery}} \sim \frac{\text{extension distance}}{\text{extension speed}}.$$

The extension distance is roughly the animal's size l. The extension speed is roughly the takeoff velocity. In the energy-limited model, the takeoff velocity is the same for all animals:

$$v_{\text{takeoff}} \propto h^{1/2} \propto l^0.$$

So

$$t_{\text{delivery}} \propto l.$$

The power required is $P \propto l^3/l = l^2$.

That proportionality is for the power itself, but a more interesting scaling is for the specific power: the power per mass. It is

I am still not comfortable with being able to drop most of the values in an equation and say that energy required to jump is proportional to l^3 so let's only use this value. Why can you do this?

Proportionality is about the relations between quantities. We're not pretending that the power IS length squared, we're saying that as length changes, the power increases by the square of that change. One simple example I like to come back to is surface area to volume ratio, which goes as 1/L. Try to think about how the quantities affect each other, as if you were moving a slider for L and watching the change in the power. Ignore the absolute values, which are affected by constant, but constants are just that, they stay the same regardless of scale.

I don't know if you can include some sort of comments section in the book but I have found that these comments really make the material much more transparent and easier to understand. I think finding a way to compile/summarize the comments and present them along with the book text would make the book GREAT!!! Maybe include them as little hints in the margins similar to the way some high school science books have random bits of information thrown in.

I think I'm beginning to really understand this idea now...to try to figure out proportionally how things compare, and commonly seeing how "per mass" factors in is quite helpful!

$$\frac{P}{m} \propto \frac{l^2}{l^3} = l^{-1}.$$

Ah, smaller animals need a higher specific power!

A model for power limits is that all muscle can generate the same maximum power density (has the same maximum specific power). So a small-enough animal cannot jump to its energy-limited height. The animal can store enough energy in its muscles, but cannot release it quickly enough.

More precisely, it cannot do so unless it finds an alternative method for releasing the energy. The click beetle, which is toward the small end in the preceding graph and data set, uses the following solution. It stores energy in its shell by bending the shell, and maintains the bending like a ratchet would (holding a structure motionless does require energy). This storage can happen slowly enough to avoid the specific-power limit, but when the beetle releases the shell and the shell snaps back to its resting position, the energy is released quickly enough for the beetle to rise to its energy-limited height.

But that height is less than the height for locusts and humans. Indeed, the largest deviations from the constant-height result happen at the low-mass end, for fleas and click beetles. To explain that discrepancy, the model needs to take into account another physical effect: drag.

4.4 Drag

4.4.1 Jumping fleas

The drag force

$$F \sim \rho A v^2$$

affects the jumps of small animals more than it affects the jumps of people. A comparison of the energy required for the jump with the energy consumed by drag explains why.

The energy that the animal requires to jump to a height h is mgh, if we use the gravitational potential energy at the top of the jump; or it is $\sim mv^2$, if we use the kinetic energy at takeoff. The energy consumed by drag is

I think this was mentioned before too... having the equation and the introduction sentence on the same page is always better.

What is specific power?

A "specific" version of a quantity is that quantity divided by the mass.

I still don't get it.

I think it's basically the power it needs divided by its mass.

specific power= power/mass

specific power = Power/Mass. Just like specific heat = (Heat Capacity)/Mass, specific volume = Volume/Mass (inverse of density), etc.

This explanation helped out a lot...as someone that hasn't dealt with any of those terms in a long time the idea of specific power confused me

Thanks for the explanation! I think it would be helpful to put a single line in for this in the reading.

By dividing by mass, it basically "normalizes" the quantity to relative to the nature of the object.

is specific power inittial power?

maybe its same as power density or power per unit of mass

yeah, what he's saying here is that we should look at specific power–power per mass. this happens to be proportional to 1/l since p is proporitonal to area (or l^2) and m is proportional to volume (l^3). divide area by volume and you get 1/l.

This sentence is just confusing to me.

Since P/m is proportional to L^-1 it means that smaller animals (those with small "L's") will have higher P/m to generate the P required to jump. Hope this helps.

$$\frac{P}{m} \propto \frac{l^2}{l^3} = l^{-1}.$$

Ah, smaller animals need a higher specific power!

A model for power limits is that all muscle can generate the same maximum power density (has the same maximum specific power). So a small-enough animal cannot jump to its energy-limited height. The animal can store enough energy in its muscles, but cannot release it quickly enough.

More precisely, it cannot do so unless it finds an alternative method for releasing the energy. The click beetle, which is toward the small end in the preceding graph and data set, uses the following solution. It stores energy in its shell by bending the shell, and maintains the bending like a ratchet would (holding a structure motionless does require energy). This storage can happen slowly enough to avoid the specific-power limit, but when the beetle releases the shell and the shell snaps back to its resting position, the energy is released quickly enough for the beetle to rise to its energy-limited height.

But that height is less than the height for locusts and humans. Indeed, the largest deviations from the constant-height result happen at the low-mass end, for fleas and click beetles. To explain that discrepancy, the model needs to take into account another physical effect: drag.

4.4 Drag

4.4.1 Jumping fleas

The drag force

$$F \sim \rho A v^2$$

affects the jumps of small animals more than it affects the jumps of people. A comparison of the energy required for the jump with the energy consumed by drag explains why.

The energy that the animal requires to jump to a height h is mgh, if we use the gravitational potential energy at the top of the jump; or it is $\sim mv^2$, if we use the kinetic energy at takeoff. The energy consumed by drag is

This is somewhat surprising.

Really? While reading this section I had a mental image of a lever arm in contact with the ground. The longer the arm, the more contact time, so the less specific power needed.

I also remember hearing from elementary school or maybe Bill Nye that if we had the same power as an ant proportional to its size, we could lift cars above our heads.

i <3 bill nye

I don't know how the Bill Nye comment relates to this context.

This passage is saying that smaller animals need a higher specific power to reach the same height, putting them at a disadvantage. The Bill Nye comment says that smaller animals, like ants, have it easier.

I think Bill Nye was referring to energy. If that were the case, then he'd be right. In fact, we would probably be able to lift more than a car, which is only the weight of 10 times our current lifting capacity.

Agreed, it makes sense, but you have to think about it more

What exactly is considered a "small animal"?

I think it's just setting up a relationship (i.e. as size decreases, required specific power increases.)

We've talked about animals as small as fleas, so I guess this could refer to bugs?

Does this ratio also show that smaller animals are more capable of moving quicker and generally being stronger because mass will have less of an affect on power because of the dimension ratio?

It does seem to be correlated. It is interesting though, that you can consider animals like ants that can carry items many times their weight.

Or as we see below, jumping fleas!

at what point does a "smaller" animal do have enough power? for example, are frogs big enough?

$$\frac{P}{m} \propto \frac{l^2}{l^3} = l^{-1}.$$

Ah, smaller animals need a higher specific power!

A model for power limits is that all muscle can generate the same maximum power density (has the same maximum specific power). So a small-enough animal cannot jump to its energy-limited height. The animal can store enough energy in its muscles, but cannot release it quickly enough.

More precisely, it cannot do so unless it finds an alternative method for releasing the energy. The click beetle, which is toward the small end in the preceding graph and data set, uses the following solution. It stores energy in its shell by bending the shell, and maintains the bending like a ratchet would (holding a structure motionless does require energy). This storage can happen slowly enough to avoid the specific-power limit, but when the beetle releases the shell and the shell snaps back to its resting position, the energy is released quickly enough for the beetle to rise to its energy-limited height.

But that height is less than the height for locusts and humans. Indeed, the largest deviations from the constant-height result happen at the low-mass end, for fleas and click beetles. To explain that discrepancy, the model needs to take into account another physical effect: drag.

4.4 Drag

4.4.1 Jumping fleas

The drag force

$$F \sim \rho A v^2$$

affects the jumps of small animals more than it affects the jumps of people. A comparison of the energy required for the jump with the energy consumed by drag explains why.

The energy that the animal requires to jump to a height h is mgh, if we use the gravitational potential energy at the top of the jump; or it is $\sim mv^2$, if we use the kinetic energy at takeoff. The energy consumed by drag is

do you mean large enough?

i agree that this is confusing. i think that it's saying that if small animals need more power, but we all produce the same amount of power, so they wouldn't have enough. it depends on your reference point i suppose.

he's not arguing at all that all animals produce the same amount of power (and that would be false). Power required scales like l^2, but specific power required scales like l^-1. So as l decreases, specific power required increases.

Small here refers to length, not mass. (Yes, I know these things have their own relationship, but the key is that P/m gets too high to achieve physically for animals as their L decreases.

this still doesn't seem right- large animals are not always the highest jumpers

I guess speed is included in the analysis, but it seems as though time specifically should have been added, since it's analyzing the amount of time over which energy is released.

This is interesting - why does this apply only to small animals?

Why is it then that some bugs can jump so high?

Never mind I should have read a couple sentences further

why is this? does is relate to muscle size?

I'm not sure if this is correct but maybe we can make an analogy to gears. If a small gear wants to release the same amount of energy as a big gear it has to spin really fast. Small animals, since they have smaller "L's" can "spin fast enough" to keep up with animals with bigger "L's". Therefore they can't release their energy fast enough and thus can't jump as high.

Did I miss this graph and data set?

Its at the end of reading 15.

I believe it was in the previous reading memo.

Sort of like skiing, you get energy out of the camber of the ski and then it releases the energy to shoot you into the next turn.

$$\frac{P}{m} \propto \frac{l^2}{l^3} = l^{-1}.$$

Ah, smaller animals need a higher specific power!

A model for power limits is that all muscle can generate the same maximum power density (has the same maximum specific power). So a small-enough animal cannot jump to its energy-limited height. The animal can store enough energy in its muscles, but cannot release it quickly enough.

More precisely, it cannot do so unless it finds an alternative method for releasing the energy. The click beetle, which is toward the small end in the preceding graph and data set, uses the following solution. It stores energy in its shell by bending the shell, and maintains the bending like a ratchet would (holding a structure motionless does require energy). This storage can happen slowly enough to avoid the specific-power limit, but when the beetle releases the shell and the shell snaps back to its resting position, the energy is released quickly enough for the beetle to rise to its energy-limited height.

But that height is less than the height for locusts and humans. Indeed, the largest deviations from the constant-height result happen at the low-mass end, for fleas and click beetles. To explain that discrepancy, the model needs to take into account another physical effect: drag.

4.4 Drag

4.4.1 Jumping fleas

The drag force

$$F \sim \rho A v^2$$

affects the jumps of small animals more than it affects the jumps of people. A comparison of the energy required for the jump with the energy consumed by drag explains why.

The energy that the animal requires to jump to a height h is mgh, if we use the gravitational potential energy at the top of the jump; or it is $\sim mv^2$, if we use the kinetic energy at takeoff. The energy consumed by drag is

that's really cool, but just goes to show that our estimations don't account for everything, and is not a reliable.

Rachets maintain posistion and/or prevent movement in the opposite direction, but not necessarily with bending or stored energy. This is more like a leaf spring.

This seems like it should either be made more important and removed from parentheses/explained more thoroughly or dropped from the text - as-is it doesn't seem to add to the example and adds one more thing to think about

Doesn't that depend on how one holds it motionless? Letting a weight sit on a table requires no energy, but holding it above the table does?

To me, a ratchet mechanism indicates not requiring energy to hold something motionless. Isn't that why people often use ratchets? To keep a shaft or a rope or something else from slipping without requiring energy?

Unless you're making a subtle distinction between energy and power (in some cases takes initial energy but no constant power to hold something motionless), I don't see how this statement is necessarily true.

Could "energy" be referring to potential energy? in the tension of the shell or whatever restoring force is acting on the beetle's shell when it is ratcheted out of resting position. That would make more sense to me.

interesting way of relating this example to a well-known mechanical tool!

what is specific power limit again?

That's what he just derived: P/m = l^-1

along these lines, I always wonder where are the bounds for which these "animals" actually follow the predicted outcome (i.e. jump height etc.) all I can think about are the exceptions, I would like more examples of what animals they actually work for

so I looked these up on you-tube and they are pretty awesome

thats amazing

$$\frac{P}{m} \propto \frac{l^2}{l^3} = l^{-1}.$$

Ah, smaller animals need a higher specific power!

A model for power limits is that all muscle can generate the same maximum power density (has the same maximum specific power). So a small-enough animal cannot jump to its energy-limited height. The animal can store enough energy in its muscles, but cannot release it quickly enough.

More precisely, it cannot do so unless it finds an alternative method for releasing the energy. The click beetle, which is toward the small end in the preceding graph and data set, uses the following solution. It stores energy in its shell by bending the shell, and maintains the bending like a ratchet would (holding a structure motionless does require energy). This storage can happen slowly enough to avoid the specific-power limit, but when the beetle releases the shell and the shell snaps back to its resting position, the energy is released quickly enough for the beetle to rise to its energy-limited height.

But that height is less than the height for locusts and humans. Indeed, the largest deviations from the constant-height result happen at the low-mass end, for fleas and click beetles. To explain that discrepancy, the model needs to take into account another physical effect: drag.

4.4 Drag

4.4.1 Jumping fleas

The drag force

$$F \sim \rho A v^2$$

affects the jumps of small animals more than it affects the jumps of people. A comparison of the energy required for the jump with the energy consumed by drag explains why.

The energy that the animal requires to jump to a height h is mgh, if we use the gravitational potential energy at the top of the jump; or it is $\sim mv^2$, if we use the kinetic energy at takeoff. The energy consumed by drag is

That's really clever.

This is cool to think about. How much difference does this alternative energy source make in the beetle's overall jump height?

Wow, that's really awesome!

It's an alternate power delivery system, energy is still conserved, it's just storing it up over a longer time and releasing it over a shorter one, like a capacitor, to bring in the electrical analogy from earlier in the course.

so then I guess since this release in energy is large compared to the total energy needed to propel the beetle far (in relation to its body size), it can jump really far. so that's why larger animals can't use this right–the energy supplied from this type of shell snapping is actually small, but large compared to the energy needed to move the beetle

there are a lot of really sweet videos of the beetle...one of the first ones I saw is http://www.youtube.co

Nice find! Interesting solution to the energy limit.

awesome. this sounds really interesting.

Awesome. I was just going to go look for something like this.

This is really cool! Are there other animal examples of this phenomenon?

Would the number of legs be proportional to the amount of power that can be delivered? For example, would increasing the number of legs allow for greater power delivery?

It might be nice to say that small animals can't jump as high rather than just calling it a discrepancy.

It's a discrepancy between our calculations and actual observations.

I like that the concepts we learn earlier come back and are linked to the current topics.

yeah I agree, it's a really good way to tie all the concepts together and make sure we don't forget them!

I feel like this comes up really often in our examples. Is that just this unit, or approximation in general or this type of problem or what?

$$\frac{P}{m} \propto \frac{l^2}{l^3} = l^{-1}.$$

Ah, smaller animals need a higher specific power!

A model for power limits is that all muscle can generate the same maximum power density (has the same maximum specific power). So a small-enough animal cannot jump to its energy-limited height. The animal can store enough energy in its muscles, but cannot release it quickly enough.

More precisely, it cannot do so unless it finds an alternative method for releasing the energy. The click beetle, which is toward the small end in the preceding graph and data set, uses the following solution. It stores energy in its shell by bending the shell, and maintains the bending like a ratchet would (holding a structure motionless does require energy). This storage can happen slowly enough to avoid the specific-power limit, but when the beetle releases the shell and the shell snaps back to its resting position, the energy is released quickly enough for the beetle to rise to its energy-limited height.

But that height is less than the height for locusts and humans. Indeed, the largest deviations from the constant-height result happen at the low-mass end, for fleas and click beetles. To explain that discrepancy, the model needs to take into account another physical effect: drag.

4.4 Drag

4.4.1 Jumping fleas

The drag force

$F \sim \rho A v^2$

affects the jumps of small animals more than it affects the jumps of people. A comparison of the energy required for the jump with the energy consumed by drag explains why.

The energy that the animal requires to jump to a height h is mgh, if we use the gravitational potential energy at the top of the jump; or it is $\sim mv^2$, if we use the kinetic energy at takeoff. The energy consumed by drag is

This always seems to come back!

And i never would have considered it in this case, were it not presented to me. Although when I think about it, it makes sense that light enough animals would be effected.

Yeah this also seemed to come out of nowhere for me too. You figure we would get better at knowing when to consider it since we've used it a few times already.

So since the area scales with L^2 are we going to show that drag is the limiting factor for larger animals? Wouldn't the surface area to volume ratio go up for small animals and then they would have a larger effect from drag?

Huh, that's surprising, I would have thought it to be the opposite, since smaller creatures have less surface area...

Yes, but I think they're also less dense. (Though, come to think of it, if we're all comprised of the same materials, then it doesn't matter). But something tells me the very very small mass, relative to the surface area causes a problem. F pAv^2, but p m/V, so m/some L (thickness maybe?) probably shows that m increases much faster than an L. Just my two cents; I could be completely wrong.

I would have thought the exact same thing.

but its interesting that the force (in the equation above) has no dependence on mass. or do you mean small in terms of size? then it makes more sense because F depends on the area

I think this refers to area.

don't small animals have less area, so why then does the drag force affect them more

My first reaction here was: "but why", and then thought about it for a minute or two before realizing you explained it below. Of course that's my fault, but it gave me pause while reading it.

For the first time ever, we were given a bried synopsis or abstract of the theory about to be presented. I like it.

$$\frac{P}{m} \propto \frac{l^2}{l^3} = l^{-1}.$$

Ah, smaller animals need a higher specific power!

A model for power limits is that all muscle can generate the same maximum power density (has the same maximum specific power). So a small-enough animal cannot jump to its energy-limited height. The animal can store enough energy in its muscles, but cannot release it quickly enough.

More precisely, it cannot do so unless it finds an alternative method for releasing the energy. The click beetle, which is toward the small end in the preceding graph and data set, uses the following solution. It stores energy in its shell by bending the shell, and maintains the bending like a ratchet would (holding a structure motionless does require energy). This storage can happen slowly enough to avoid the specific-power limit, but when the beetle releases the shell and the shell snaps back to its resting position, the energy is released quickly enough for the beetle to rise to its energy-limited height.

But that height is less than the height for locusts and humans. Indeed, the largest deviations from the constant-height result happen at the low-mass end, for fleas and click beetles. To explain that discrepancy, the model needs to take into account another physical effect: drag.

4.4 Drag

4.4.1 Jumping fleas

The drag force

$$F \sim \rho A v^2$$

affects the jumps of small animals more than it affects the jumps of people. A comparison of the energy required for the jump with the energy consumed by drag explains why.

The energy that the animal requires to jump to a height h is mgh, if we use the gravitational potential energy at the top of the jump; or it is $\sim mv^2$, if we use the kinetic energy at takeoff. The energy consumed by drag is

This part reads a bit awkwardly. Perhaps remove the commas before the 'if's. Or, in the spirit of course 6, write it as if...then... statements?

Or break it out of sentence form into a set of two equations with notes: E mgh using PE at peak E mv^2 using KE at takeoff That would keep it in line with the next part about Edrag.

I like the second solution here. I think as long as this is in sentence form then it'll be awkward. It's a lot easier to follow if it takes the form of the Edrag equation.

This is a good point. I'm not sure I would have made this connection left to my own devices.

$$E_{\text{drag}} \sim \underbrace{\rho v^2 A}_{F_{\text{drag}}} \times h.$$

The ratio of these energies measures the importance of drag. The ratio is

$$\frac{E_{\text{drag}}}{E_{\text{required}}} \sim \frac{\rho v^2 A h}{m v^2} = \frac{\rho A h}{m}.$$

Since A is the cross-sectional area of the animal, Ah is the volume of air that it sweeps out in the jump, and ρAh is the mass of air swept out in the jump. So the relative importance of drag has a physical interpretation as a ratio of the mass of air displaced to the mass of the animal.

To find how this ratio depends on animal size, rewrite it in terms of the animal's side length l. In terms of side length, $A \sim l^2$ and $m \propto l^3$. What about the jump height h? The simplest analysis predicts that all animals have the same jump height, so $h \propto l^0$. Therefore the numerator ρAh is $\propto l^1$, the denominator m is $\propto l^3$, and

$$\frac{E_{\text{drag}}}{E_{\text{required}}} \propto \frac{l^2}{l^3} = l^{-1}.$$

So, small animals have a large ratio, meaning that drag affects the jumps of small animals more than it affects the jumps of large animals. The missing constant of proportionality means that we cannot say at what size an animal becomes 'small' for the purposes of drag. So the calculation so far cannot tell us whether fleas are included among the small animals.

The jump data, however, substitutes for the missing constant of proportionality. The ratio is

$$\frac{E_{\text{drag}}}{E_{\text{required}}} \sim \frac{\rho A h}{m} \sim \frac{\rho l^2 h}{\rho_{\text{animal}} l^3}.$$

It simplifies to

$$\frac{E_{\text{drag}}}{E_{\text{required}}} \sim \frac{\rho}{\rho_{\text{animal}}} \frac{h}{l}.$$

As a quick check, verify that the dimensions match. The left side is a ratio of energies, so it is dimensionless. The right side is the product of two dimensionless ratios, so it is also dimensionless. The dimensions match.

Now put in numbers. A density of air is $\rho \sim 1\,\text{kg}\,\text{m}^{-3}$. The density of an animal is roughly the density of water, so $\rho_{\text{animal}} \sim 10^3\,\text{kg}\,\text{m}^{-3}$. The

Comments on page 3

Still unsure about why the h is here.

This equation is looking at the energy lost from the drag force not the the force itself. This seems pretty simple to me.

Because energy is force through a distance (E = F * d). Since we are trying to find the energy consumed by drag, we use the drag force: E_drag = F_drag * d.

keep in mind that the drag force is changing with height, and that it also happens to be the case that integral(F*dy) v^2*h, as well.

As a course 6 major, I'm unfamiliar with a lot of these calculations about drag. I was curious if drag is affected by gravity? For example, would a jumping animal experience a different amount of drag on the moon? Is it that all animals would still jump to the same height on the moon, but that height would be higher on the moon than it is on the earth?

I think gravity figures into the density equation, so it is implicit that air density on earth is higher than say the ambient density on the moon, due to gravity.

Drag is a phenomenon that depends on an atmosphere. The moon has no atmosphere, therefore no drag exists. Think of it as friction from rubbing on gas.

Another way of thinking about drag is what Sanjoy mentioned one or two readings ago: The drag force is proportional to the amount of fluid displaced by the object.

That way, denser medium will yield a larger drag force, simply because you have to displace more mass per volume as you move through it.

thank you 10:33 & 10:52!

Agreed, thanks. This stuff is far less familiar to CS majors than the MechEs.

Total reversal from the beginning when we were talking about UNIX...

and those of us not meche or cs are still totally floundering.

That's a really cool way of measuring drag.

(and by measuring I mean thinking of.)

wow, this is great, I never thought about it this way.

wow, this is great, I never thought about it this way.

$$E_{drag} \sim \underbrace{\rho v^2 A}_{F_{drag}} \times h.$$

The ratio of these energies measures the importance of drag. The ratio is

$$\frac{E_{drag}}{E_{required}} \sim \frac{\rho v^2 A h}{m v^2} = \frac{\rho A h}{m}.$$

Since A is the cross-sectional area of the animal, Ah is the volume of air that it sweeps out in the jump, and ρAh is the mass of air swept out in the jump. So the relative importance of drag has a physical interpretation as a ratio of the mass of air displaced to the mass of the animal.

To find how this ratio depends on animal size, rewrite it in terms of the animal's side length l. In terms of side length, $A \sim l^2$ and $m \propto l^3$. What about the jump height h? The simplest analysis predicts that all animals have the same jump height, so $h \propto l^0$. Therefore the numerator ρAh is $\propto l^1$, the denominator m is $\propto l^3$, and

$$\frac{E_{drag}}{E_{required}} \propto \frac{l^2}{l^3} = l^{-1}.$$

So, small animals have a large ratio, meaning that drag affects the jumps of small animals more than it affects the jumps of large animals. The missing constant of proportionality means that we cannot say at what size an animal becomes 'small' for the purposes of drag. So the calculation so far cannot tell us whether fleas are included among the small animals.

The jump data, however, substitutes for the missing constant of proportionality. The ratio is

$$\frac{E_{drag}}{E_{required}} \sim \frac{\rho A h}{m} \sim \frac{\rho l^2 h}{\rho_{animal} l^3}.$$

It simplifies to

$$\frac{E_{drag}}{E_{required}} \sim \frac{\rho}{\rho_{animal}} \frac{h}{l}.$$

As a quick check, verify that the dimensions match. The left side is a ratio of energies, so it is dimensionless. The right side is the product of two dimensionless ratios, so it is also dimensionless. The dimensions match.

Now put in numbers. A density of air is $\rho \sim 1\,\mathrm{kg\,m^{-3}}$. The density of an animal is roughly the density of water, so $\rho_{animal} \sim 10^3\,\mathrm{kg\,m^{-3}}$. The

wow, this is great, I never thought about it this way.

This type of ratio makes up a lot of important named parameters in fluid mechanics (Reynolds number, Prandtl number, etc.) Reynolds is kinetic forces vs. viscous forces. Maybe we'll come back to this in dimensional analysis.

wow, this is great, I never thought about it this way.

Would you need some sort of coefficient in order to take the shape of the animal into account?

I think that trying the accuracy gained by trying to use a coefficient to make this animal shape more accurate than a simple box would be canceled out by all of the other simplifying assumptions we have made, and ultimately would not be worth the effort.

Unless the animal has holes in it or very different cross-sectional areas, pAh will still be the same mass of air swept out by its passing through. You're right, further refinement would involve things like birds and whales being streamlined, since that helps them not have to displace all the air they would if they were all cylinders.

Hmm this is also interesting and makes complete sense. Never would have thought of it that way.

Also gives a calculation why exactly jumping higher is harder....sort of

I really like this explanation of drag.

Yeah, this is really interesting...never would have thought of this...

right– it's a balance of mass to surface area.

yeah this sentence makes the idea of drag in this example really clear and easy to visualize.

This relationship is true for all size animals though

$$E_{drag} \sim \underbrace{\rho v^2 A}_{F_{drag}} \times h.$$

The ratio of these energies measures the importance of drag. The ratio is

$$\frac{E_{drag}}{E_{required}} \sim \frac{\rho v^2 A h}{m v^2} = \frac{\rho A h}{m}.$$

Since A is the cross-sectional area of the animal, Ah is the volume of air that it sweeps out in the jump, and ρAh is the mass of air swept out in the jump. So the relative importance of drag has a physical interpretation as a ratio of the mass of air displaced to the mass of the animal.

To find how this ratio depends on animal size, rewrite it in terms of the animal's side length l. In terms of side length, $A \sim l^2$ and $m \propto l^3$. What about the jump height h? The simplest analysis predicts that all animals have the same jump height, so $h \propto l^0$. Therefore the numerator ρAh is $\propto l^1$, the denominator m is $\propto l^3$, and

$$\frac{E_{drag}}{E_{required}} \propto \frac{l^2}{l^3} = l^{-1}.$$

So, small animals have a large ratio, meaning that drag affects the jumps of small animals more than it affects the jumps of large animals. The missing constant of proportionality means that we cannot say at what size an animal becomes 'small' for the purposes of drag. So the calculation so far cannot tell us whether fleas are included among the small animals.

The jump data, however, substitutes for the missing constant of proportionality. The ratio is

$$\frac{E_{drag}}{E_{required}} \sim \frac{\rho A h}{m} \sim \frac{\rho l^2 h}{\rho_{animal} l^3}.$$

It simplifies to

$$\frac{E_{drag}}{E_{required}} \sim \frac{\rho}{\rho_{animal}} \frac{h}{l}.$$

As a quick check, verify that the dimensions match. The left side is a ratio of energies, so it is dimensionless. The right side is the product of two dimensionless ratios, so it is also dimensionless. The dimensions match.

Now put in numbers. A density of air is $\rho \sim 1\,\mathrm{kg\,m^{-3}}$. The density of an animal is roughly the density of water, so $\rho_{animal} \sim 10^3\,\mathrm{kg\,m^{-3}}$. The

I don't understand how this assumption makes sense, unless you simply didn't want to have l^3/l^3

it makes sense based on the analysis we did previous (forgot whether it was in class or in the readings). it's not what we "want" it makes sense because the forces that we overcome by jumping are directly proportional to the capacity with we have to jump with...sort of

Just think about it in terms of a cube. Imagine them as little boxes with tiny legs. It's pretty standard physics to think of everything as a cube or a sphere.

I think this assumptions is too leading. Obviously, a beetle can't jump 2 feet in the air like a human can, but does that mean it doesn't have a proportional amount of power as a human?

What he's basically saying here is that the jump height is independent of the animal's "length" l. It might help if he explicitly stated it though.

I thought we were ignoring the density?

I feel like this will be dealt with once we start doing proportions in that it will probably be canceled out or be unnecessary.

we still are. the l comes from h

Is this supposed to be L^2 here?

I think so because right below, its L^2 on the numerator

I never would have guessed this, I would have guessed the more surface area, the more drag

You're sort of right; more surface area contributes more to drag, but heavier objects are intuitively more "immune" to drag. So it really comes down to surface area to volume ratio. Smaller animals have a larger area to volume ratio, despite having a smaller area overall.

$$E_{\text{drag}} \sim \underbrace{\rho v^2 A}_{F_{\text{drag}}} \times h.$$

The ratio of these energies measures the importance of drag. The ratio is

$$\frac{E_{\text{drag}}}{E_{\text{required}}} \sim \frac{\rho v^2 A h}{m v^2} = \frac{\rho A h}{m}.$$

Since A is the cross-sectional area of the animal, Ah is the volume of air that it sweeps out in the jump, and $\rho A h$ is the mass of air swept out in the jump. So the relative importance of drag has a physical interpretation as a ratio of the mass of air displaced to the mass of the animal.

To find how this ratio depends on animal size, rewrite it in terms of the animal's side length l. In terms of side length, $A \sim l^2$ and $m \propto l^3$. What about the jump height h? The simplest analysis predicts that all animals have the same jump height, so $h \propto l^0$. Therefore the numerator $\rho A h$ is $\propto l^1$, the denominator m is $\propto l^3$, and

$$\frac{E_{\text{drag}}}{E_{\text{required}}} \propto \frac{l^2}{l^3} = l^{-1}.$$

So, small animals have a large ratio, meaning that drag affects the jumps of small animals more than it affects the jumps of large animals. The missing constant of proportionality means that we cannot say at what size an animal becomes 'small' for the purposes of drag. So the calculation so far cannot tell us whether fleas are included among the small animals.

The jump data, however, substitutes for the missing constant of proportionality. The ratio is

$$\frac{E_{\text{drag}}}{E_{\text{required}}} \sim \frac{\rho A h}{m} \sim \frac{\rho l^2 h}{\rho_{\text{animal}} l^3}.$$

It simplifies to

$$\frac{E_{\text{drag}}}{E_{\text{required}}} \sim \frac{\rho}{\rho_{\text{animal}}} \frac{h}{l}.$$

As a quick check, verify that the dimensions match. The left side is a ratio of energies, so it is dimensionless. The right side is the product of two dimensionless ratios, so it is also dimensionless. The dimensions match.

Now put in numbers. A density of air is $\rho \sim 1\,\text{kg}\,\text{m}^{-3}$. The density of an animal is roughly the density of water, so $\rho_{\text{animal}} \sim 10^3\,\text{kg}\,\text{m}^{-3}$. The

If this is true, why can smaller animals often jump so much farther than larger ones?

they jump far compared to their body size–a beetle can't actually jump farther than a human (just think about a human long jump vs. a bug jumping–the bug just looks like it jumps far). Plus, different animals have different ways of getting around drag (here we're just making estimates for animals in general, not the few freak cases)

The long jump doesn't apply here - that is a running start very different from jumping from a standstill. My cat just lumped like 3 feet - and she has my jump height beat for sure.

they don't actually (if you and a beetle were in a jumping contest, you would win)

This fact comes intuitively to me and it's really interesting to think about this with calculations.

This is logical to me too.

I agree. It is definitely nice to be able to explain things that you see with mathematical models.

does this have to do with the drag coefficient at all?

$$E_{drag} \sim \underbrace{\rho v^2 A}_{F_{drag}} \times h.$$

The ratio of these energies measures the importance of drag. The ratio is

$$\frac{E_{drag}}{E_{required}} \sim \frac{\rho v^2 A h}{m v^2} = \frac{\rho A h}{m}.$$

Since A is the cross-sectional area of the animal, Ah is the volume of air that it sweeps out in the jump, and $\rho A h$ is the mass of air swept out in the jump. So the relative importance of drag has a physical interpretation as a ratio of the mass of air displaced to the mass of the animal.

To find how this ratio depends on animal size, rewrite it in terms of the animal's side length l. In terms of side length, $A \sim l^2$ and $m \propto l^3$. What about the jump height h? The simplest analysis predicts that all animals have the same jump height, so $h \propto l^0$. Therefore the numerator $\rho A h$ is $\propto l^1$, the denominator m is $\propto l^3$, and

$$\frac{E_{drag}}{E_{required}} \propto \frac{l^2}{l^3} = l^{-1}.$$

So, small animals have a large ratio, meaning that drag affects the jumps of small animals more than it affects the jumps of large animals. The missing constant of proportionality means that we cannot say at what size an animal becomes 'small' for the purposes of drag. So the calculation so far cannot tell us whether fleas are included among the small animals.

The jump data, however, substitutes for the missing constant of proportionality. The ratio is

$$\frac{E_{drag}}{E_{required}} \sim \frac{\rho A h}{m} \sim \frac{\rho l^2 h}{\rho_{animal} l^3}.$$

It simplifies to

$$\frac{E_{drag}}{E_{required}} \sim \frac{\rho}{\rho_{animal}} \frac{h}{l}.$$

As a quick check, verify that the dimensions match. The left side is a ratio of energies, so it is dimensionless. The right side is the product of two dimensionless ratios, so it is also dimensionless. The dimensions match.

Now put in numbers. A density of air is $\rho \sim 1\,\mathrm{kg\,m^{-3}}$. The density of an animal is roughly the density of water, so $\rho_{animal} \sim 10^3\,\mathrm{kg\,m^{-3}}$. The

Would it be useful to estimate it?

It's not a question of whether it would be useful (which it would be), but whether it is worth the trouble.

Here, Sanjoy cleverly evaded using a constant of proportionality by indirectly plugging in known numbers to get a ratio instead. Since it is a ratio, the constant of proportionality appears on both top and bottom, and cancels, eliminating the need to ever know what it was.

The ratio itself, tells us valuable information about "to what degree is the flea affected by drag", which is something that having a constant of proportionality would've also been able to do. The problem is that finding a constant of proportionality probably requires experimental data to "fit" the equation of interest, whereas doing the ratio here is a shortcut to getting some useful interpretation right away.

> Where you say "the constant of proportionality appears on top and bottom, and cancels" is false for any of the equations written on this page.
>
> What the equation above says, is that E=C/l, so E is proportional to 1/l. That constant is still there, hidden, and does not cancel. It is alluded to by the "proportional" sign.
>
> The constants do cancel, however, when you want to compare the magnitude of this ratio for a human to the magnitude of the ratio for a flea, for example. Since the human has a length scale of a thousand times that of a flea, this ratio, Edrag/Erequired, is a thousand times smaller for a human than for a flea. That doesn't give us any idea about its magnitude for a flea, unless we knew its magnitude for a human, and thus could derive C, the constant of proportionality.

why the difference (as in drag is no longer trivial in comparison to each) compared to the earlier drag lecture we had with the cones?

$$E_{\text{drag}} \sim \underbrace{\rho v^2 A}_{F_{\text{drag}}} \times h.$$

The ratio of these energies measures the importance of drag. The ratio is

$$\frac{E_{\text{drag}}}{E_{\text{required}}} \sim \frac{\rho v^2 A h}{m v^2} = \frac{\rho A h}{m}.$$

Since A is the cross-sectional area of the animal, Ah is the volume of air that it sweeps out in the jump, and $\rho A h$ is the mass of air swept out in the jump. So the relative importance of drag has a physical interpretation as a ratio of the mass of air displaced to the mass of the animal.

To find how this ratio depends on animal size, rewrite it in terms of the animal's side length l. In terms of side length, $A \sim l^2$ and $m \propto l^3$. What about the jump height h? The simplest analysis predicts that all animals have the same jump height, so $h \propto l^0$. Therefore the numerator $\rho A h$ is $\propto l^1$, the denominator m is $\propto l^3$, and

$$\frac{E_{\text{drag}}}{E_{\text{required}}} \propto \frac{l^2}{l^3} = l^{-1}.$$

So, small animals have a large ratio, meaning that drag affects the jumps of small animals more than it affects the jumps of large animals. The missing constant of proportionality means that we cannot say at what size an animal becomes 'small' for the purposes of drag. So the calculation so far cannot tell us whether fleas are included among the small animals.

The jump data, however, substitutes for the missing constant of proportionality. The ratio is

$$\frac{E_{\text{drag}}}{E_{\text{required}}} \sim \frac{\rho A h}{m} \sim \frac{\rho l^2 h}{\rho_{\text{animal}} l^3}.$$

It simplifies to

$$\frac{E_{\text{drag}}}{E_{\text{required}}} \sim \frac{\rho}{\rho_{\text{animal}}} \frac{h}{l}.$$

As a quick check, verify that the dimensions match. The left side is a ratio of energies, so it is dimensionless. The right side is the product of two dimensionless ratios, so it is also dimensionless. The dimensions match.

Now put in numbers. A density of air is $\rho \sim 1\,\text{kg}\,\text{m}^{-3}$. The density of an animal is roughly the density of water, so $\rho_{\text{animal}} \sim 10^3\,\text{kg}\,\text{m}^{-3}$. The

So basically this says that we forgot to put in the density of the animal into our original calculations, and this fixes it?

Not so much that we forgot, but that this is how one would go about using the proportionality calculations we ran through above into calculations of an actual limit (this is more of an equation, whereas the above derivation is getting us a relationship)

It almost seems like a way to throw out constants and other information to arrive at proportionalities and then somehow fix for the dropped things at the end...mildly hand-wavy but really cool

Where the top density is the density of our fluid, here air, right?

Oops, never mind. I see it.

I'm sure this works well for air. However, I feel that when I am in the water (say a swimming pool), I can jump higher despite the drag. Does that have to do with the boyancy of water?

Yes. It equates to a reduction in acceleration. While before we assumed that acceleration was a constant (g=9.8m/ss), in water your acceleration is a=F/m, but here F=|Wbody-Wwater|. Our analysis could have kept the variable for acceleration, and that would have explained your question.

Keep in mind water has a different density than air, of course, so the rho.fluid changes, too. I'm not sure if water's viscosity comes into play or not.

Yeah, it would also be interesting to see how the density of the environment affects the result- i.e. can you jump higher at sea level than on top of mt everest?

You'd be able to jump higher on top of Mt. Everest because lower rho in the numerator means lower drag energy ratio. (baseballs fly farther even in Denver, cyclists ride faster in Mexico City...)

Yeah! It gets even weirder than that. So many factors affect drag. The depth of a swimming pool, for example, can effect how fast you swim through the surface. That's so not-intuitive.

yeah, good point. let's just lump all that into 'edge effects' :)

$$E_{\text{drag}} \sim \underbrace{\rho v^2 A}_{F_{\text{drag}}} \times h.$$

The ratio of these energies measures the importance of drag. The ratio is

$$\frac{E_{\text{drag}}}{E_{\text{required}}} \sim \frac{\rho v^2 A h}{m v^2} = \frac{\rho A h}{m}.$$

Since A is the cross-sectional area of the animal, Ah is the volume of air that it sweeps out in the jump, and $\rho A h$ is the mass of air swept out in the jump. So the relative importance of drag has a physical interpretation as a ratio of the mass of air displaced to the mass of the animal.

To find how this ratio depends on animal size, rewrite it in terms of the animal's side length l. In terms of side length, $A \sim l^2$ and $m \propto l^3$. What about the jump height h? The simplest analysis predicts that all animals have the same jump height, so $h \propto l^0$. Therefore the numerator $\rho A h$ is $\propto l^1$, the denominator m is $\propto l^3$, and

$$\frac{E_{\text{drag}}}{E_{\text{required}}} \propto \frac{l^2}{l^3} = l^{-1}.$$

So, small animals have a large ratio, meaning that drag affects the jumps of small animals more than it affects the jumps of large animals. The missing constant of proportionality means that we cannot say at what size an animal becomes 'small' for the purposes of drag. So the calculation so far cannot tell us whether fleas are included among the small animals.

The jump data, however, substitutes for the missing constant of proportionality. The ratio is

$$\frac{E_{\text{drag}}}{E_{\text{required}}} \sim \frac{\rho A h}{m} \sim \frac{\rho l^2 h}{\rho_{\text{animal}} l^3}.$$

It simplifies to

$$\frac{E_{\text{drag}}}{E_{\text{required}}} \sim \frac{\rho}{\rho_{\text{animal}}} \frac{h}{l}.$$

As a quick check, verify that the dimensions match. The left side is a ratio of energies, so it is dimensionless. The right side is the product of two dimensionless ratios, so it is also dimensionless. The dimensions match.

Now put in numbers. A density of air is $\rho \sim 1\,\text{kg}\,\text{m}^{-3}$. The density of an animal is roughly the density of water, so $\rho_{\text{animal}} \sim 10^3\,\text{kg}\,\text{m}^{-3}$. The

I understand why this is important because of prior exposure to dimensional analysis. On the other hand, you may want to comment on why this is important in case the reader has not idea why dimensions matching matters.

This is a fair point - some readers may not understand why dimensions need to match.

I feel like if this is intended for an MIT audience it would not need to be explained. Also, I'm fairly sure the next unit is dimensional analysis, so this might be a sort of lead-in.

I agree, anyone who is reading through a book which analyzes the mass and drag of an animal should understand the concept of matching up dimensions.

I seem to remember that even in middle school and high school we would check answers based on if the dimensions matched... it certainly isn't a new concept for me that I"m learning in the class. It is cool to see how ti works in things like proportionality; it was cool how on the homework we got P Ev for example.

I think it is good that this line is here. It's like a good lead-in to the next unit.

I think dimensional analysis should have been included all along; it's really valuable as a "checking" method for your assumptions.

$$E_{\text{drag}} \sim \underbrace{\rho v^2 A}_{F_{\text{drag}}} \times h.$$

The ratio of these energies measures the importance of drag. The ratio is

$$\frac{E_{\text{drag}}}{E_{\text{required}}} \sim \frac{\rho v^2 A h}{m v^2} = \frac{\rho A h}{m}.$$

Since A is the cross-sectional area of the animal, Ah is the volume of air that it sweeps out in the jump, and $\rho A h$ is the mass of air swept out in the jump. So the relative importance of drag has a physical interpretation as a ratio of the mass of air displaced to the mass of the animal.

To find how this ratio depends on animal size, rewrite it in terms of the animal's side length l. In terms of side length, $A \sim l^2$ and $m \propto l^3$. What about the jump height h? The simplest analysis predicts that all animals have the same jump height, so $h \propto l^0$. Therefore the numerator $\rho A h$ is $\propto l^1$, the denominator m is $\propto l^3$, and

$$\frac{E_{\text{drag}}}{E_{\text{required}}} \propto \frac{l^2}{l^3} = l^{-1}.$$

So, small animals have a large ratio, meaning that drag affects the jumps of small animals more than it affects the jumps of large animals. The missing constant of proportionality means that we cannot say at what size an animal becomes 'small' for the purposes of drag. So the calculation so far cannot tell us whether fleas are included among the small animals.

The jump data, however, substitutes for the missing constant of proportionality. The ratio is

$$\frac{E_{\text{drag}}}{E_{\text{required}}} \sim \frac{\rho A h}{m} \sim \frac{\rho l^2 h}{\rho_{\text{animal}} l^3}.$$

It simplifies to

$$\frac{E_{\text{drag}}}{E_{\text{required}}} \sim \frac{\rho}{\rho_{\text{animal}}} \frac{h}{l}.$$

As a quick check, verify that the dimensions match. The left side is a ratio of energies, so it is dimensionless. The right side is the product of two dimensionless ratios, so it is also dimensionless. The dimensions match.

Now put in numbers. A density of air is $\rho \sim 1\,\text{kg}\,\text{m}^{-3}$. The density of an animal is roughly the density of water, so $\rho_{\text{animal}} \sim 10^3\,\text{kg}\,\text{m}^{-3}$. The

So the denser the animal, the less it is influenced by drag? Or do I have it backwards?

I am also confused by that.

yes. this is indicated by the animal's density appearing in the denominator. Think of tossing a baseball straight up into the air, and then a paper ball of the same size into the air with the same initial velocity. The baseball will go higher, and only because it is less influenced by drag.

i get it! that's so weird!

That's weird? Doesn't it make perfect sense?

I don't think there's a big range of animal densities, aside from cases where air volume is a significant fraction of internal volume. (Think of how you can change your buoyancy in water by having full or empty lungs.) I think for animals reducing cross-section is more effective.

To follow up, I found a classic book on this. Their values range from 0.99 to 1.1 specific gravity for mammals, and they mention the effect of air in the body. http://books.google.com/books?id=SV8rAAAAYAAJ&lpg=PA154&ots=wc5KfvUf

Whoa, I think this reasoning is flawed. If the baseball and paper have the same initial velocity, then the baseball will have more energy (making it go higher), since it has a higher mass. If the two balls have the same cross-section, mass and initial velocity, they'll behave the same way. Maybe if you tossed with equal energies this logic would hold up, since a less dense paper ball would lose more of its energy to drag, meaning it wouldn't go as high? The underlying physics of a tossed baseball and a jumping animal are different. We just went through a lot to find the relation of jump height to energy and length.

That's an interesting concept. I don't think it's fully accurate, as I would think drag is so much more influenced by sufrace area for instance that density itself might be negligible.

$$E_{drag} \sim \underbrace{\rho v^2 A}_{F_{drag}} \times h.$$

The ratio of these energies measures the importance of drag. The ratio is

$$\frac{E_{drag}}{E_{required}} \sim \frac{\rho v^2 A h}{m v^2} = \frac{\rho A h}{m}.$$

Since A is the cross-sectional area of the animal, Ah is the volume of air that it sweeps out in the jump, and $\rho A h$ is the mass of air swept out in the jump. So the relative importance of drag has a physical interpretation as a ratio of the mass of air displaced to the mass of the animal.

To find how this ratio depends on animal size, rewrite it in terms of the animal's side length l. In terms of side length, $A \sim l^2$ and $m \propto l^3$. What about the jump height h? The simplest analysis predicts that all animals have the same jump height, so $h \propto l^0$. Therefore the numerator $\rho A h$ is $\propto l^1$, the denominator m is $\propto l^3$, and

$$\frac{E_{drag}}{E_{required}} \propto \frac{l^2}{l^3} = l^{-1}.$$

So, small animals have a large ratio, meaning that drag affects the jumps of small animals more than it affects the jumps of large animals. The missing constant of proportionality means that we cannot say at what size an animal becomes 'small' for the purposes of drag. So the calculation so far cannot tell us whether fleas are included among the small animals.

The jump data, however, substitutes for the missing constant of proportionality. The ratio is

$$\frac{E_{drag}}{E_{required}} \sim \frac{\rho A h}{m} \sim \frac{\rho l^2 h}{\rho_{animal} l^3}.$$

It simplifies to

$$\frac{E_{drag}}{E_{required}} \sim \frac{\rho}{\rho_{animal}} \frac{h}{l}.$$

As a quick check, verify that the dimensions match. The left side is a ratio of energies, so it is dimensionless. The right side is the product of two dimensionless ratios, so it is also dimensionless. The dimensions match.

Now put in numbers. A density of air is $\rho \sim 1\,\mathrm{kg\,m^{-3}}$. The density of an animal is roughly the density of water, so $\rho_{animal} \sim 10^3\,\mathrm{kg\,m^{-3}}$. The

Sorry to correct your correction (9:58), but the baseball analogy is correct. Ignoring drag, a baseball and a paper ball will reach the same height because KE.initial=1/2mv^2 is equal to PE.final=mgh. The m's cancel and you get h=v^2/(2g).

But now add drag into the mix. Say the baseball loses 1/4 of its energy to drag, and the paper ball 1/2 of its energy. then the baseball will reach height 3h/4, but the paper ball will only reach a height of h/2.

Once they leave the ground, according to our model, animals and baseballs/paper balls behave in the same way.

> from 9:58 poster: You're right, I missed the m's cancelling, so that classic ball throwing problem from the first day of physics still works and vo determines h in the absence of drag.

Does this apply to humans as well?

Yes - I don't remember if the argument was made in this class or I saw it somewhere else, but humans are more than half made of water, so it's not a terrible rough estimate.

typical jump height – which is where the data substitutes for the constant of proportionality – is 60 cm or roughly 1 m. A flea's length is about 1 mm or $l \sim 10^{-3}$ m. So

$$\frac{E_{\text{drag}}}{E_{\text{required}}} \sim \frac{1\,\text{kg}\,\text{m}^{-3}}{10^{3}\,\text{kg}\,\text{m}^{-3}}\,\frac{1\,\text{m}}{10^{-3}\,\text{m}} \sim 1.$$

The ratio being unity means that if a flea would jump to 60 cm, overcoming drag would require roughly as much as energy as would the jump itself in vacuum.

Drag provides a plausible explanation for why fleas do not jump as high as the typical height to which larger animals jump.

Sometimes a summary table might be useful as a way to list the quantities that have been derived.

Completely agreed - I had to scroll back up the page to remember some.

This was also something I have been thinking about while reading some of the more numerical sections.

I agree - especially since it is hard for up to scroll between sections when reading on NB.

it's probably be cool to present, at the beginning or end of the book, a "common sense table" for obvious metrics (ex: time to fly to california, what 100km is like...etc). i know it makes more sense to internalize something that makes sense to us but it'd be cool just to have one anyway (kind of an extended version of what we had for our pretest)

it's probably be cool to present, at the beginning or end of the book, a "common sense table" for obvious metrics (ex: time to fly to california, what 100km is like...etc). i know it makes more sense to internalize something that makes sense to us but it'd be cool just to have one anyway (kind of an extended version of what we had for our pretest)

Are all these concepts useful to animal researchers in any way? I feel like this section is good in terms of analyzing drag once more, but in terms of practicality I find it a bit loose

I would like to see this same procedure applied to different animals so that we can compare the results.

typical jump height – which is where the data substitutes for the constant of proportionality – is 60 cm or roughly 1 m. A flea's length is about 1 mm or $l \sim 10^{-3}$ m. So

$$\frac{E_{\text{drag}}}{E_{\text{required}}} \sim \frac{1\,\text{kg}\,\text{m}^{-3}}{10^{3}\,\text{kg}\,\text{m}^{-3}} \frac{1\,\text{m}}{10^{-3}\,\text{m}} \sim 1.$$

The ratio being unity means that if a flea would jump to 60 cm, overcoming drag would require roughly as much as energy as would the jump itself in vacuum.

Drag provides a plausible explanation for why fleas do not jump as high as the typical height to which larger animals jump.

So does this mean that in order for a flea to jump 60cm, they would have to exert double the amount of force? (the force to accomplish the jump in a vacuum plus the force to overcome drag)

Good point. Does it scale linearly? I would guess yes, for our purposes, but I'm not sure.

I don't understand your question. If overcoming drag requires as much energy as the jump inside a vacuum, doesn't that imply that force needed is double the force within a vacuum.

Yeah I believe so. I had to reread it again to figure out that it didn't mean that the drag force was essentially zero. I think putting it in terms of doubling the amount of energy (like you mentioned) would have made it clearer.

yeah your comment helped me figure out what he actually meant by that.

It does definitely sound like he is saying that the drag force is zero. Perhaps it should be spelled out that a flea has to exert double the "vacuum force" in order to jump to 60 cm.

Yea I agree with the comment above. This would help clarify what was meant. I think just slight rewording would help like " if a flea would jump to 60cm, just overcoming the drag force requires the same amount of energy as performing the entire jump in a vacuum."

one too many "as" here

I think your explanation for this section makes sense.

Here, when you mention 60 cm again, I got a little confused. Perhaps make it more clear that 60 cm 1 m, and since we used 1 m in our equation, I think you should refer to it as 1 m the second time around.

Here, when you mention 60 cm again, I got a little confused. Perhaps make it more clear that 60 cm 1 m, and since we used 1 m in our equation, I think you should refer to it as 1 m the second time around.

At one point we assumed h to be independent of l, to continue in our calculation. Now that we know it's not true, it would be interesting to take a better estimate of h, and refine our estimate further.

typical jump height – which is where the data substitutes for the constant of proportionality – is 60 cm or roughly 1 m. A flea's length is about 1 mm or $l \sim 10^{-3}$ m. So

$$\frac{E_{\text{drag}}}{E_{\text{required}}} \sim \frac{1\,\text{kg}\,\text{m}^{-3}}{10^{3}\,\text{kg}\,\text{m}^{-3}}\,\frac{1\,\text{m}}{10^{-3}\,\text{m}} \sim 1.$$

The ratio being unity means that if a flea would jump to 60 cm, overcoming drag would require roughly as much as energy as would the jump itself in vacuum.

Drag provides a plausible explanation for why fleas do not jump as high as the typical height to which larger animals jump.

I will say that although we have looked at drag a few times in this course already, this one was the easiest example for me to understand.

I thought that flees could fly a little bit as well as just jump. How would you account for the animals that can jump, but can also fly, or sort of fly to help them jump further, such as a beetle, flying fish or flying squirrel. Is there any way to apply the same formulas to their movements?

I would think that since we're only considering height, gliding ability wouldn't come into effect.

Is there a way we can figure out how high they can jump by using this, or can we only say that they don't jump as far?

I feel like we've ignored a lot of constants along the route of proportionality so that the estimate would be quite rough.

true, we have been ignoring constants, but in finding proportionality, we don't need to worry about constants. for example, if we're finding the ratio of how high a human jumps vs a fly, we don't need constants that stay the same (like g) since they cancel out when we divide.

The above thread talks about how a flea would need double the energy that larger animals need since it needs energy to get to the height (mgh) and it needs an equal proportion of energy to overcome drag.

By this reasoning, fleas should be able to jump to half the height of larger animals, which seems a lot more realistic from what I remember in movies/other info.

I agree with this reasoning and the idea that its a rough guess based on dropping constants. It seems like some of the final proportionality we used (the ratio of densities) helped make this guess a bit more reasonable. I wonder if there is a way to combine divide and conquer and proportional reasoning to add in accounting for constants

wow Drag is indeed very important!

Thats amazing.

I definitely expected this result from the similarity to surface area to volume scaling.

I guess you were clever about it, I would definitely not have thought this was the case though. Great conclusion!

typical jump height – which is where the data substitutes for the constant of proportionality – is 60 cm or roughly 1 m. A flea's length is about 1 mm or $l \sim 10^{-3}$ m. So

$$\frac{E_{\text{drag}}}{E_{\text{required}}} \sim \frac{1\,\text{kg}\,\text{m}^{-3}}{10^3\,\text{kg}\,\text{m}^{-3}}\,\frac{1\,\text{m}}{10^{-3}\,\text{m}} \sim 1.$$

The ratio being unity means that if a flea would jump to 60 cm, overcoming drag would require roughly as much as energy as would the jump itself in vacuum.

Drag provides a plausible explanation for why fleas do not jump as high as the typical height to which larger animals jump.

Still, considering their size, it's impressive how high they can jump!

For sure, I think someone told me that if you look at the ratio of jump height/height of animal, a flea could jump three stories if it were the height of a human.

What about larger animals that don't jump, like bears or elephants?

They just weren't designed t jump. This analysis is only useful for similarly designed animals. This is similar to not comparing lift on a person to that of the lift on a plane.

I wonder if there is some optimal size for jumping? Clearly larger animals jump higher, as seen in this example, but at what point do they get too big and start to lose jump height? I think this would be interesting to explore.

Probably when they are large / heavy enough to hurt themselves when they jump - like he was talking about with an elephant - jumping would be enough pressure to crush its bones.

I wonder what other animals of similar size jump; would it be proportional, or are fleas just a special case?

So I completely spaced on doing this reading before Friday's lecture, but I find that doing it afterwards I get more out of it because you've already explained it in person once.

I like how this lecture goes from the most basic scenario, and adds in other factors one by one