5.4 Bending of light by gravity

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This was definitely one of my favorite chapters so far, I've always been incredibly interested by this material but as a course 2 don't get to see it too much. I keep being impressed with the broad spectrum of material we cover that is relevant to everyday life

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Read this section for the memo due on Sunday at the end of spring break.

this is an interesting question to start the chapter

I'm confused by this - I thought light did feel the effect of gravity, especially when you consider phenomena like black holes? Or is that something different entirely?

I agree, interesting questions. I mean I guess you could argue that you feel light as energy, but not so much as something like drag. It doesn't retard any force

I was confused by the wording of this at first as well... I don't think the question is stating that light doesn't feel the effect of gravity, its more like "how does gravity affect light?"

Light is affected by gravity. But since we normally think of it as "massless photons" we assume that something that acts on mass, ie: gravity, will not affect it. This is an incorrect assumption.

so does this mean that everything is affected by gravity? I mean I thought light was a form of electrog-magnetic radiation which is a type of energy. So does this mean that energy is affected by gravity (even magnetic energy or electric energy)?

I've never even thought of that question.

I seem to think about this question a lot, most importantly when it refers to stars. I don't think we would see stars at all if light were affected by gravity.

This is such a great question to begin the section with. It attracts my attention because it is something that I have never ever thought of.

I agree - its also a question that the reader can start to try to reason about...what do we know about black holes? do we ever hear about light bending?

It's already very attention-grabbing, but maybe even mentioning black holes in the reading would be cool.

I agree about black holes, or maybe just mention something in class about them. I'm pretty interested in them and think it would be nice to go into a bit of detail since they are related to this topic

I've wondered this myself.

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I am kind of confused by this... the first part says light is not effected and the second part says we will analyze how it is effected...

It's a writing / teaching technique to say something that most people will agree with, then prove them wrong.

Is this similar to how the cloaking/invisibility technology theoretically works?

Could someone give some intuition into how gravity bends light?

Ever hear of a black Hole? Black holes suck in light because the mass is so dense that the escape velocity is greater than the speed of light. If you understand this concept, it may be easier for you to justify deflection from gravity.

I'm not too versed in physics, but the way I understand it at a subatomic level, mass and energy and mass can be thought of as interchangeable values (remember E=mc^2). Anything made up of energy (which is everything) has a mass. Please correct me if I am wrong.

So gravity bends light because, it turns out that mass just bends "space time" into different shapes, and both massive and massless particles follow geodesics, or the shortest path between two points. Since gravity in the general relativistic picture isn't just a force between two masses, but instead is just things moving in "straight lines," light bends just like anything else.

So are we going to calculate how gravity bends light, in order to see that it doesn't affect light unless the gravitational force is extremely strong? Because I know a black hole does this.

Why has this concept never been introduced in Physics classes?

I'm really interested in the applications to black holes.

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I've never even heard of these!

Neither had I! According to Wikipedia, a geodesic equation in math is one that generalizes the notion of a straight line to curved spaces. The Wikipedia articles has a great figure to refer to and a more detailed explanation.

Ah, thanks for the explanation. Saved me a Google search.

I was wondering if these were related to geodesic domes (think of Spaceship Earth at Epcot). The wikipedia article on geodesic domes is also really interesting.

I think putting a little more explanation leading into this equation in the reading would be helpful

I get the impression that my "huh? what?" thoughts for this paragraph will make the analysis more awesome.

It might be helpful to explain a few terms here. "geodesic" and "metric tensor" are completely new terms for me, allowing me to get lost in just the 2nd sentence.

I think that's the point. It's supposed to demonstrate how confusing and complex the situation is.

I agree that definitions might be helpful. However, the point is not to use the equations but to show how we can get around them. In that sense, definition is not absolutely necessary.

I understand that these equations are generally listed to demonstrate how confusing certain problems are, but I don't know how necessary this is to do every time (we had another example before, either with flow or pressure, can't remember exactly).

This is pretty cool stuff that you never learn in engineering courses (at least at MIT). It think it would be nice to explain this stuff, even if it is not essential to the problem we are trying to solve.

No, because then we get complaints about how confusing and unnecessary the definitions of equations and terms of throw-away examples are. This way, people can just look up terms they feel impede their understanding without messing with the flow.

What about the beta mu and nu?

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pretty sure I've never seen these before

Similar to notes posted elsewhere: I don't think many have, and that is the point. They are supposed to look "mean and scary" so as to make our, hopefully easier, method look nice.

Yeah, but this one is a new level of scariness.

Coming at the end of spring break, this looks quite frightening!

Yeah..I don't even understand the formula..

spelling error

you repeat the "whose evaluation require solving" twice...maybe phrase it differently

i believe it's done to emphasize the fact that it will be difficult to solve this equation.

what is the difference between a metric tensor and say the tensor used in structures?

If the intent of this is to repel us from solving it traditionally/accurately, it is succeeding admirably.

I believe it is and you are most correct.

My takeaway from this paragraph was to be prepared to cry if you have to solve these for any non-trivial case

Indeed! There are very few cases where the solutions are known. One of the few cases is for a spherically symmetric, nonrotating mass – the Schwarzschild solution (derived while Schwarzschild was in the German army in WW1). It was such a feat of equation solving that even Einstein did not expect an exact solution.

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if the point of this paragraph was to say that these equations are normally very difficult and hard to understand, then job well done. :)

hahaha agreed

Indeed, though to analyze this question in terms of syntax, the double use of phrases that start with 'whose' seems awkward.

I agree. Mentioning Christoffel symbols and curvature equations is unnecessary and detracts from your point, which is that these equations are hard to solve, and quite likely not something the reader even knows how to approach.

I agree - you use this same technique of telling us how complicated solving out these giant equations is in previous sections. I think it is a tactic that only really needs to be included once because realizing how complicated brute force is isn't really specific to each equation - it is more of a general concept that we learned in a previous section. I think most people can recognize its application after just looking at each new jumbled mess of symbols.

Giving a description in really complicated terms kind of makes me not want to read the section though. I like the point you're making, but I like the method of saying "solving this would really suck" and then going into a better way to approach the problem. Like you do in the next paragraph.

I think this paragraph and the beginning section makes me want to skim. I'm wide-eyed from reading it and it just feels cumbersome. I feel like it could be simplified quite a bit, and maybe a side note to an appendix for more information (for the actual book). For example, "There are really difficult calculations here, but we can use the familiar principle..." A bit more than that definitely, but the way it is just feels hard to read.

i think that showing this complicated equation and talking about the ten non-linear partial-differential equations is a great way to let the reader understand why the approximation we are about to do is so powerful and how it makes life so much easier!

yeah! this paragraph is scary! I think terms like metric tensor, relativity curvature, geodesic, ten non linear partial differential equations really brings out how tough this problem is to solve actually. to top that off, some frankensteinish name like Christoffel really tops it off.

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Ten partial-differential equations! Thank god I am not a physicist.

....And they aren't even linear equations.

I'm a little confused about how it could be 10. Normally these kinds of things represent 3 or 9 because there's 3 directions which can be represented with respect to the same 3 directions. I definitely don't know enough to actually think the 10 is wrong, it just surprises me.

So there are ten if you're talking about 3 space dimensions and one time dimension because it is not a vector equation, it's dealing with tensor-like things. The reason it is ten is more involved... but that's why it's not three.

Agreed, this equation sounds insane and double agreed on the physicist comment.

Without symmetry, it would be 16 equations: 4 possibilities for the first index (mu) times 4 possibilities for the second index (nu). Then symmetry reduces the number from 16 to 10. The symmetry operation is switching index 1 and 2, and it doesn't change anything physical, i.e. it doesn't give you a new equation.

This wording seems off to me. Maybe "The equations are numerous, and worse, nonlinear." I'm not really sure (I'm not a literary genius), but it does not quite read right to me.

what;s gravity

so this is explaining the "brute force" approach to solving the problem?

what about numerical approximations like the Runga-Kutta method?

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My new reply for when my parents ask what I'm learning in school.

but don't you think that's a weird thing to put in a textbook? i wouldn't say we're lowering standards, i would say we're looking for a different answer.

I read it as kind of a joke. It does sound a little weird for a textbook if taken out of context but here I think its clear that we are lowering the expectations of accuracy in the interest of saving ourselves from spending 10 years learning the math behind it.

I agree. Its a trade, we get decreased accuracy, but it can actually be solved in a class period

I like how this ties back to a previous chapter- this book flows really nicely

I agree but I don't think it would be a bad idea to mention what former chapter it does refer to.

that still seems kinda high

I think you're off by a factor of 10 or so.

Yeah this is strange...one thousand pages? I thought we were supposed to make the problem significantly easier by sacrificing some accuracy?

Misner-Thorne-Wheeler, the great bible of GR is 1215 pages. Maybe we are just measuring against that.

yeah i think the idea is to say that it'll be fewer than the "10 years, 1000 pages, 10 non-linear diff eqns." in this case, it will be FAR fewer.

I didn't catch the reference to this great bible of GR, thanks for the note.

I didn't get that either. Thanks

What is that?

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There are 4 steps listed below.

haha. Nice catch. Completely missed that.

In another note, have these "three/four steps" been mentioned before in this way? It's been a week since I've read a memo but I don't remember seeing them spelled out this clearly before. If not, maybe this should be in an earlier section.

I don't think they've been numbered like this but we have used these steps before

I think its a nice little summary of what we've been doing.

I think enumerating the steps in this way is very useful and should be added when dimensional analysis is introduced.

Of the four steps, I still feel like this one is the most difficult. It requires a lot of initial thinking, whereas manipulation of the dimensionless groups seems less thought-intensive.

I can already see how this is going to be more useful than it was in the last unit

So we're not using the equations at all?

Well, we are, just not that particularly nasty one.

Right, I think we're going to form dimensionless groups which will show us the important ratios we should care about.

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i like the fact that you listed out the steps...aside from the fact that there are 4 listed steps here and you wrote "three"

I agree. Steps are necessary for any method and I don't believe they were enunciated before.

Yeah, these steps would be nice to see in the previous chapter (the one that introduces dimensionless analysis)

I agree that the steps help. I usually list steps when learning a new method for approaching a problem. Could you list steps for other methods? I think it would help to understand them.

Yeah I really like how the steps summarize what we will be doing in this chapter. It makes it easier to follow. Also, this provides a start to solving problems that may seem daunting, and I think using this will raise my confidence when dealing with such problems.

I, also, really like the fact that you listed the steps here...i think that doing something similar would work really well in a couple of the more complicated examples from earlier.

how does one define "most general" here? i'm not sure what that means in math terms.

Before, we've tried to incorporate as many terms as possible, so perhaps it's something involving that. Then 'most general' would mean not neglecting a parameter that shows up in the dimensionless groups.

most general basically means describing the phenomena using the most simple and fundamental variables (which are arranged into dimensionless groups)

How do we know what the most general dimensionless statements are?

By "most general" I mean the form in which one dimensionless group is a function of all the other dimensionless groups. That's as general a statement as you can make. (Then you add physical knowledge to restrict the statement.)

Several of you suggested that I include the four-step (not three-step!) recipe in earlier sections. That's a good idea, and it would also make it clearer here what I mean by "most general" dimensionless statement.

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The first step in a dimensional analysis is to decide what physical parameters the bending angle can depend on. For that purpose I often start with an unlabeled diagram, for it prods me into thinking of labels; and many of the labels are parameters of the problem.

If possible, I think it would be nice to include more explanation about this point before this reading. I believe in the previous reading, this corresponds to the section where you debated including c or h in the dimensionless value, and I was very confused where all of that was coming from. Perhaps a reordering of sections would make these points more clear.

Is this kind of like divide and conquer? using items you already know to assist in making the dimensional analysis of the unknown.

I think you have to take a little from divide and conquer for just about every problem you ever do, however this is a more detailed approach to answering questions.

great process.

"are followed" or "are covered"?

"We use these steps in the following sections"? ..."are done" does not read right.

I don't think this last comment is that necessary.

I think it actually guides the reader.

I think you should say somewhere above that the thing you are going to figure out is the angle.

I'm kinda having trouble with the idea of gravity bending a photon. Could you maybe put a little in about that concept?

5.4 Bending of light by gravity

Rocks, birds, and people feel the effect of gravity. So why not light? The analysis of that question is a triumph of Einstein's theory of general relativity. We could calculate how gravity bends light by solving the so-called geodesic equations from general relativity:

$$\frac{d^2x^\beta}{d\lambda^2} + \Gamma^\beta_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} = 0,$$

wwhere $\Gamma^\beta_{\mu\nu}$ are the Christoffel symbols, whose evaluation requires solving for the metric tensor $g_{\mu\nu}$, whose evaluation requires solving the general-relativity curvature equations $R_{\mu\nu} = 0$.

The curvature equations are themselves a shorthand for ten partial-differential equations. The equations are rich in mathematical interest but are a nightmare to solve. The equations are numerous; worse, they are nonlinear. Therefore, the usual method for handling linear equations – guessing a general form for the solution and making new solutions by combining instances of the general form – does not work. One can spend a decade learning advanced mathematics to solve the equations exactly. Instead, apply a familiar principle: When the going gets tough, lower your standards. By sacrificing some accuracy, we can explain light bending in fewer than one thousand pages – using mathematics and physics that you (and I!) already know.

The simpler method is dimensional analysis, in the usual three steps:

1. Find the relevant parameters.
2. Find dimensionless groups.
3. Use the groups to make the most general dimensionless statement.
4. Add physical knowledge to narrow the possibilities.

These steps are done in the following sections.

5.4.1 Finding parameters

The first step in a dimensional analysis is to decide what physical parameters the bending angle can depend on. For that purpose I often start with an unlabeled diagram, for it prods me into thinking of labels; and many of the labels are parameters of the problem.

This is an interesting approach to solving an abstract problem. It kind of makes sense. It stirs ideas and makes the thinking process more exciting.

I agree. It reminds me of the usefulness of Free Body Diagrams.

It is often hard to visualize all of the parameters of a problem in one's mind, but a diagram can bring many things to light.

I like this because it lets you think about all kinds of variables that could be causing the bend while keeping track of things like direction. Really reminds me of those free body diagrams from 8.01

this is how I started all my physics problems...!

This approach makes a lot of sense, and it is something that I find I do very often since I am a visual learner. It just makes more sense to draw things out.

I think all primates, including humans, are visual learners. We have so much brain hardware devoted to visual processing, so we are very smart when we use pictures and diagrams. People who call themselves visual learners are people who have realized this out already!

labels == quantities?

i think this might refer to variables?

I think it's closer to variables. As in, he is not actually drawing on the force lines and "labeling" which force causes the light to do what. Just a general picture of what it is doing.

He is trying to identify the important dimensions present.

yeah, I read his paper on teaching first year physics in college and how these diagrams are the best to conceptualize the problem, I think that's what he's doing here.

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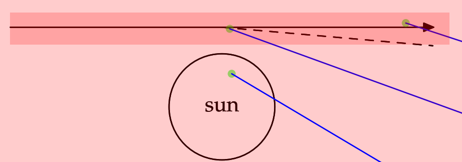

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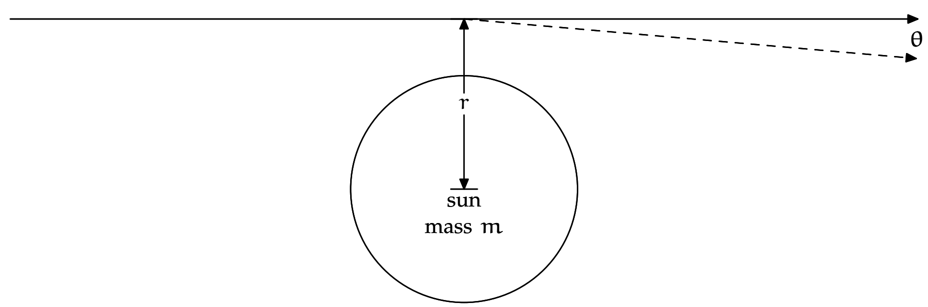

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r	distance from center of sun	L

On a completely unrelated note to this memo, I love the auto adjust when you scroll to a new page.

Sacha will be very glad to know that! (I'll point him to this thread.)

why is light represented as a straight line? how are you quantifying the amount of light included in this "line"?

What does this line represent?? Is it light? If so where is it coming from if that's the sun? Other stars?

when i read the text leading into the diagram, my assumption was that we were thinking in terms of the earth's gravity...thinking more about it, i get why the sun is _so_ much better, but it might be nice to say sun earlier...or explain why the sun works better.

The paragraph above was kind of lost on me until I saw this diagram - maybe having it before the page break would be good.

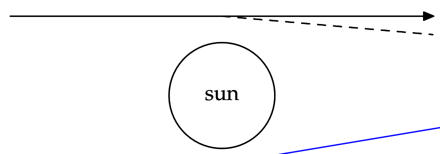
we see the light from other stars and galaxies in the night sky. is this same light coming from these distant places assumed to be straight and unaffected by other suns (including our own) or blackholes this far away from their origin?

I would like to see an example of a parameter that you might think it useful but is actually not and the reason for that. I liked in class when you asked us to list parameters in the "Catholic falling into the manure" example. Then, you went through the ones suggested by the class and gave reason as to why they would or would not be useful. This helped me to understand why some parameters are important while others are irrelevant.

Is it bad to include too many parameters? Or will they come out of the dimensional analysis at the end?

Including too many parameters will make the dimensional analysis very difficult. You'll end up with a statement of the form, "this group is a function of these other five groups", and have a hard time figuring out the function of five variables.

So, toss out parameters early, as if they were deadweight in a lifeboat. If you overdo it, no worries: add one or two back later to fix it up. In short, err on the side of too few than too many.



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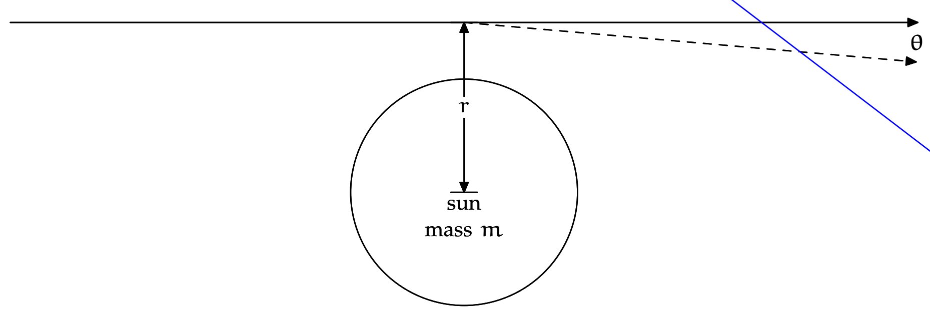

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I think you meant to add in "are" between "Here" and "various" to read "Here are various..."

Does it help to start with the equations we know about gravity then eliminate ones that aren't relevant? I'm not so good at just thinking about parameters for a situation without an equation or something as a starting point.

I think the point of the dimensional analysis is that we don't really know what the equations are or how to use them. For this situation, we don't know how to calculate the force of gravity on light, what would you use for the mass of light? Dimensional analysis allows us to not worry about the physics.

> Right. And if you use the Einstein equations naively, you'll often miss an important quantity, namely G (item 4 in the list), because the equations are often written in a unit system in which c and G are both set to 1.

I think that's pretty much what he's doing here–he's using the diagram to come up with parameters that affect gravity (and these parameters he's listing are in the gravity equations we know). basically he uses the diagram as his starting point.

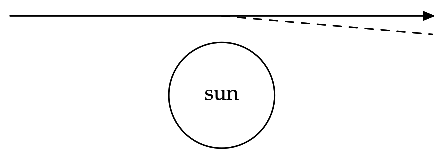
After making a diagram like this, maybe the light's not having an easy mass is apparent, and so the usual Gm1m2/R^2 doesn't apply. To some degree that's why we're trying this problem, since it baffles me how you would start with Newtonian gravity.

It was nice to see this reasoning worked out. I feel like identifying the relevant parameters is probably the most daunting aspect when using dimensional analysis to solve problems.

We've mentioned this in class, but I oftentimes get too focused on other parameters that I forget the parameter that I want in the end!

that's why it is helpful to list it first always... at least for me.

not a "have to" thing but the sentence could be clearer if it said "Since we are solving for theta, the list obviously must include theta" or something more direct



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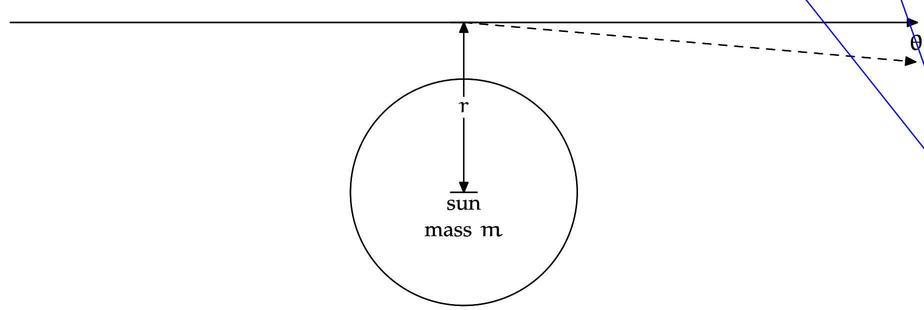

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how exactly do we simplify the problem to break it down to just theta? doesnt light and all mass want to orbit as opposed to simply bending linearly?

If you think of it in terms of his diagram even without knowing the physics, it's clear that there is a bending angle theta. And uh, still without knowing the physics, but just experience tells us that light will bend without orbiting (it escapes that)

I think it should be stated earlier the relationship between curvature and theta. I, for some reason, thought they were different things, and in the explanation at the start, theta is not mentioned.

Of course this is the classical example, but some motivation for using the sun might be nice.

I concur, at first I read back to see if I missed something on why we chose this particular star of ours instead of some other stellar body.

the sun is the celestial body that is easiest for us to relate to and explain this phenomena. Everyone knows about the sun, while others may not know as much about other celestial bodies.

This question reminds me that there is another reason to use the sun: We move a lot relative to the sun, so we can take pictures of the sky with and without the sun (to do it with the sun requires an eclipse). Whereas relative to other stars, we hardly move at all, so we wouldn't get with-and-without pictures.

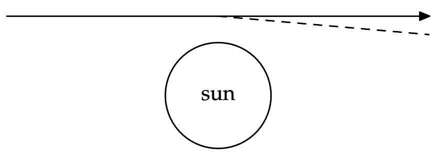
If I'm understanding this correctly, perhaps use bullets to represent that they are different groups? Or adding "and"?

I'm not sure what you mean by this.

A numbered list implies a step-by-step process (i.e. the points are ordered) whereas bullets do not.

I can guess that the mass definitely has to do with it, but how large are black holes compared with other objects in our galaxy? I have no intuition about this.

I feel like this sentence doesn't flow too well after the previous one. Perhaps with a better transition or parallel structure, this sentence would flow more smoothly in the text. How about "The huge mass of a black hole is probably the reason it can deflect light."



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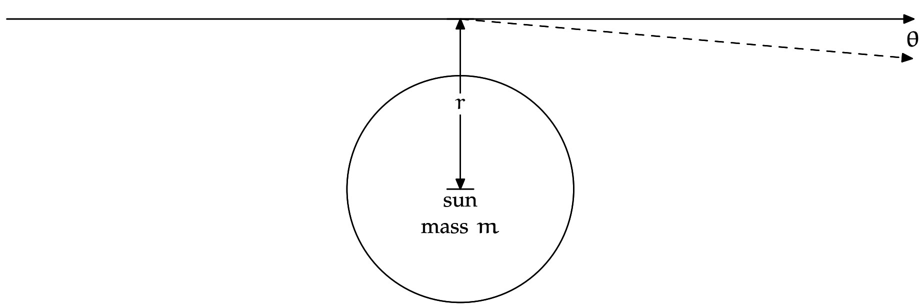

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Deflect makes sense, but i had to read the sentence twice to understand it. When I think of a black hole, I usually think of it attracting light, not deflecting it, because of its mass. Maybe use the word bend instead?

i agree. i had the same mental image.

I really liked how you included the parameters, so are you saying that light from the sun is bent? Because if the gravitational force of the sun doesn't bend light, why are you including its mass in this calculation?

So we include quantities to solve for and the values that affect those quantities?

whether the mass is concentrated in a star, or if it is concentrated in a point mass of a black hole, it is still the same mass with the same gravitational force.

how do you rationalize using these parameters and not others?

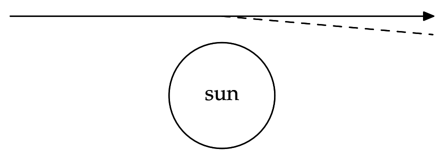
I really like the enumeration and reasoning listed here. It helps me think about these types of problems properly. To answer the above question, I think you just need to limit yourself with a certain number of parameters. Obviously you could go on and on listing other parameters leading to a more precise solution, but that defeats the purpose.

It seems like you need to include things that would affect your first-order solution, and then include things such that the dimensions work out.

Aren't these reasons the rationalization? What others would you suggest?

you could use others if you wanted as long as they make sense. these seem to make logical sense to me, but if you had other ones that appealed to you and made sense I'm confident you would find a similar result.

Now that I think about it, his parameters are not hard to reason. Of course, without the knowledge that gravity bends light, we can assume that light acts like a mass particle and the analogy would still be correct.



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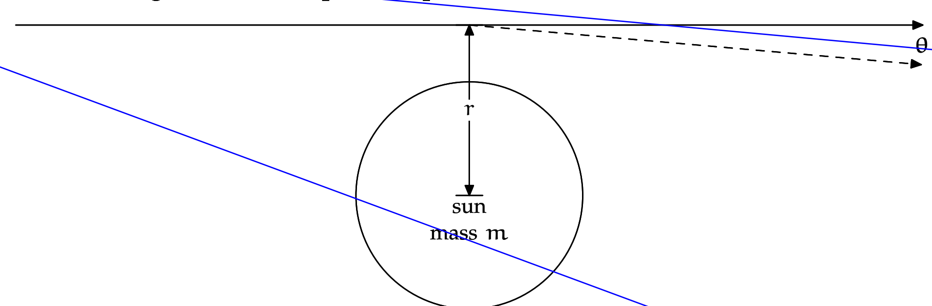

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technically this still doesn't define r that well, since the light will start bending (slightly) before it reaches what would have been the point of closest approach (i.e. its path isn't quite a line with a sharp corner, as drawn below).

is this the same as the line that's perpendicular to the light ray that goes through the center of the sun?

yes, i think it means that line. The dotted line representing the bending isnt supposed to be straight, but since we are approximating this, we can neglect the curvature.

I wish I had read this earlier. I feel like it would have helped me a lot with other classes.

This personification of dimensional analysis is kind of strange.

Could we be sure though that light uses the same G that affects macroscopic physics?

Can you perhaps explain why you include a constant in your list of parameters? I agree that it's important for dimensional analysis, but it is fundamentally different from the previous 3 items and it seems a little out of place on this list. I would have made a special note or a separate list of related constants in this problem.

I think we have to include G here for the dimensional analysis (otherwise we can't get rid of units if we only look at angle, mass, and distance)

I was also a little confused by including the constant G, although it does seem to make sense I'm not sure where it fits in.

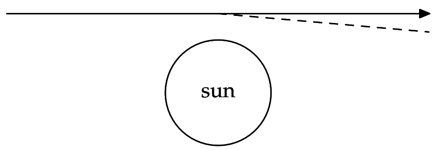
nice diagram

I really appreciate how its a copy of the one just above with the labels added. Something like this would've been useful in the previous sections where we built upon trees and the like but only showed the final diagram.

Yeah, adding pictures in step-by-step is incredibly useful when reading.

Good to know! I'll do that more often. Tufte, a master of information presentation, discusses a similar principle that he calls "Small multiples" (_Envisioning Information_, pp. 28-33).

I really like how this is presented...the same information in 3 different ways! it's awesome.



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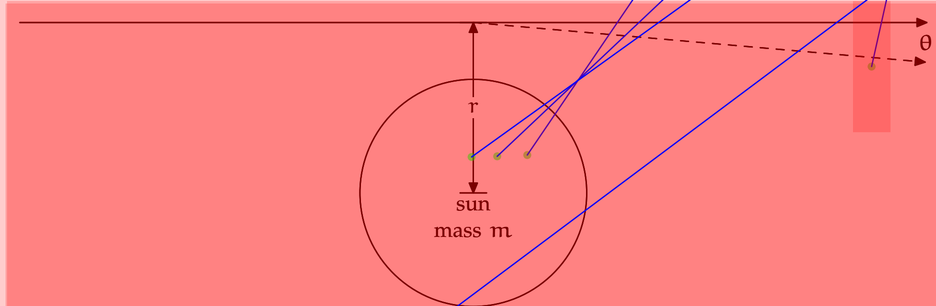

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I like how this diagram is here to easily see the different parameters.

nice summary

So I've been reading that a photon has been estimated to have a rest mass on the order of 10ˆ-27 eV. Is the gravity acting on that mass? or does it have something to do with the idea that as something's speed approaches the speed of light it's mass becomes infinite?...

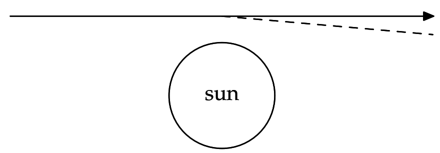
Neither, actually. Think of a marble rolling on a rubber sheet: normally it would travel in a straight line, but if a rock is placed on the sheet, it will warp the sheet and cause the marble to change direction as it goes past the rock.

where would you included G, or since it is a constant it is not on the diagram. I just think I would have forgotten it, is there some way to think about these constants and not forget them?

I really like this diagram to show how we are using each parameter.

These types of diagrams are super useful when doing physics problems.

I really like the coupling of the explanatory list, table, and diagram! 3 times is a charm.



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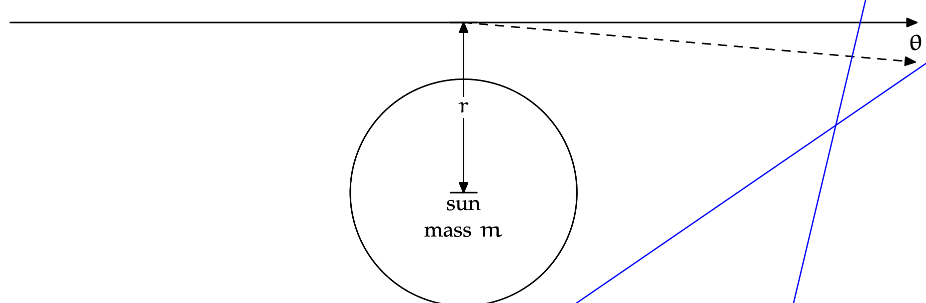

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is this method less effective when the dimensions are not as simple as those in mechanics?

I've only done problems using mass, length, time, and temperature, but I don't see why you couldn't also use this method for other dimension. What types of things were you thinking of?

why doesn't the angle have a dimension of degrees?

See above, angles are dimensionless. We sort of covered this in class in the inclined plane problem, where gsinTheta was lumped together and had the same units as g.

Here is where the distinction between units and dimensions is helpful. Angles are dimensionless: They are the ratio of two lengths (arc length divided by radius). But they have units: radians, degrees, gradians (my ancient calculator has those). The universe cares about dimensions, not our system of units, so reason using dimensions. Often, reasoning using units and dimensions leads you to the same place; but angles are one area where one must be careful.

the whole reason we "lower our standards" is so that we can use these dimensions that we're familiar with...so i think if you hit a situation with really complicated parameters you'd try to express them in terms of these simple ones...

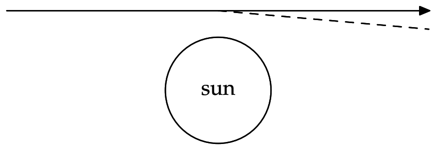
This was really useful to have tabulated here. Though, I have to ask, is angle really also dimensionless?

I think this goes back to a previous section, where the he discussed a difference between dimensions and units.

Where was that?

This was, I believe, covered in class. The only problem this represents is making me think that since it is dimensionless, I can also arbitrarily multiply it when I try and cancel things.

When I think back on trig, I think of angles as length over length (dimensionless), which is why you can use operations like sine on it. Otherwise you'd have some weird dimension like sine-length.



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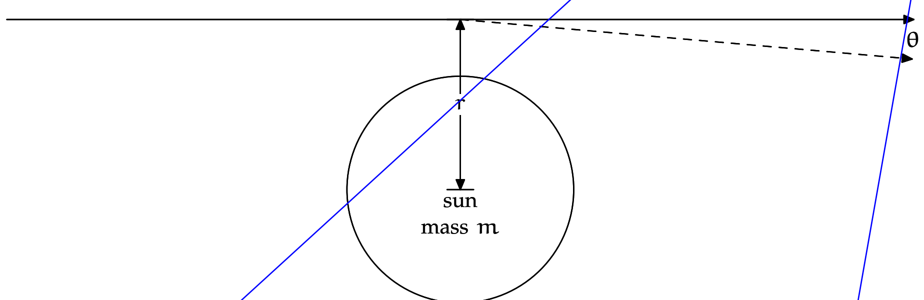

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Should not speed be taken into account? would cancel the time values

I thought the same, at this point in the reading. Since it's addressed later, this is answered. However, I wonder why you chose to not include speed from the start. Perhaps, it's just because we're getting too familiar with these sorts of problems from having done a few in previous readings that utilized "c". Maybe a first-time reader of this section wouldn't necessarily think of using "c". Then again, this is the third reading in a 4 part series, people should have read the first 2 and gotten exposure to c already. Why choose a more roundabout way of accounting for c in the third reading?

How do you know that these are all of the parameters you'll need? How do you know when to stop looking for more variables?

I think you can figure it all from the fundamental equations of the system, which in this case, include these parameters.

where L, M, and T represent the dimensions of length, mass, and time, respectively.

5.4.2 Dimensionless groups

The second step is to form dimensionless groups. One group is easy: The parameter θ is an angle, which is already dimensionless. The other variables, G, m, and r, cannot form a second dimensionless group. To see why, following the dimensions of mass. It appears only in G and m, so a dimensionless group would contain the product Gm, which has no mass dimensions in it. But Gm and r cannot get rid of the time dimensions. So there is only one independent dimensionless group, for which θ is the simplest choice.

Without a second dimensionless group, the analysis seems like nonsense. With only one dimensionless group, it must be a constant. In slow motion:

$$\theta = \text{function of other dimensionless groups},$$

but there are no other dimensionless groups, so

$$\theta = \text{constant}.$$

This conclusion is crazy! The angle must depend on at least one of m and r. Let's therefore make another dimensionless group on which θ can depend. Therefore, return to Step 1: Finding parameters. The list lacks a crucial parameter.

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Here is the latest table of parameters and dimensions:

Comments on page 3

Is there any way that this could go on the same page as the chart?

is there any way this info could go in the previous sections? it seems like a lot of this is too late in the text.

this method reminds me a lot of state space equations from 6.011

there seems to be alot of cross over once you get down to such a level

This seems pretty similar to the sliding block on a spring problem we did during lecture...

i'm not sure what this is saying because it doesn't make sense to me grammatically

Should it be "follow the dimensions of mass" ?

Agreed...what does this mean? Even if it was "follow the dimensions of mass," I think it could be made more explicit.

I think he means look at all the terms that have the dimension of mass somewhere in them.

Perhaps this sentence could be reworded to say "To see why, examine those variables which contain mass in their dimensions." Then you could start the next sentence with "Mass appears only in..." It might also be helpful (albeit a bit redundant) to put the dimension of G and m again to show how the product Gm has no mass dimensions in it. But if you clarify the first sentence I mentioned above, it should definitely make this easier to follow.

I don't understand why you want to "follow" mass to begin with...you can see right away that it could be canceled out. why not 'follow' T right from the start?

I feel like this whole part could be worded better for better understanding.

where L, M, and T represent the dimensions of length, mass, and time, respectively.

5.4.2 Dimensionless groups

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Without a second dimensionless group, the analysis seems like nonsense. With only one dimensionless group, it must be a constant. In slow motion:

$$\theta = \text{function of other dimensionless groups},$$

but there are no other dimensionless groups, so

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what if you accidentally put in speed of light there, then you can form a dimensionless group. so is there a way to check whether this dimensionless group is right or wrong

Agreed. I would have almost certainly included the speed of light. That determines what I would consider the 'duration' that gravity acts on the light. Just like a planet hurtling past will be less affected (smaller theta) at higher speeds, right? (At least intuitively, to me)

Oops, this is addressed entirely at the bottom of this page.

That would be a useful accident, and you'd be well on the way to a solution (see the upcoming text).

this paragraph didnt make any sense to me, but i know what you're trying to say without it.

All it really takes to understand it is to look back at the chart any time a parameter is mentioned. More often than not, you can piece together what's being said based on how the units interact.

don't quite show of this statement? I know you need a second dimensionless group but couldn't i use other methods to finish off the analysis?

Er? I think I'm confused by what this means.

haha I agree...taking it slower so we can understand?

Yeah just a step by step run through of what we're doing.

When I read this I had to smile; I got where you were going immediately, and although I think the earlier explanation is fine, this was a nice summary!

haha yeah, I didn't get it at first, I thought time was going to get slowed down or something lol

While this seems obvious now, esp doing a lot of dimensional analysis in 2.006, I think I've been confused by this before, and why we know this. Perhaps justifying this would help.

I agree, we never really explained why one dimensionless group is a function of the others

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I think I lost track of the logic here – theta is a function of other dimensionless groups, there are no other dimensionless groups... so why must it be a constant?

I think it means that we were trying to approximate theta as a function of other dimensionless groups in the analysis, but because the only dimensionless group is theta, then there's no other varying "function" that theta could depend on. Therefore, theta must be equal to a constant value.

I'm confused too... Why does theta have to be a function of something without dimensions?

how do you know that you didn't just miss a term? that would make the most sense in this situation.

this part was great! it really showed how to conceptualize the thinking in terms of dimensions well, and what was going on in your head.

!!!!!!!!!!!!!!!!!! wow. this seems so hard to believe.

so without enough dimensionless groups, it's possible to come to wrong conclusions? How do you know when you have enough dimensionless groups?

yeah i'm confused about this same point too

I think the point to take away from this is that you need at least two dimensionless groups to solve the problem. Some times you may get more than two dimensionless groups, then you should reduce your number of dimensionless groups to a subset which are independent, meaning you can't make any one group in your set from a combination of the other groups.

Without the correct parameters, you will not come to a viable conclusion - you have to identify all of them! If you end up with only 1 and you are sure of it, then you have a very boring problem.

Thanks! This explanation really helped.

Right – in this problem, physical reasoning tells you that you need at least two dimensionless groups. In other problems, one is enough: for example, in estimating how long it takes an object to free fall from a height h (no air resistance).

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I wouldn't say it's crazy, just very trivial...like concluding 1= 1?

> It's different than 1 = 1, which is indeed trivial. 1=1 is a mathematical tautology, and is always true (just boring). Whereas theta=constant is a physical prediction, and it is almost certainly bogus (although it is interesting).

So i guess the process is iterative. Find some parameters that look like they're relevant. If they don't fit then go back and find more parameters.

This process is still a little ambiguous to me, is there any set of procedures we can use to see if we have found all of the required constants? and what exactly is 'Free Associating'

> I agree...free associating may not work for me sometimes. Is there a more formal process to find dimensionless groups?
>
> > Ooh yeah I def. agree.
> >
> > > Yeah, I definitely agree. The idea of using the speed of light seems obvious now because you just stated that we should, but I wouldn't have a clue about where to start or how to find other necessary values if I was in this position. Is there a more defined way we can go about finding these parameters?
> > >
> > > > I think guess and check is really the only way to go about it. The only way to complete the problem formulaicly is through the ugly physics which we're trying to avoid.
> > > >
> > > > > Ugly? Really? Maybe extremely difficult, but ugly?
>
> Yeah, this phrase is a bit ambiguous as to the meaning of this sentence.
>
> Could we go over free associating in class?

Random and awesome.

hahaha, true and a cute observation

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I don't think humans existed millions of years ago... So they probably weren't studying rocks.

> However, our ancestors were studying rocks, at least implicitly. The primates who had bad abilities at predicting didn't do so well and are less likely to be our ancestors. So, evolution studied it (if I can personify evolution), and built some of that knowledge into our brains.

I'd move this sentence between "... rock." and "Therefore ..." ... it makes more sense/flows better there.

Never thought of it like that!

This is cool - turning light into a projectile

Yeah I never would have thought about light in this way, definitely a cool way to look at it.

this goes back to the constant thing I just talked about-for some reason I feel like they shouldn't matter since they are constant for all while the other variables are the one that are changing and would actually have an effect on the output

now it seems so obvious why did we not include this.

it seems obvious and this shows one of the shortcomings of our diagram - it showed light as a line and not some moving thing so we didnt think to add c to the list of parameters

A little confusing on why c is added in the section of dimensionless constants? Shouldn't it be somewhere else?

I think the addition of c brings the size of the differences into perspective. Also, it works perfectly because we are analyzing the speed of light.

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I feel dumb for not guessing that

Yeah, i was wondering in the above section on finding parameters, whether or not we needed to include the speed of light. but then I thought to myself that the speed of light is constant for all light, so it shouldn't change anything. Except in this case we need to include it to find a dimensionless group.

yeah from this point, it seems that some variable with speed should have been included, I think it's great that you left this to be concluded from the second group's lack of dimensionlessness.

So do we need to begin considering light as mass?

yeah, I was thinking, why do we chose c (the speed of light) to further characterize the phenomena rather than using the mass of light?

edit: updated

Parameter	Meaning	Dimensions
θ	angle	–
m	mass of sun	M
G	Newton's constant	$L^3T^{-2}M^{-1}$
r	distance from center of sun	L
c	speed of light	LT^{-1}

Length is strewn all over the parameters (it's in G, r, and c). Mass, however, appears in only G and m, so the combination Gm cancels out mass. Time also appears in only two parameters: G and c. To cancel out time, form Gm/c^2. This combination contains one length, so a dimensionless group is Gm/rc^2.

5.4.3 Drawing conclusions

The most general relation between the two dimensionless groups is

$$\theta = f\left(\frac{Gm}{rc^2}\right).$$

Dimensional analysis cannot determine the function f, but it has told us that f is a function only of Gm/rc^2 and not of the variables separately.

Physical reasoning and symmetry narrow the possibilities. First, strong gravity – from a large G or m – should increase the angle. So f should be an increasing function. Now apply symmetry. Imagine a world where gravity is repulsive or, equivalently, where the gravitational constant is negative. Then the bending angle should be negative; to make that happen, f must be an odd function: namely, $f(-x) = -f(x)$. This symmetry argument eliminates choices like $f(x) \sim x^2$.

The simplest guess is that f is the identity function: $f(x) \sim x$. Then the bending angle is

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But there is probably a dimensionless constant in f. For example,

$$\theta = 7\frac{Gm}{rc^2}$$

and

Comments on page 4

Why is t his not degrees or radians?

Degrees and radians are still dimensionless

The table is very helpful. (allows us to keep track of the dimensionless parameters).

It seems like these constants were constructed for the sole purpose of generating dimensionless groups. Like people were trying to think of how quantities were related, and just thought of a new quantity that would conveniently cancel out any bothersome dimensions. How, for example, were the dimensions of G found?

You find the dimensions of G from any other equation that you trust with G in it, e.g. F=GMm/r^2.

Why don't we consider the mass of light? Doesn't the gravitation equation require two masses? Would account for this mass, even though it is incredibly small, change the proportionality of the equation?

I think we ignore it because the mass cancels out in Gm anyway.

Also, that may be getting into the nitty-gritty physics which we are trying to avoid.

Correct, in lowering on standards on accuracy, we can ignore the really picky and more or less insignificant quantities that would be just extra work.

so are we omitting it because the mass is insignificantly small or because it cancels out anyways?

I would say that the mass of light is relatively insignificant and like said above by lowering accuracy we can leave out some things. It doesn't look like the mass actually cancels out though since here we've just made a non-dimensional equation, not canceled anything out.

In gravity problems, the mass always cancels out (the force is GMm/r^2 and the acceleration is GM/r^2, which is independent of m). I think I should discuss this point earlier in the text when the list is being constructed.

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I really like this paragraph. I like how it goes through the process step by step. It cleared up my questions when it came to dimensionless groups.

Couldn't you have written the inverse of rc^2/Gm?

include a table of the dimensionless constants since there are 2 of them?

The previous two sections are very good; the explanation is clear and following is easy. It feels very similar to lecture, which is also easy to follow. I wouldn't alter it too much.

This is so cool how you can figure out how to put together the equation. But I still don't understand if we are treating light as a mass or not?

I was really surprised you could come to this conclusion without any calculus or physics.

I took me reading this a few times to see the 'function of' notation there...kept thinking, why aren't you saying this and then proving it.

i'm confused by this function business.

Well, theta could equal Gm/rc^2, or the sin(theta) could, or we could square the entire right side, or...

All we know is that the angle relates to some function of that "group" although we do not know if it is linear with the group, etc.

I really liked the example in class where you showed us a complex physics phenomena where you don't need to memorize individual quantities but only their combination. It's really cool to see the parallels between this text and class and helps to solidify my understanding of these concepts.

do you have any idea of what the function is? is it usually linear, or polynomial or could it really be anything? is that something you become better with over time?

I'm a bit confused, if we know from earlier that the end result is non-linear, do these analysis still hold?

The non-linearity may come in in the form of the function f, but we can still say that it must be a function of this particular product, I think.

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this answers my previous question

I wasn't sure how we would go about determining the functions of out dimensionless values but this makes a lot of sense. I was confused about this in class a bit so I'm glad we're covering it now.

meaning it has a positive slope? but what parameters would be the independent one?

I would have stopped here, but like when you make me realize how much further I can take my estimation.

In this case is negative theta a "pushing away" effect, like the force would cause the angle of deflection to be away from the sun?

but it shouldn't necessarily be equal. Since the bending occurs over a window of time, the object bending towards the sun will be closer (and therefore more affected) by gravity at the end of that timeframe than the object bending away.

The extreme example, is that if the object is moving too slowly, it will spiral into the sun. There is no equivalent 'spiraling away', only being repelled somewhat linearly.

My point is that f(-x) shouldn't intuitively equal f(x), at least not for large-ish x.

I'm not sure I understand what it means for the bending angle to be negative? Also, the point given by the student above is also quite interesting.

But what if you think of the sun as being on the opposite side of the light ray? This would demonstrate a negative gravity from the perspective below the light ray, and also show a negative theta, where the light would bend up, away from anything below the ray.

I like this explanation of why it can't be an odd function.

no...it can't be an even function. maybe the explanation didn't work that well...

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I like this reasoning to determine f(x)

Yeah, this is really interesting reasoning. I never would have thought of this, but it makes total sense.

Can we go over an example where f(x) /= x? It seems like for everything we have done so far we can approximate them as the same and I would be interested to see and example where physical reasoning leads us to another answer

Yeah I really enjoy this. I would have never thought of using this sort of thinking. It's extremely useful.

It is a good guess, but I think it'd be nice to add why guessing x^3 doesn't make sense. I know it's pretty intuitive, but a sentence or two would make the thinking clear.

In this case, I think the example actually detracts. My first instinct was that the correct answer had a 7 in it.

It might be better to write "const." x Gm/(rc^2) like you do on the board sometimes.

Disagree. I would have no idea what constant to use so 7 or 0.3 seem equally likely. Although the confusion about the twiddle versus the proportional sign below does confuse me; I thought twiddle also meant "approximately" and not "proportional to"

Proportional to means the dimensions don't have to agree. A single twiddle means that the dimensions agree but there is a constant of proportionality. A double twiddle means approximately equal (the constant of proportionality is about 1)

I think I shouldn't use 7. It's such a friendly number; not only is it, in popular culture, a lucky number, it is also not irrational. So, I'll use all implausible constants.

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and

maybe instead of having an actual number it might be better to have a "k"?

I don't think it matters what number/variable he uses, since it's an arbitrary constant. It will be removed anyways with the use of twiddle later.

I think that gives you more of a reason to use "k" here rather than an actual number. Or maybe, after using k = 7 and then k = 0.3, you could put a third equation using just k. Then it'll flow nicely into the twiddle later.

I was going to nitpick on the 7 as well. Being an MIT student, I prefer constants in letter form rather than specific cases. But then I read a little farther and saw that he addressed it, so I think in this case it's fine... it appeals both to people who like specificity and generality.

Should this be 'or'?

these could be put on the same line, just saying

$$\theta = 0.3\frac{Gm}{rc^2}$$

are also possible. This freedom means that we should use a twiddle rather than an equals sign:

$$\theta \sim \frac{Gm}{rc^2}.$$

5.4.4 Comparison with exact calculations

All theories of gravity have the same form for the result, namely

$$\theta \sim \frac{Gm}{rc^2}.$$

The difference among the theories is in the value for the missing dimensionless constant:

$$\theta = \frac{Gm}{rc^2} \times \begin{cases} 1 & \text{(simplest guess);} \\ 2 & \text{(Newtonian gravity);} \\ 4 & \text{(Einstein's theory).} \end{cases}$$

Here is a rough explanation of the origin of those constants. The 1 for the simplest guess is just the simplest possible guess. The 2 for Newtonian gravity is from integrating angular factors like cosine and sine that determine the position of the photon as it moves toward and past the sun.

The most interesting constant is the 4 for general relativity, which is double the Newtonian value. The fundamental reason for the factor of 2 is that special relativity puts space and time on an equal footing to make spacetime. The theory of general relativity builds on special relativity by formulating gravity as curvature of spacetime. Newton's theory is the limit of general relativity that considers only time curvature; but general relativity also handles the space curvature. Most objects move much slower than the speed of light, so they move much farther in time than in space and see mostly the time curvature. For those objects, the Newtonian analysis is fine. But light moves at the speed of light, and it therefore sees equal amounts of space and time curvature; so its trajectory bends twice as much as the Newtonian theory predicts.

5.4.5 Numbers!

At the surface of the Earth, the dimensionless gravitational strength is

does this mean that it could be multiplied by any dimensionless constant

I sincerely hope that's the real word to describe that shape. I don't remember your using it in class the day you went over the four symbols, and it seems too fun to be real, but I could have just missed that part.

It's technically called a tilde, but I think twiddle is an accepted 'nickname'. However, i think tilde is more universally known...and maybe it should be changed to that.

And why not a proportional sign, again? This seems to be exactly the time to use a proportional sign – when we are explicitly omitting a constant of proportionality. I feel like there is a lack of consistency throughout these readings on this point.

I thought the twiddle meant approximately, which is roughly the same as the proportional sign. The proportional sign is confusing because it looks like an alpha.

A proportional sign hides a dimensioned constant. We have none missing here. A twiddle hides a dimensionless constant.

This is mentioned a few sentences later

I had not realized the reason for this distinction before. I've been treating twiddles as equivalent to proportionality signs up until now. It would've been nice to explain the difference much earlier on (unless I didn't see it when it was mentioned).

we've been approximating all along. why bring this up now?

I agree, I think we talked about this point in class at some time but it would have been nice to see earlier in writing.

but you don't really know whether the exponent is really 1 though, it might be 2, 3,... how can you assume it's 1 and use a twiddle here

Well it can't be 2... since that's an even function. But I agree otherwise.

That's a very good point that didn't cross my mind. How can we tell if it's 1, 3, etc?

He said earlier that he's just using x^1 because it's the most simple even function. It could in theory be 3,5,etc. but that complexity would take away from the approximation I think.

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What do you mean "all theories?" Are you just referring to the various ways we describe gravity?

I think he's referring to the fact that all the ways that we describe gravity are just theories because we don't really have any way of knowing if these theories are correct. We just come up with theories and their corresponding formulas to describe what we observe, but we can't prove which one if correct. Therefore, they're all theories.

Sorry – I meant, "all of the proposed theories of gravity", including Newtonian gravity, general relativity, and the Brans-Dicke theory. Actually, I probably should not say "all", since I don't know all of them, and there may be some pretty crazy ones out there. But the list above is a reasonable subset of the reasonable theories.

Is this really the answer we would get after solving the original equation and the subsequent 10 non linear equations? That's crazy that we could arrive at the same thing after going through a page of dimensional analysis

The factor of 4 is what you get after solving the horrible equations. So, the dimensional analysis with a bit of guessing (for the function f) gives you almost all of the GR solution, just not the actual value of the constant.

by simplest guess do you mean what would make it easiest? or did someone say that it was 1

Where did these numbers come from?

I was confused too before reading the paragraph below.

They are constants from different theories. It's a favorite story in physics classes about how Newton was off by a factor of two.

At least to me, this is a silly sentence construction.

Agreed - why is this sentence even here?

I think the sentence is here because he's listing out the reasons for each of the constants (simplest guess, newtonian, ...) and it would seem strange to leave one of them out. But I agree it could use rewording. Even placing "simplest guess" in quotes to show that we're referring to the named constant in the expression above could be helpful.

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are also possible. This freedom means that we should use a twiddle rather than an equals sign:

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Does the Newtonian approach work here? We can't use the mass... I guess we are really just deflecting the momentum of the photon?

Yeah, that's my interpretation as well. Kind of interesting to see the differences in the factors.

I definitely did not know this- that's REALLY cool

Yeah it is. And although it is something that is extremely complex, this paragraph does a good job of explaining it to non physics majors like myself.

cool!

A diagram illustrating how spacetime is curved might be helping here. I'm thinking in particular of the classic "bowling ball on a sheet" illustration that represents the warping of spacetime around a sun/planet.

it's almost comical to try to explain special relativity in an approximation textbook in 7 sentences.

But it is interesting and relevant - we don't need a perfect understanding but a general background is helpful.

Regardless of how funny you might think it is, I think it's really nice that Sanjoy puts the background of different fields into his book.

I am simply amazed at how much he knows...

How are units compared here? I feel like we just had a section talking about how we can't make this statement.

I'm also confused, what does it mean to move "farther" in time than in space?

Does everything move the same in time? If so that would explain why slower objects move less far in space than in time.

this is really fascinating.

I think this could be reworded to sound less redundant.

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This is pretty cool.

I would never have guessed that with my naive understanding of physics.

Huh, I didn't realize it worked that way (I still don't understand the physics entirely, but I didn't realize it just doubles like that. I'm sure there's a lot of hidden math here though...)

SO which one is right?!

I don't know much about special relativity or advanced physics but this was a very good and clear explanation. However, how do we know that this is actually the reasoning. Maybe I'm just a skeptic but it would be nice to have some references here.

$$\frac{Gm}{rc^2} \sim \frac{6.7\cdot 10^{-11}\,\mathrm{m^3\,s^{-2}\,kg^{-1}} \times 6.0\cdot 10^{24}\,\mathrm{kg}}{6.4\cdot 10^{6}\,\mathrm{m} \times 3.0\cdot 10^{8}\,\mathrm{m\,s^{-1}} \times 3.0\cdot 10^{8}\,\mathrm{m\,s^{-1}}} \sim 10^{-9}.$$

This miniscule value is the bending angle (in radians). If physicists want to show that light bends, they had better look beyond the earth! That statement is based on another piece of dimensional analysis and physical reasoning, whose result is quoted without proof: A telescope with mirror of diameter d can resolve angles roughly as small as λ/d, where λ is the wavelength of light; this result is based on the same physics as the diffraction pattern on a CDROM (Section 1.1). One way to measure the bending of light is to measure the change in position of the stars. A lens that could resolve an angle of 10^{-9} has a diameter of at least

$$d \sim \lambda/\theta \sim \frac{0.5\cdot 10^{-6}\,\mathrm{m}}{10^{-9}} \sim 500\,\mathrm{m}.$$

Large lenses warp and crack; one of the largest existing lenses has $d \sim 6\,\mathrm{m}$. No practical mirror can have $d \sim 500\,\mathrm{m}$, and there is no chance of detecting a deflection angle of 10^{-9}.

Physicists therefore searched for another source of light bending. In the solar system, the largest mass is the sun. At the surface of the sun, the field strength is

$$\frac{Gm}{rc^2} \sim \frac{6.7\cdot 10^{-11}\,\mathrm{m^3\,s^{-2}\,kg^{-1}} \times 2.0\cdot 10^{30}\,\mathrm{kg}}{7.0\cdot 10^{8}\,\mathrm{m} \times 3.0\cdot 10^{8}\,\mathrm{m\,s^{-1}} \times 3.0\cdot 10^{8}\,\mathrm{m\,s^{-1}}} \sim 2.1\cdot 10^{-6} \approx 0.4''.$$

This angle, though small, is possible to detect: The required lens diameter is roughly

$$d \sim \lambda/\theta \sim \frac{0.5\cdot 10^{-6}\,\mathrm{m}}{2.1\cdot 10^{-6}} \sim 20\,\mathrm{cm}.$$

The eclipse expedition of 1919, led by Arthur Eddington of Cambridge University, tried to measure exactly this effect.

For many years – between 1909 and 1916 – Einstein believed that a correct theory of gravity would predict the Newtonian value, which turns out to be 0.87 arcseconds for light that grazes the surface of the sun. The German mathematician Soldner derived the same result in 1803. Fortunately for Einstein's reputation, the eclipse expeditions that went to test his (and Soldner's) prediction got rained or clouded out. By the time an expedition got lucky with the weather (Eddington's in 1919), Einstein had invented

this is really small, but then again what if theta = (Gm/rc^2) ^(- power, then the angle would be big how can you make sure the exponent is not negative i am a bit confused about this

That wouldn't make sense. We know it has to be a positive power because of proportionality. If G increased, then theta would also have to increase.

since you're looking at earth first, shouldn't your diagram on pg97 say earth? ...this would work for my prev. comment too.

I'd like to see a "theta " at the beginning here.

While the rest of this section seems sort of like a long side note about our approximation, it was very interesting! I'm glad I read it and learned a few things about the history of gravity and general relativity.

this should probably be explained at the beginning (if at all) since we started out saying it's a dimensionless value. otherwise, we know it's radians.

actually, this makes me wonder, how did we decide it was in radians and not degrees?

doesn't light bending also have a lot to do with density of the atmosphere and such? are we just disregarding that?

If your question is about whether telescopes on earth will see light bending more due to the atmosphere than due to gravity, then the answer has to be yes, because no telescopes that exist can see bending of light due to gravity anyway.

If your question is a general question about whether we're ignoring atmosphere throughout the whole section, the answer is still yes. The atmosphere is relatively thin (thin as in height above the surface of the earth, not density) and most of the light that passes by the planet won't go through the atmosphere where it's dense enough to matter. And the light that does go through the atmosphere gets diffracted so much that it ceases to matter anyway. More importantly, that's just not the effect we're interested in studying here.

I don't really see how measuring this would work, what is the change being compared to?

I'm a little confused on how they actually measure this, like how does measuring the change in position of the stars identify the angle?

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This sentence saying how to measure the bending of light seems out of place sandwiched between two sentences on how and why we can't measure bending light on earth. Perhaps it should be relocated to the end of the next paragraph, where you actually talk about this?

I like this position because it helps the reader understand the use for much of the math that is being used. Moving this sentence to the next paragraph would make the last sentence of this paragraph difficult to understand.

Interesting thing I found out just now from wikipedia: to be completed in 2018, the Giant Magellan Telescope will have 7 8.4m mirrors that, together, have the resolving power of a single 24.5m mirror. Also, for reference, the hubble has a 2.5m mirror.

can you show how the two equations for the angle would relate to each other?

is there a way to magnify the result of a small lens to somehow emulate the result of a d 500m lens?

Well, one could use a much shorter wavelength of light, since the minimal angle is lambda/d

This is cool to think about. If one explanation does fit, find another!

I like how from here on out you include some history...it's interesting and very relevant to the section!

why did they choose the largest mass? ...it seems like the densest planet would be a better choice (largest mass with smallest radius)...but I'm not sure.

Wow this surprised me. I didn't think that the light from the Sun would be bent. Also, would using a comparison to slope help explain why light doesn't bend?

It might be nice to note that this is in arcseconds.

That's not that small...aren't most older telescopes this size anyways?

Possibly, but this implies a perfect lens

So even here the value of the angle is really small.

I've never heard of him.

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I'm a bit confused on how an eclipse might prove the result?

I think it was so they could look at the stars behind the sun, which would not normally be visible due to the suns brightness, but I'm not 100% sure.

It's really interesting looking at the history going on a this time as well. According to 8.225, this was one of several efforts to measure this. I think the first one failed because the person sent to measure the eclipse got caught in the war and couldn't actually make the measurements until much later.

If you can insert the images of the starfield and the starfield during the eclipse, that would be really helpful here.

how do you get arcseconds?

An arcsecond is just 1/60 of a degree.

It's interesting to think about how much luck/chance has in scientific discovery.

wow. weather is so important. this is why we should get pretty days all the time! :)

a new theory of gravity – general relativity – and it predicted a deflection of 1.75 arcseconds.

The goal of Eddington's expedition was to decide between the Newtonian and general relativity values. The measurements are difficult, and the results were not accurate enough to decide which theory was right. But 1919 was the first year after the World War in which Germany and Britain had fought each other almost to oblivion. A theory invented by a German, confirmed by an Englishman (from Newton's university, no less) – such a picture was comforting after the trauma of war. The world press and scientific community saw what they wanted to: Einstein vindicated!

A proper confirmation of Einstein's prediction came only with the advent of radio astronomy, in whichsmall deflections could be measured accurately. Here is then a puzzle: If the accuracy (resolving power) of a telescope is λ/d, where λ is the wavelength and d is the telescope's diamater, how could radio telescopes be more accurate than optical ones, since radio waves have a much longer wavelength than light?

It's funny how life works out.

I like the history lesson of the discovery. It makes it more interesting.

i like the history lesson actually

agreed - it's interesting

I'm not sure I understand why that particular piece of information is relevant. It's interesting, of course, but does it somehow make the Englishman more English?

As scientists taught by scientists, I don't think our inclination is to believe that non-scientific forces can affect scientific outcome. I agree that I like the history anecdote.

This is an interesting comment.

that is really funny!

minor grammar? either take out the "to" or add a word, such as "see" after

(typo)

I don't understand this completely. How can you measure light diffraction off a planet or the sun with radio waves?

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Does this have to do with the fact that arrays of radio telescopes can have effective diameters that are incredibly large?

It could be due to the enormous size of radio telescopes (unless I'm confused on what rad teles are) compared to optical ones. Alternately, radio waves may be less susceptible to distortion in ways that would make them more useful / accurate over long distances?

Yep; it's called interferometry, so you can have radio telescopes with an effective diameter the length of the Earth, so it winds up having a higher resolution.

I like how throughout this class we've been realizing why all these little things are the way they are. I never really would have questioned how they determined that particular dimension, but now we know.

I just visited the largest radiotelescope in the world, and seeing this in this week's reading was fantastic!

It also helps that lamda_radio/lamda_visible_light is between 10^7 and 10^9

I also visited the largest radio telescope this past week. The dish itself had a diameter of 300m because you don't have to worry about lenses. So the ability to have a much larger base gives a better resolution.

sweet, I was just reading about this the other day!

VLA?

I like it when you wrap up each section with a concluding paragraph or a take-home message about how to solve these problems. Especially since these reading was a bit longer than usual, would it be possible to add a conclusion to recap what we learned in this section? I'm feeling a bit scattered right now.

I agree. Even though we're currently in a history-laden section, some sort of recap of the principals/methods discussed would be helpful - at least to help solidify the important points.

Yeah, I was expecting something different with the last few paragraphs.

I'm alright with it as long as in the next section there is some explanation of the puzzle. I think the history is alright after all the calculation explanation, and this ending just leads me to believe the section isn't over. I think this may have been a bad place to end a reading, since I'm left to want to know more (and am slightly confused since I don't know the answer to the end).