

Massachusetts Institute of Technology  
Department of Electrical Engineering and Computer Science  
6.061 Supplementary Notes 5  
Introduction To Load Flow

J.L. Kirtley Jr.

February 25, 2001

## 1 Introduction

Even though electric power networks are composed of components which are (or can be approximated to be) *linear*, electric power flow, real and reactive, is a *nonlinear* quantity. The calculation of load flow in a network is the solution to a set of nonlinear equations. The purpose of this note is to describe how network load flows may be calculated.

This is only an elementary treatment of this problem: there is still quite a bit of activity in the professional literature concerning load flow algorithms. The reason for this is that electric utility networks are often quite large, having thousands of buses, so that the amount of computational effort required for a solution is substantial. A lot of effort goes into doing the calculation efficiently. This discussion, and the little computer program at the end of this note, uses the crudest possible algorithm for this purpose. However, for the relatively simple problems we will be doing, it should work just fine.

## 2 Power Flow

To start, we view the power system as being a collection of *buses*, connected together by *lines*. At each of the buses, which we may regard as *nodes*, we may connect equipment which will supply power to or remove power from the system. (Note: in speaking of *power* here, we are really referring to *complex* power, with both real and reactive components). If we have made a connection to a given system node (say with a generator), the complex power flow *into* the network is:

$$\mathbf{S}_k = P_k + jQ_k = \mathbf{V}_k \mathbf{I}_k^* \quad (1)$$

## 3 Bus Admittance

Now, if the network itself is linear, interconnections between buses and between buses and ground can all be summarized in a multiport *bus impedance matrix* or its inverse, the *bus admittance matrix*. As it turns out, the admittance matrix is easy to formulate.

The network consists of a number  $N_b$  of buses and another number  $N_\ell$  of lines. Each of the lines will have some (generally complex) impedance  $Z$ . We form the *line admittance matrix* by

placing the admittance (reciprocal of impedance) of each line on the appropriate spot on the main diagonal of an  $N_\ell \times N_\ell$  matrix:

$$\mathbf{Y}_\ell = \begin{bmatrix} \frac{1}{\mathbf{Z}_1} & 0 & 0 & \cdots \\ 0 & \frac{1}{\mathbf{Z}_2} & 0 & \cdots \\ 0 & 0 & \frac{1}{\mathbf{Z}_3} & \cdots \\ \vdots & \vdots & & \ddots \end{bmatrix} \quad (2)$$

Interconnections between buses is described by the *bus incidence matrix*. This matrix, which has  $N_\ell$  columns and  $N_b$  rows, has two entries for each line, corresponding to the buses at each end. A “direction” should be established for each line, and the entry for that line, at location  $(n_b, n_\ell)$  in the *node incidence matrix*, is a 1 for the “sending” end and a  $-1$  at the “receiving” end. Actually, it is not important which end is which. The bus incidence matrix for the network described by Figure 1 below is:

$$\underline{\underline{NI}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 1 & 1 & 0 \\ 0 & 0 & -1 & -1 \\ 0 & -1 & 0 & 0 \end{bmatrix}$$

It is not difficult to show that the *bus admittance matrix* is given by the easily computed expression:

$$\underline{\underline{\mathbf{Y}}} = \underline{\underline{\mathbf{NI}}} \quad \underline{\underline{\mathbf{Y}_\ell}} \quad \underline{\underline{\mathbf{NI}'}} \quad (3)$$

The elements of the *bus admittance matrix*, the self- and mutual- admittances, are all of the following form:

$$\mathbf{Y}_{jk} = \frac{\mathbf{I}_k}{\mathbf{V}_j} \quad (4)$$

with all other voltages equal to zero.

Thus an alternative way to estimate the bus admittance matrix is to:

- Assume that all nodes (buses) are shorted to ground,
- Assume that one node is unshorted and connected to a voltage source,
- Calculate all node currents resulting from that one source.
- Do this for each node.

We may observe:

- Reciprocity holds:

$$\mathbf{Y}_{jk} = \mathbf{Y}_{kj} \quad (5)$$

- Driving point admittance  $\mathbf{Y}_{kk}$  is just the sum of all admittances of lines connected to bus  $k$ , including any fixed impedances connected from that bus to ground.
- Mutual admittance  $\mathbf{Y}_{jk}$  is *minus* the sum of the admittances of all lines connected *directly* between buses  $j$  and  $k$ . Usually there is only one such line.

Network currents are then given by:

$$\mathbf{I} = \underline{\mathbf{Y}} \mathbf{V} \quad (6)$$

Where  $\mathbf{I}$  is the vector of bus currents (that is, those currents *entering* the network at its buses.  $\mathbf{V}$  represents the bus voltages and  $\underline{\mathbf{Y}}$  is the *bus admittance matrix*. We will have more to say about estimating the bus admittance matrix in another section. For the moment, note that an individual bus current is given by:

$$\mathbf{I}_k = \sum_{j=1}^N \mathbf{Y}_{jk} \mathbf{V}_j \quad (7)$$

where  $N$  is the number of buses in the network. Then complex power flow at a node is:

$$\mathbf{S}_k = \mathbf{V}_k \sum_{j=1}^N \mathbf{Y}_{jk}^* \mathbf{V}_j^* \quad (8)$$

Now, the typical load flow problem involves buses with different constraints. It is possible to specify six quantities at each bus: real and reactive parts of voltage, current and complex power. These are, of course, inter-related so that any two of these are specified by the other four, and the network itself provides two more constraints. Thus it is necessary to, in setting up a load flow problem, specify two of these six quantities. Typical combinations are:

- **Generator Bus:** Real power and terminal voltage magnitude are specified.
- **Load Bus:** Real and reactive power are specified.
- **Fixed Impedance:** A fixed, linear impedance connected to a bus constrains the relationship between voltage and current. Because it constrains both magnitude and angle, such an impedance constitutes two constraints.
- **Infinite Bus:** This is a voltage source, of constant magnitude and phase angle.

The load flow problem consists of solving [8] as constrained by the terminal relationships.

One bus in a load flow problem is assigned to be the “slack bus” or “swing bus”. This bus, which is taken to be an “infinite bus”, since it does not have real nor reactive power constrained, accommodates real power dissipated and reactive power stored in network lines. This bus is necessary because these losses are not known *a priori*. Further, one phase angle needs to be specified, to serve as an origin for all of the others.

## 4 Gauss–Seidel Iterative Technique

This is one of many techniques for solving the nonlinear load flow problem. It should be pointed out that this solution technique, while straightforward to use and easy to understand, has a tendency to use a lot of computation, particularly in working large problems. It is also quite capable of converging on incorrect solutions (that is a problem with nonlinear systems). As with other iterative techniques, it is often difficult to tell when the correct solution has been reached. Despite these shortcomings, Gauss–Seidel can be used to get a good feel for load flow problems without excessive numerical analysis baggage.

Suppose we have an initial estimate (ok: guess) for network voltages. We may partition [8] as:

$$\mathbf{S}_k = \mathbf{V}_k \sum_{j \neq k} \mathbf{Y}_{jk}^* \mathbf{V}_j^* + \mathbf{V}_k \mathbf{Y}_{kk}^* \mathbf{V}_k^* \quad (9)$$

Noting that  $\mathbf{S}_k = P_k + jQ_k$ , we can solve for  $\mathbf{V}_k^*$  and, taking the complex conjugate of that, we have an expression for  $\mathbf{V}_k$  in terms of all of the voltages of the network,  $P_k$  and  $Q_k$ :

$$\mathbf{V}_k = \frac{1}{\mathbf{Y}_{kk}} \left( \frac{P_k - jQ_k}{\mathbf{V}_k^*} - \sum_{j \neq k} \mathbf{Y}_{jk} \mathbf{V}_j \right) \quad (10)$$

Expression [10] is a better estimate of  $\mathbf{V}_k$  than we started with. The solution to the set of nonlinear equations consists of carrying out this expression, repeatedly, for all of the buses of the network.

An iterative procedure in which a correction for each of the voltages of the network is computed in one step, and the corrections applied all at once is called *Gaussian Iteration*. If, on the other hand, the improved variables are used immediately, the procedure is called *Gauss–Seidel Iteration*.

Note that [10] uses as its constraints P and Q for the bus in question. Thus it is useable directly for *load* type buses. For other types of bus constraints, modifications are required. We consider only two of many possible sets of constraints.

For *generator* buses, usually the *real* power and terminal voltage *magnitude* are specified. At each time step it is necessary to come out with a terminal voltage of specified magnitude: voltage phase angle and reactive power Q are the unknowns. One way of handling this situation is to:

1. Generate an *estimate* for reactive power Q, then
2. Use [10] to generate an estimate for terminal voltage, and finally,
3. Holding voltage phase angle constant, adjust magnitude to the constraint.

At any point in the iteration, reactive power is:

$$Q_k = \text{Im} \left( \mathbf{V}_k \sum_{j=1}^N \mathbf{Y}_{jk}^* \mathbf{V}_j^* \right) \quad (11)$$

This sounds like a pretty flaky calculation, leading to justifiable skepticism. Remember, however, that in an iterative computation it is essential only to guarantee that each guess will be better than the prior estimate. The technique described seems to work.

For buses loaded by constant impedance, it is sufficient to lump the load impedance into the network. That is, the *load admittance* goes directly in parallel with the *driving point admittance* at the node in question.

These three bus constraint types, generator, load and constant impedance are sufficient for handling most problems of practical importance.

## 5 Example: Simple–Minded Program

Attached to this note is a MATLAB script which will set up carry out the Gauss–Seidel procedure for networks with the simple constraints described here. The script is self-explanatory and carries out the load flow described by the simple example below.

Note that, as with many nonlinear equation solvers, success sometimes requires having an initial guess for the solution which is reasonably close to the final solution.

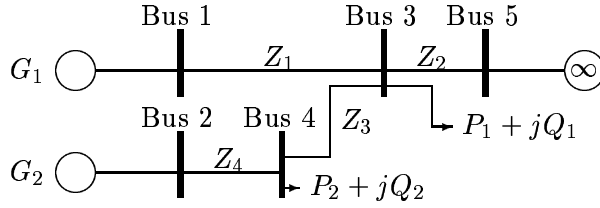


Figure 1: Sample System

## 6 Example

Consider the system shown in Figure 1. This simple system has five buses (numbered 1 through 5) and four lines. Two of the buses are connected to generators, two to loads and bus 5 is the “swing bus”, represented as an “infinite bus”, or voltage supply.

For the purpose of this exercise, assume that the line impedances are:

$$\begin{aligned}
 \mathbf{Z}_0 &= .05 + j.1 \\
 \mathbf{Z}_1 &= .05 + j.05 \\
 \mathbf{Z}_2 &= .15 + j.2 \\
 \mathbf{Z}_3 &= .04 + j.12
 \end{aligned}
 \tag{12}$$

We also specify real power and voltage magnitude for the generators and real and reactive power for the loads:

- Bus 1: Real power is 1, voltage is 1.05 per-unit
- Bus 2: Real power is 1, voltage is 1.00 per-unit
- Bus 3: Real power is -.9 per-unit, reactive power is 0.
- Bus 4: Real power is -1, reactive power is -.2 per-unit.

Note that load power is taken to be negative, for this simple-minded program assumes all power is measured *into* the network.