



# Sudoku Solver

11.06.2018

Magson Gao, Shreeyam Kacker





# Sudoku Primer

## Project Description

### What is Sudoku?



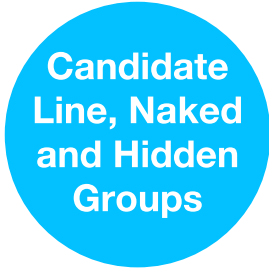
Fill 9x9  
Grid with  
81 × 1-9

### Why FPGA?



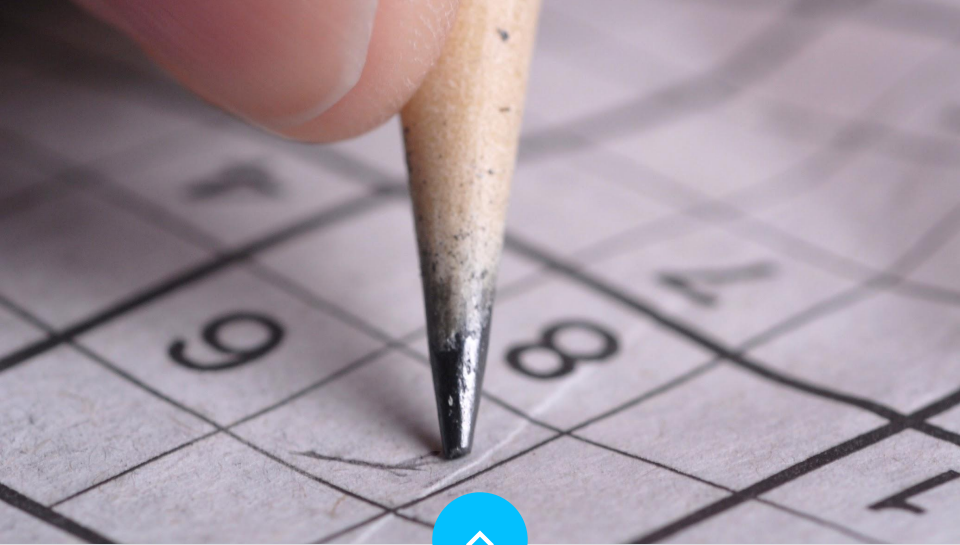
Parallelism

### Techniques?



Candidate  
Line, Naked  
and Hidden  
Groups



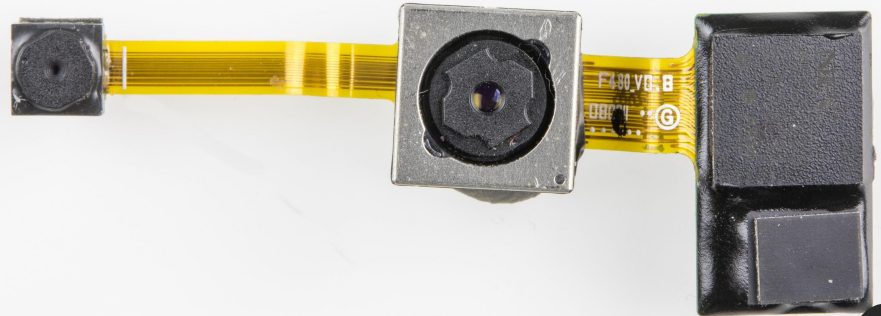


## Goals

- Solve Sudoku using human techniques
- Read Sudoku using a camera
- Stretch Goal: include a tutorial mode which displays the steps for any solve

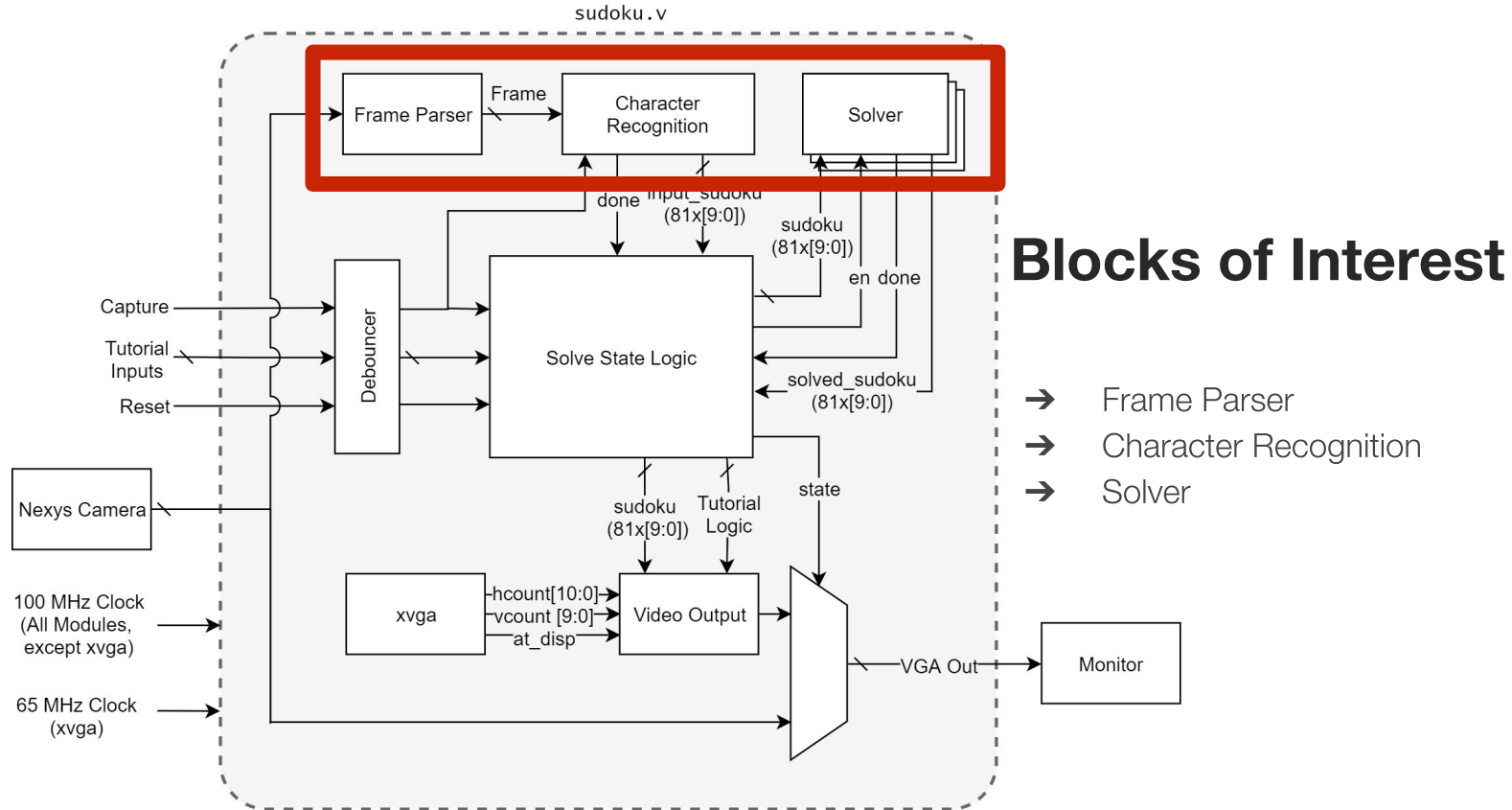
## Human Techniques

- Single position and single candidate
- Candidate line
- Hidden group
- Many more... (but only we have 8 minutes)





# Block Diagram

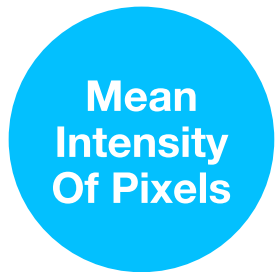




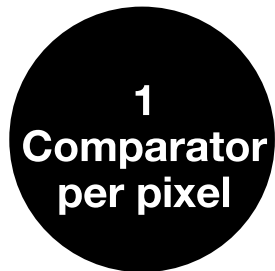
# Frame Parsing

## Reducing the number of gates for Character Recognition

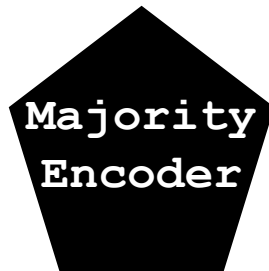
**Calculate  
Threshold**



**Convert to  
Black and  
White**



**Resize**





# Character Recognition

## Template Matching

Store flattened  
B/W values of #s

ROM size:  
9x  
pixels/char  
(bits)

Compare with  
processed  
frames

9 XOR  
operations/cycle  
9 result  
registers

Select

Comparison  
operation  
using LUTs



# Solving a Sudoku



# Keywords

			2	6		7		1
6	8		7				9	
1	9				4	5		
8	2		1				4	
		4	6		2	9		
	5				3		2	8
		9	3				7	4
	4			5			3	6
7		3		1	8			

Squares

Rows

Columns



# Single Candidate/Position



# What can go here?

1,2,3,4,5,6,7,8,9

			2	6		7		1
6	8		7				9	
1	9				4	5		
8	2		1				4	
		4	6		2	9		
	5				3		2	8
		9	3				7	4
	4			5			3	6
7		3		1	8			





# What can go here?

1,2,3,4,5,6,7,8,**9**

			2	6		7		1
6	8		7				9	
1	9				4	5		
8	2		1				4	
		4	6		2	9		
	5				3		2	8
		9	3				7	4
	4			5			3	6
7		3		1	8			



# What can go here?

1,2,3,4,5,6,7,**8**,**9**

			2	6		7		1
6	8		7				9	
1	9				4	5		
8	2		1				4	
		4	6		2	9		
	5				3		2	8
		9	3				7	4
	4			5			3	6
7		3		1	8			





# What can go here?

1,2,3,4,5,6,**7**,**8**,**9**

			2	6		7		1
6	8		7				9	
1	9				4	5		
8	2		1				4	
		4	6		2	9		
	5				3		2	8
		9	3				7	4
	4			5			3	6
7		3		1	8			



# What can go here?

1,2,3,4,5,**6**,**7**,**8**,**9**

			2	6		7		1
6	8		7				9	
1	9				4	5		
8	2		1				4	
		4	6		2	9		
	5				3		2	8
		9	3				7	4
	4			5			3	6
7		3		1	8			





# What can go here?

1,2,3,4,5,6,7,8,9

			2	6		7		1
6	8		7				9	
1	9				4	5		
8	2		1				4	
		4	6		2	9		
	5				3		2	8
		9	3				7	4
	4			5			3	6
7		3		1	8			





# What can go here?

1,2,3,4,5,6,7,8,9

			2	6		7		1
6	8		7				9	
1	9				4	5		
8	2		1				4	
		4	6		2	9		
	5				3		2	8
		9	3				7	4
	4			5			3	6
7		3		1	8			





# What can go here?

1,2,3,4,5,6,7,8,9

			2	6		7		1
6	8		7				9	
1	9				4	5		
8	2		1				4	
		4	6		2	9		
	5				3		2	8
		9	3				7	4
	4			5			3	6
7		3		1	8			



# What can go here?

**1,2,3,4,5,6,7,8,9**

5 is the **Single Candidate**

			2	6		7		1
6	8		7				9	
1	9				4	5		
8	2		1				4	
		4	6		2	9		
	5				3		2	8
		9	3				7	4
	4			5			3	6
7		3		1	8			





# Where can 9 go?

			2	6		7		1
6	8		7				9	
1	9				4	5		
8	2		1				4	
		4	6		2	9		
	5				3		2	8
		9	3				7	4
	4			5			3	6
7		3		1	8		5	





# Where can 9 go?

This is the **Single Position**

			2	6		7		1
6	8		7				9	
1	9				4	5		
8	2		1				4	
		4	6		2	9		
	5				3		2	8
		9	3				7	4
	4			5			3	6
7		3		1	8		5	





# How can a bunch of gates do Sudoku?

## One-hot Representation

- Simplifies logic significantly
- Empty Cells = 0
- Determining single candidates is a simple AND operation with masks of possible values
- Determining single positions is a NOR and AND operation between masks





# What can go here?

			2	6		7		1
6	8		7				9	
1	9				4	5		
8	2		1				4	
		4	6		2	9		
	5				3		2	8
		9	3				7	4
	4			5			3	6
7		3		1	8			





# What can go here?

			2	6		7		1
6	8		7				9	
1	9				4	5		
8	2		1				4	
		4	6		2	9		
	5				3		2	8
		9	3				7	4
	4			5			3	6
7		3		1	8			

987654321

9= 100000000

4= 000001000

2= 000000010

7= 001000000

3= 000000100

|= 101001110

~= 010110001

**8,6,5 or 1**





# What can go here?

			2	6		7		1
6	8		7				9	
1	9				4	5		
8	2		1				4	
		4	6		2	9		
	5				3		2	8
		9	3				7	4
	4			5			3	6
7		3		1	8			

987654321

7= 001000000

3= 000000100

1= 000000001

8= 010000000

|= 011000101

~= 100111010 not 1 and not 8

& 010110001

= 000110000





# What can go here?

			2	6		7		1
6	8		7				9	
1	9				4	5		
8	2		1				4	
		4	6		2	9		
	5				3		2	8
		9	3				7	4
	4			5			3	6
7		3		1	8			

987654321

7= 001000000

4= 000001000

3= 000000100

6= 000100000

|= 001101100

~= 110010011

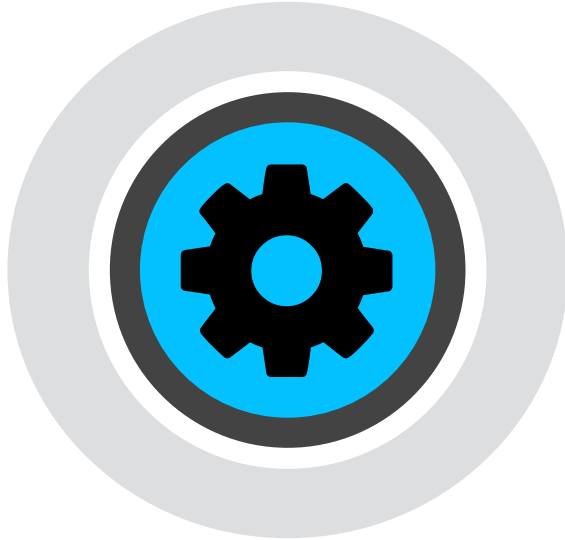
& 000110000

Must be a 5





# Hidden Group Setup



- Suppose we have **N** numbers which can **only** be placed in **N** positions
- Then **all N positions must contain one of the N numbers.**



# What can't go here?

The empty cells contain the possible values:

{4,9}

{4,6}

{2,4}

{1,3}

{2,6}

{1,2,3} -> {1,3}

We have only 2 places where 1 and 3 can go, so it is impossible for 2 to be in the last cell

8	2,5	1	2,7	3,5	6	3,7	9	4
3	2,5	4,6	2,4,7	1,5	9	1,6,7	8	1,2,7
4,9	7	4,6	2,4	8	1,3	5	2,6	1,2,3
5	4	7	8,9	6	2	1,8	3	1,9
6	3	2	8,9	1,4	1,4	7,8	5	7,9
1	9	8	3	7	5	2	4	6
4,7	8	3	6	2	4,7	9	1	5
4,7	6	5	1	9	8	3,4,7	2,7	2,3,7
2	1	9	5	3,4	3,4,7	4,6,7	6,7	8



**The digital algorithm very  
cool! The Q&A is coming up  
\*wink\* \*wink\***

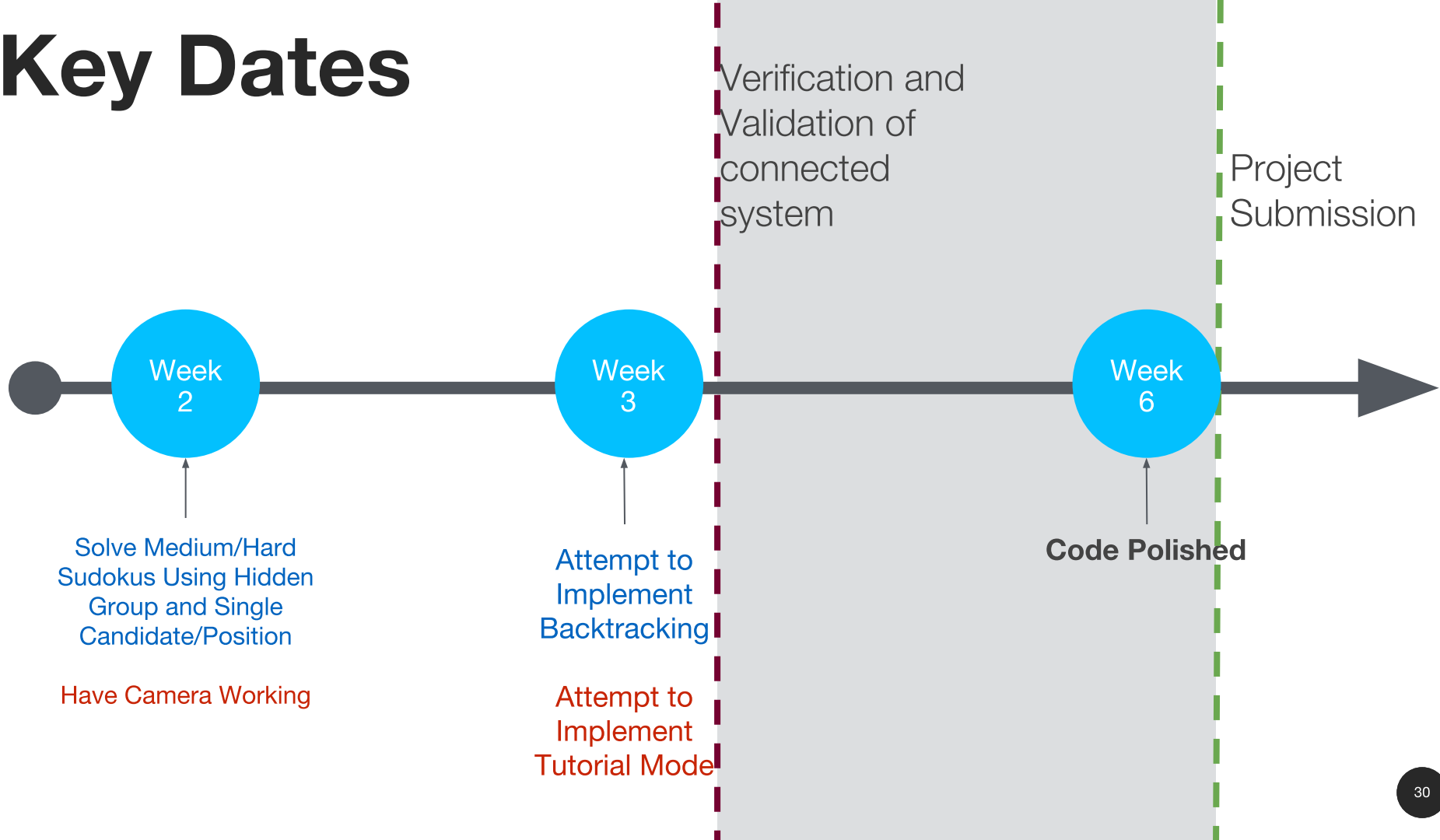





# Project Timeline



# Key Dates







# Thank you for your attention

11.06.2018

Magson Gao, Shreeyam Kacker





# What can't go here?

The empty cells contain the possible values:

987654321

{4,9} = 100001000

{4,6} = 000101000

{2,4} = 000001010

{1,3} = 000000101

{2,6} = 000100010

{1,2,3} = 000000111

In this algorithm we iterate over each set

8	2,5	1	2,7	3,5	6	3,7	9	4
3	2,5	4,6	2,4,7	1,5	9	1,6,7	8	1,2,7
4,9	7	4,6	2,4	8	1,3	5	2,6	1,2,3
5	4	7	8,9	6	2	1,8	3	1,9
6	3	2	8,9	1,4	1,4	7,8	5	7,9
1	9	8	3	7	5	2	4	6
4,7	8	3	6	2	4,7	9	1	5
4,7	6	5	1	9	8	3,4,7	2,7	2,3,7
2	1	9	5	3,4	3,4,7	4,6,7	6,7	8



# What can't go here?

Steps\*

- Add up the number of 1s of the mask we are working on (let's call the number of 1s **X** and the mask we are working on **Y**).
- AND mask **Y** with every other possibilities mask (this includes the ones that are solved, though **these always give 0**). Let us call the results of the AND operations **A**.
- If the number of 0s in **A** are equal to  $(9-x)$  then a **hidden group** is detected
- If the number of masks in **A** with  $(\# \text{ of } 1\text{s} = \mathbf{X})$  is equal to  $(x-1)$  a **naked group** exists between the **Y** and the masks in these positions.
- For every mask in the **hidden group**, replace it with the mask stored in **A**. For every mask outside the **naked group** replace the mask with  $(\text{mask} \ \& \ \sim Y)$ .

8	2,5	1	2,7	3,5	6	3,7	9	4
3	2,5	4,6	2,4,7	1,5	9	1,6,7	8	1,2,7
4,9	7	4,6	2,4	8	1,3	5	2,6	1,2,3
5	4	7	8,9	6	2	1,8	3	1,9
6	3	2	8,9	1,4	1,4	7,8	5	7,9
1	9	8	3	7	5	2	4	6
4,7	8	3	6	2	4,7	9	1	5
4,7	6	5	1	9	8	3,4,7	2,7	2,3,7
2	1	9	5	3,4	3,4,7	4,6,7	6,7	8



# What can't go here?

8	2,5	1	2,7	3,5	6	3,7	9	4
3	2,5	4,6	2,4,7	1,5	9	1,6,7	8	1,2,7
4,9	7	4,6	2,4	8	1,3	5	2,6	1,2,3
5	4	7	8,9	6	2	1,8	3	1,9
6	3	2	8,9	1,4	1,4	7,8	5	7,9
1	9	8	3	7	5	2	4	6
4,7	8	3	6	2	4,7	9	1	5
4,7	6	5	1	9	8	3,4,7	2,7	2,3,7
2	1	9	5	3,4	3,4,7	4,6,7	6,7	8

The empty cells contain the possible

values: 987654321 A: 987654321

{4,9} = 100001000 -> 000000000

{4,6} = 000101000 -> 000000000

{2,4} = 000001010 -> 000000000

{1,3} = 000000101 = Y

{2,6} = 000100010 -> 000000000

{1,2,3} = 000000111 -> 000000101

X = 2,

Number of masks = 0 in A = 4 + 3

Number of masks with 2x1s = 1



# Naked Group Setup



- Suppose we have **N** numbers which are the **only** possible values for **N** positions
- Then **all N positions must contain one of the N numbers. No other position in the row/column/square can contain the N numbers.**



# What can't go here?

The empty cells contain the possible values:

987654321 A: 987654321

**{4,9}** = 100001000 -> 000001000

**{4,7}** = 001001000 = Y

**{4,7}** = 001001000 -> 001001000

X = 2, ~Y = 110110111

Number of masks = 0 in A = 0 + 6

Number of masks with 2x1s = 1

The 3rd cell must be a **9**

8	2,5	1	2,7	3,5	6	3,7	9	4
3	2,5	4,6	2,4,7	1,5	9	1,6,7	8	1,2,7
9	7	4,6	2,4	8	1,3	5	2,6	1,2,3
5	4	7	8,9	6	2	1,8	3	1,9
6	3	2	8,9	1,4	1,4	7,8	5	7,9
1	9	8	3	7	5	2	4	6
4,7	8	3	6	2	4,7	9	1	5
4,7	6	5	1	9	8	3,4,7	2,7	2,3,7
2	1	9	5	3,4	3,4,7	4,6,7	6,7	8