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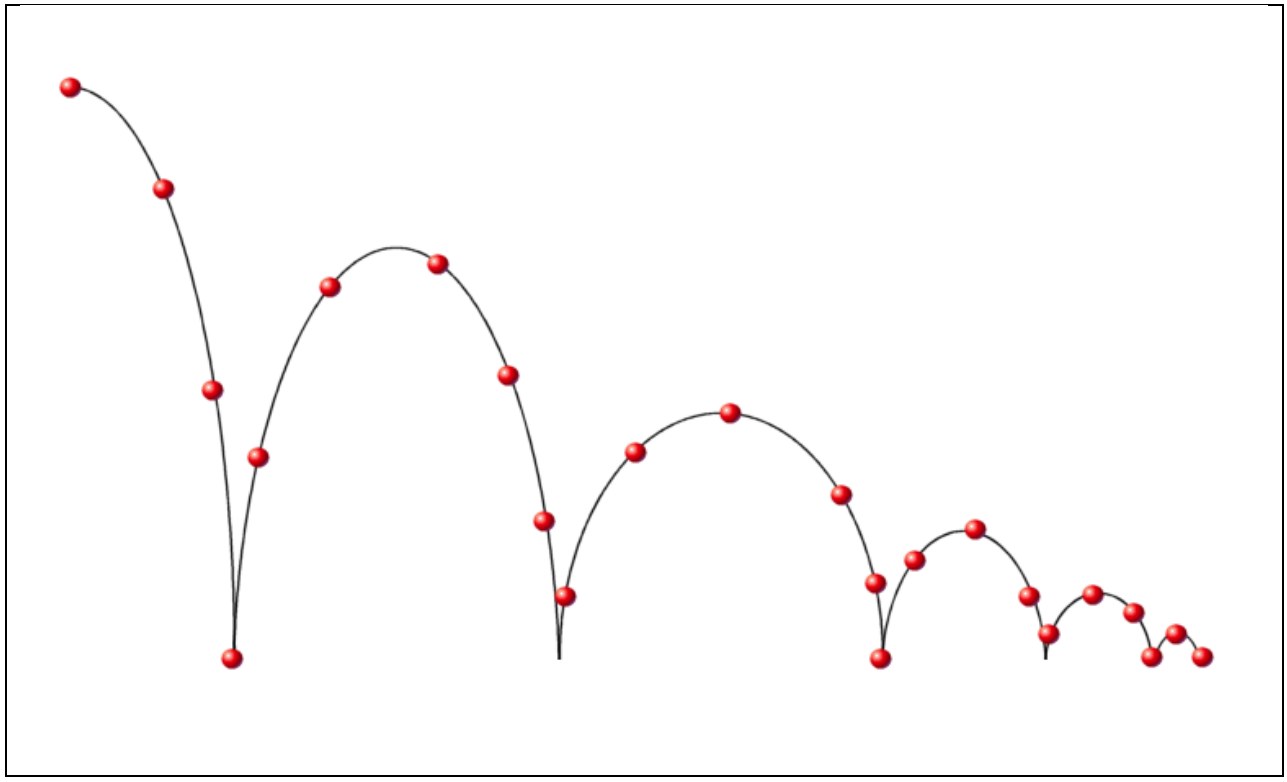
Final Project Proposal

This project enlisted on the 6.101 website as analog computer with display, but I called the analog simulation of a bouncing ball, will try to model the motion of a ball falling from some initial height with some initial velocity and then bouncing on a concrete surface until it loses energy. The design will contain three parts, the motion of the ball in the y-axis over time, the horizontal motion of the ball with time, and displaying the ball itself on an oscilloscope screen in a xy mode. This design which is similar to a paper published by TELEFUNKEN, a German radio and television apparatus company, in 1968 will hopefully demonstrate one of the rather cool things we could do with analog computing, a mainly forgotten technology. . In this design, I want the ball to be contained within small box of walls at h and –h in both dimensions.

Modeling the t vs y motion of the ball.

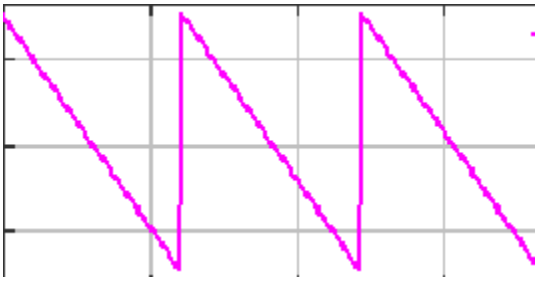
The vertical motion of the ball with time is quite simple. It depends on three variables, the elasticity of the ball, the gravitational force, and the air resistance. The elasticity (the coefficient of restitution) of the ball will determine how high the ball bounces back and the gravitational force will act on the acceleration of the ball while the coefficient of the air resistance will be proportional to the velocity of the ball. Summing this up the motion of the ball will look like a decaying full-wave rectified sinusoid in the most realistic fashion of those coefficients. However, there will be room for moving away from reality by applying abnormal values of these coefficients and absorbing the trajectory of the ball. The motion of the ball can be described by the following equation, where h is the height of the wall:

$$\ddot{y} = -g + \delta\dot{y}\begin{cases}(c/m)(|y| + h)\ if\ y < -h \\ -\left(\frac{c}{m}\right)(y - h)\ if\ y > h\end{cases}$$



Modeling the horizontal motion of the ball.

Since I want to limit the ball to bounce around inside a box, the motion of the ball will look like a charging/discharging capacitor. In other words, the horizontal motion of the ball depends on the velocity with constant initial condition that decays slowly as the ball bounces back and forth.



If the ball starts with some initial velocity of v_0 and there is some coefficient of friction μ and by integrating that and subtracting from the initial condition, and

summing that with the initial condition we have the following equation. $\dot{x(t)} = -(\int \mu dt - v_0)$.

Displaying a ball on the screen.

To display a ball on an oscilloscope screen is need to superimpose sine/cosine pair. We can achieve that by integrating the following second-order equation $\ddot{y} + \omega^2 y = 0$ with some analog circuit. Since the components will not be ideal. We will need to constantly compensate the amplitude.

The components.

The circuit will mainly use components that are available in the 6.101 lab and its stock room. Most of them will be operational amplifiers, in variety of configurations, capacitors, diodes, resistors, etc. However, I may extend the project to playing the ball on a wall and that will need galvanometer, and other electromechanical actuators.

To conclude, the design is not finished yet and I am expecting a bumpy ride with this experiment. The main challenge ahead now is choosing components mainly operational amplifiers.