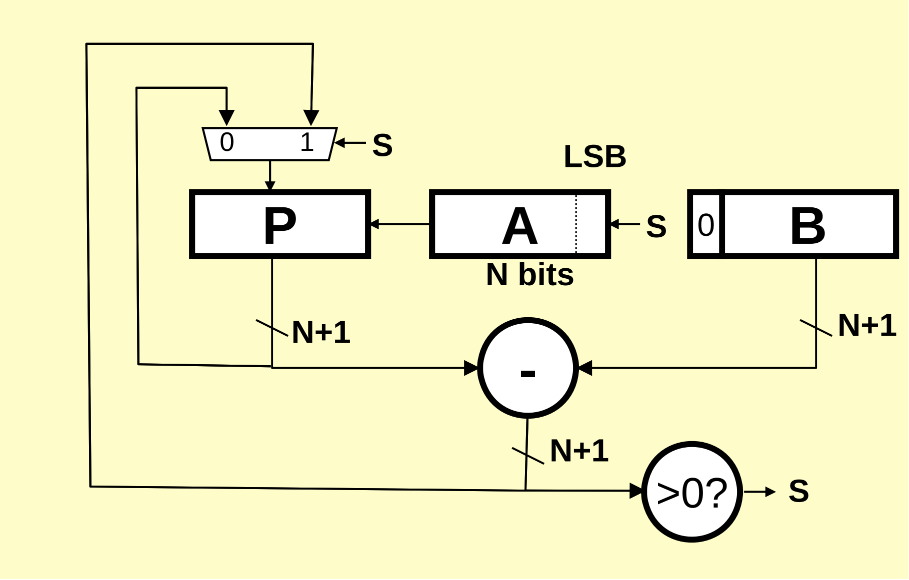
Pipelining & Verilog

- Division
- Latency & Throughput
- Pipelining to increase throughput
- Retiming
- Verilog Math Functions

Sequential Divider

Assume the Dividend (A) and the divisor (B) have N bits. If we only want to invest in a single N-bit adder, we can build a sequential circuit that processes a single subtraction at a time and then cycle the circuit N times. This circuit works on unsigned operands; for signed operands one can remember the signs, make operands positive, then correct sign of result.



```
Init: P←0, load A and B
Repeat N times {
    shift P/A left one bit
    temp = P-B
    if (temp > 0)
       {P←temp, A_LSB←1}
    else A_LSB←0
}
Done: Q in A, R in P
```

Verilog divider.v

```
// The divider module divides one number by another. It
// produces a signal named "ready" when the quotient output
// is ready, and takes a signal named "start" to indicate
// the the input dividend and divider is ready.
// sign -- 0 for unsigned, 1 for twos complement

// It uses a simple restoring divide algorithm.
// http://en.wikipedia.org/wiki/Division_(digital)#Restoring_division

module divider #(parameter WIDTH = 8)
 (input clk, sign, start,
  input [WIDTH-1:0] dividend,
  input [WIDTH-1:0] divider,
  output reg [WIDTH-1:0] quotient,
  output [WIDTH-1:0] remainder;
  output ready);

  reg [WIDTH-1:0]  quotient_temp;
  reg [WIDTH*2-1:0] dividend_copy, divider_copy, diff;
  reg negative_output;

  wire [WIDTH-1:0] remainder = (!negative_output) ?
        dividend_copy[WIDTH-1:0] : ~dividend_copy[WIDTH-1:0] + 1'b1;

  reg [5:0] bit;
  reg del_ready = 1;
  wire ready = (!bit) & ~del_ready;

  wire [WIDTH-2:0] zeros = 0;
  initial bit = 0;
  initial negative_output = 0;
```

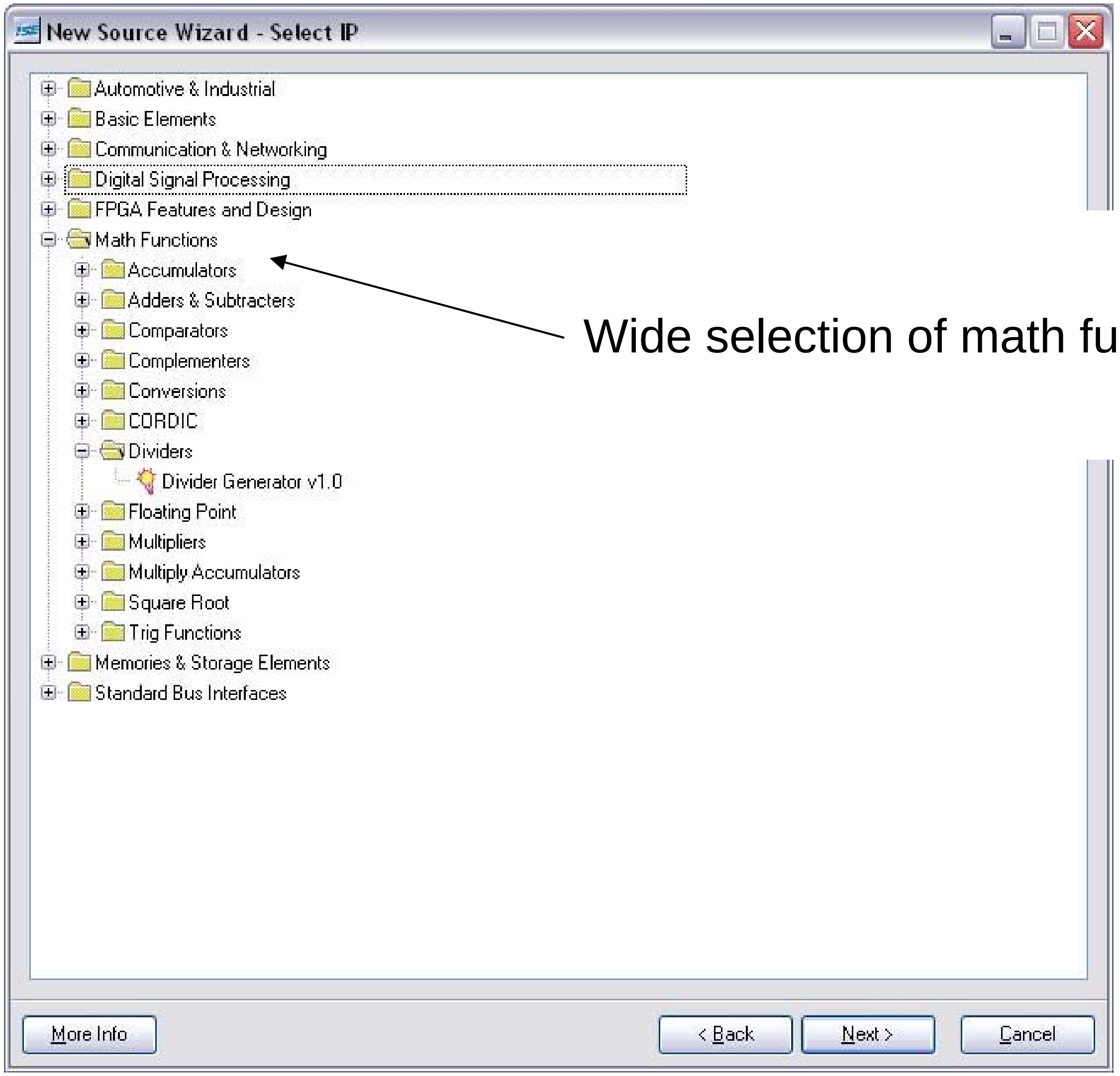
```
 always @( posedge clk ) begin
    del_ready <= !bit;
    if( start ) begin

      bit = WIDTH;
      quotient = 0;
      quotient_temp = 0;
      dividend_copy = (!sign || !dividend[WIDTH-1]) ?
                {1'b0,zeros,dividend} :
                {1'b0,zeros,~dividend + 1'b1};
      divider_copy = (!sign || !divider[WIDTH-1]) ?
                             {1'b0,divider,zeros} :
                             {1'b0,~divider + 1'b1,zeros};

      negative_output = sign &&
                  ((divider[WIDTH-1] && !dividend[WIDTH-1])
                   ||(!divider[WIDTH-1] && dividend[WIDTH-1]));
     end
    else if ( bit > 0 ) begin
      diff = dividend_copy - divider_copy;
      quotient_temp = quotient_temp << 1;
      if( !diff[WIDTH*2-1] ) begin
        dividend_copy = diff;
        quotient_temp[0] = 1'd1;
      end
      quotient = (!negative_output) ?
             quotient_temp :
             ~quotient_temp + 1'b1;
      divider_copy = divider_copy >> 1;
      bit = bit - 1'b1;
    end
  end
endmodule
```

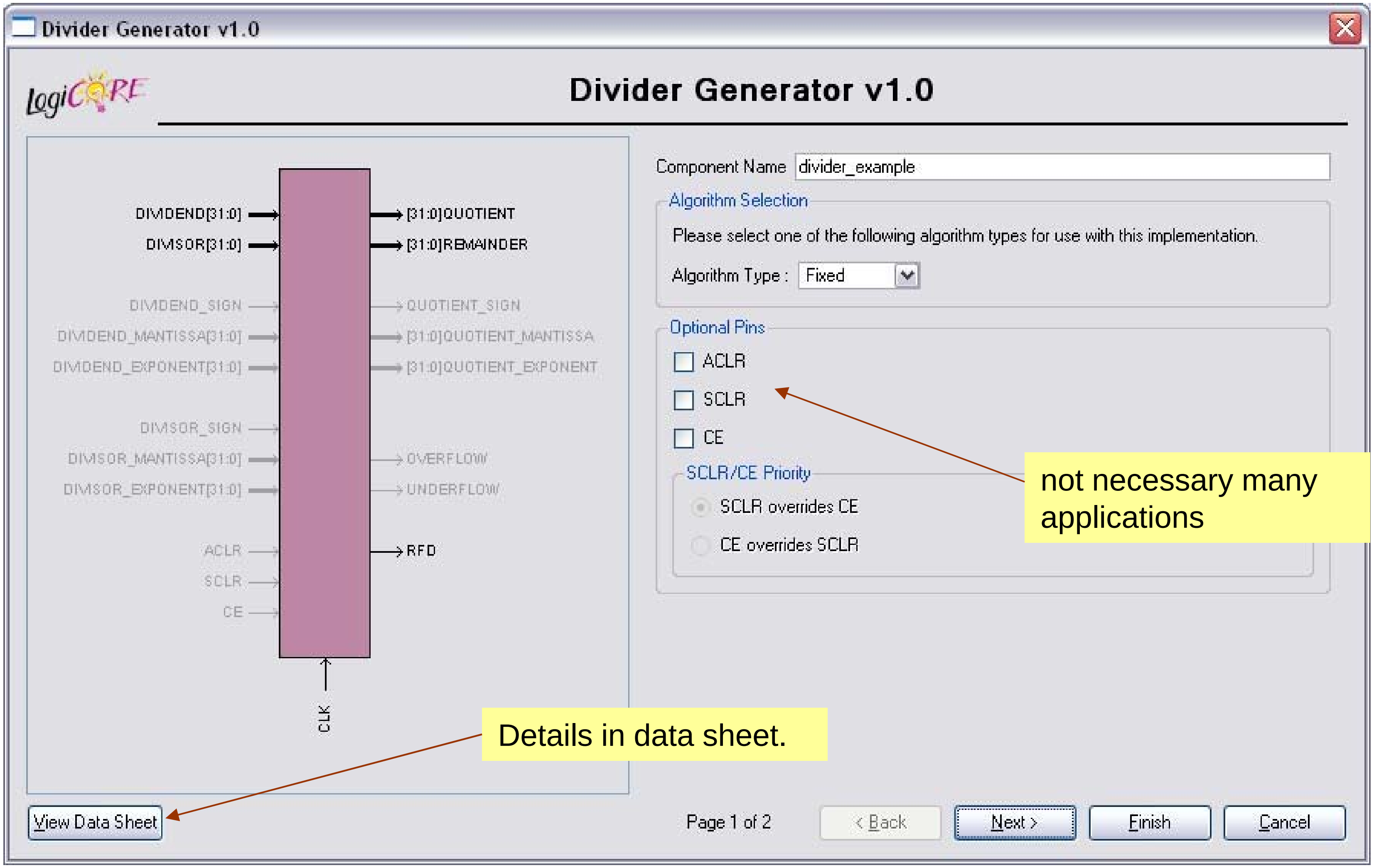
L. Williams MIT '13

Math Functions in Coregen



Wide selection of math functions available

Coregen Divider



Component Name: divider_example

Algorithm Selection

Please select one of the following algorithm types for use with this implementation.

Algorithm Type : Fixed

Optional Pins

☐ ACLR

☐ SCLR

☐ CE

SCLR/CE Priority

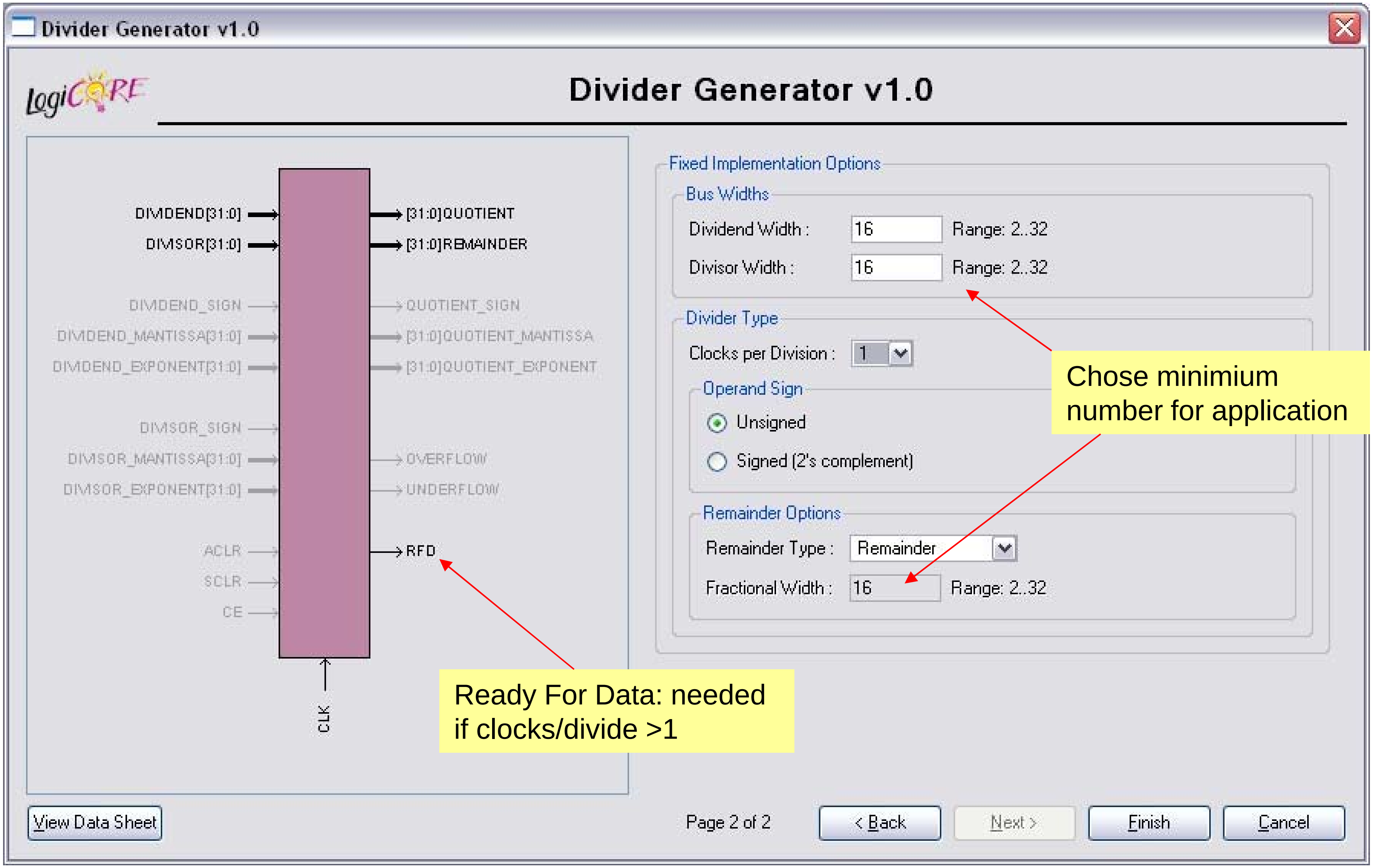
● SCLR overrides CE

○ CE overrides SCLR

not necessary many applications

Details in data sheet.

Coregen Divider



Divider Generator v1.0

Fixed Implementation Options

Bus Widths

Dividend Width : 16 Range: 2..32

Divisor Width : 16 Range: 2..32

Divider Type

Clocks per Division : 1

Operand Sign

● Unsigned

○ Signed (2's complement)

Remainder Options

Remainder Type : Remainder

Fractional Width : 16 Range: 2..32

Chose minimium number for application

Ready For Data: needed if clocks/divide >1

Performance Metrics for Circuits

Circuit Latency (L):	time between arrival of new input and generation of corresponding output. For combinational circuits this is just t_{PD}.
Circuit Throughput (T):	Rate at which new outputs appear. For combinational circuits this is just $1/t_{PD}$ or $1/L$.

Coregen Divider Latency

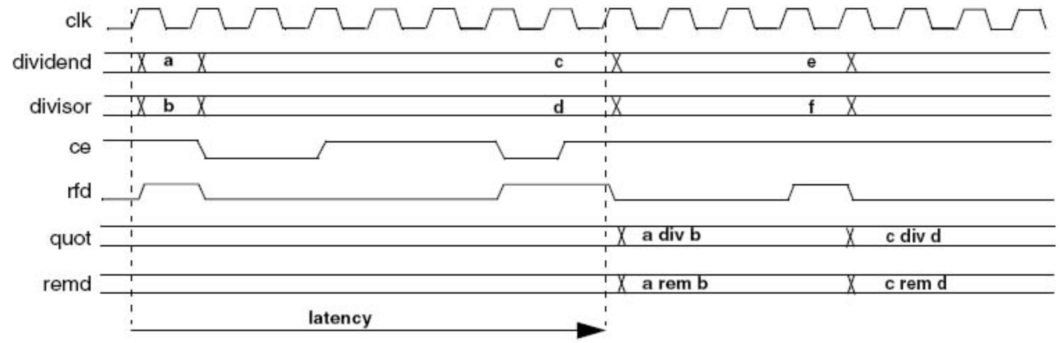


Figure 2: **Latency Example (Clocks per Division = 4)**

Table 4: **Latency of Fixed-point Solution Based on Divider Parameters**

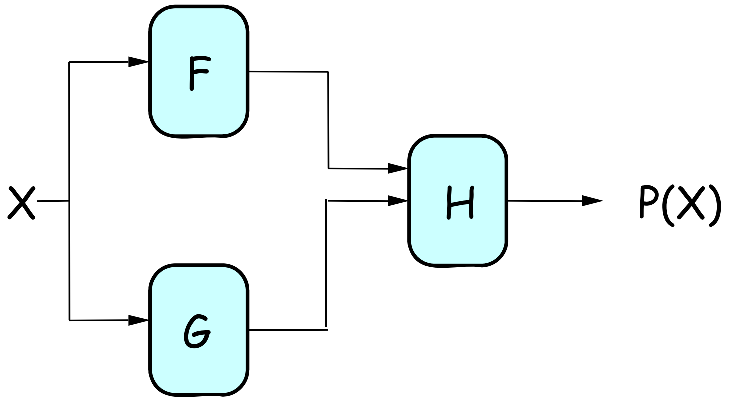
Signed	Fractional	Clks/Div	Latency
False	False	1	M+2
False	False	>1	M+3
False	True	1	M+F+2
False	True	>1	M+F+3
True	False	1	M+4
True	False	>1	M+5
True	True	1	M+F+4
True	True	>1	M+F+5

Note: M=dividend width, F=fractional remainder width.

Latency dependent on dividend width + fractioanl reminder width

The divclk_sel parameter allows a range of choices of throughput versus area. With divclk_sel = 1, the core is fully pipelined, so it will have maximal throughput of one division per clock cycle, but will occupy the most area. The divclk_sel selections of 2, 4 and 8 reduce the throughput by those respective factors for smaller core sizes.

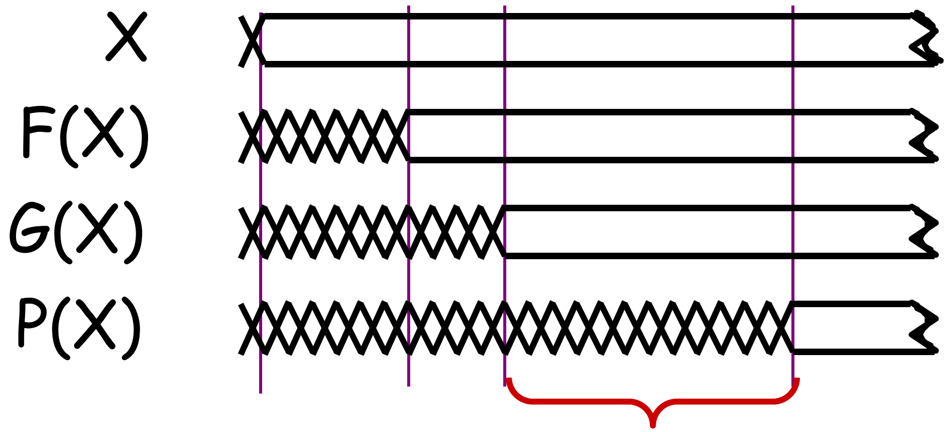
Performance of Combinational Circuits



For combinational logic:

$L = t_{PD}$,
$T = 1/t_{PD}$.

We can't get the answer faster, but are we making effective use of our hardware at all times?

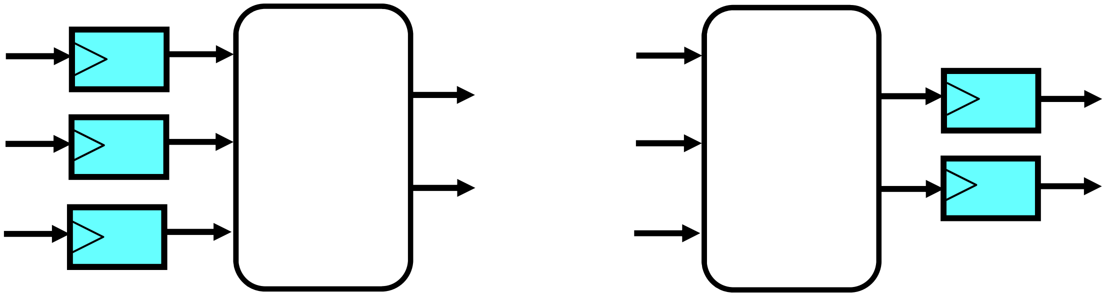


F & G are "idle", just holding their outputs stable while H performs its computation

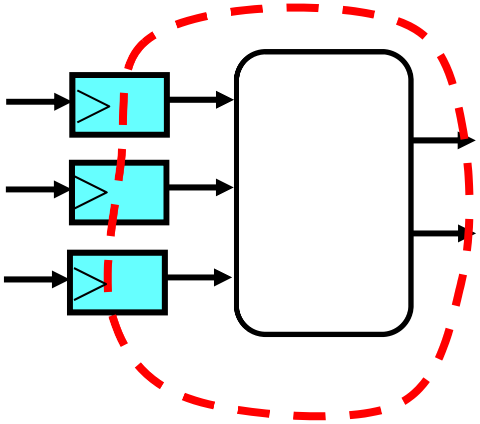
Retiming: A very useful transform

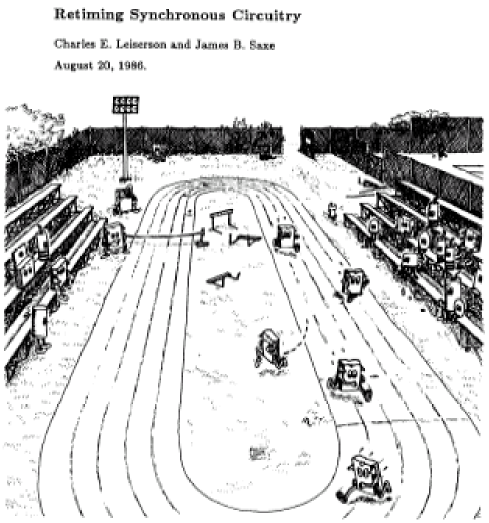
Retiming is the action of moving registers around in the system

- Registers have to be moved from ALL inputs to ALL outputs or vice versa



Cutset retiming: A cutset intersects the edges, such that this would result in two disjoint partitions of the edges being cut. To retime, delays are moved from the ingoing to the outgoing edges or vice versa.





Retiming Synchronous Circuitry

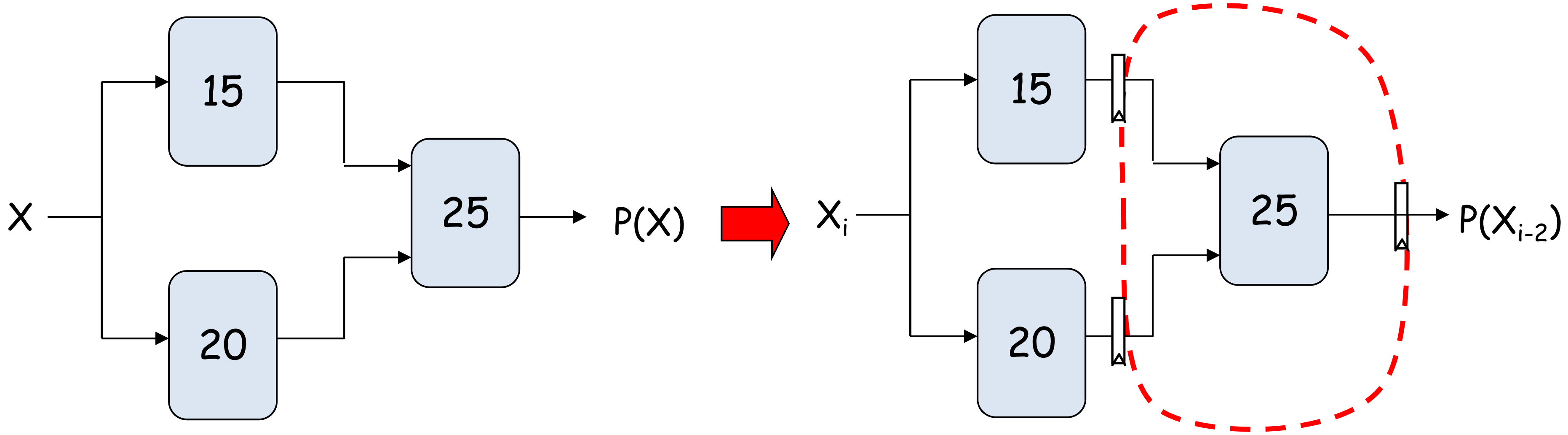
Charles E. Leiserson and James B. Saxe

August 20, 1986.

Benefits of retiming:

- Modify critical path delay
- Reduce total number of registers

Retiming Combinational Circuits aka "Pipelining"

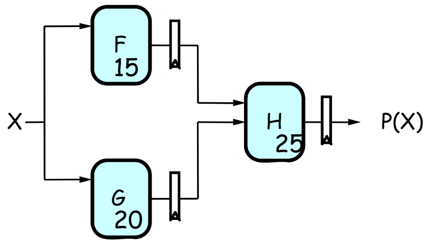


Assuming ideal registers:
i.e., $t_{PD} = 0$, $t_{SETUP} = 0$ →

$t_{CLK} = 25$
$L = 2 * t_{CLK} = 50$
$T = 1/t_{CLK} = 1/25$

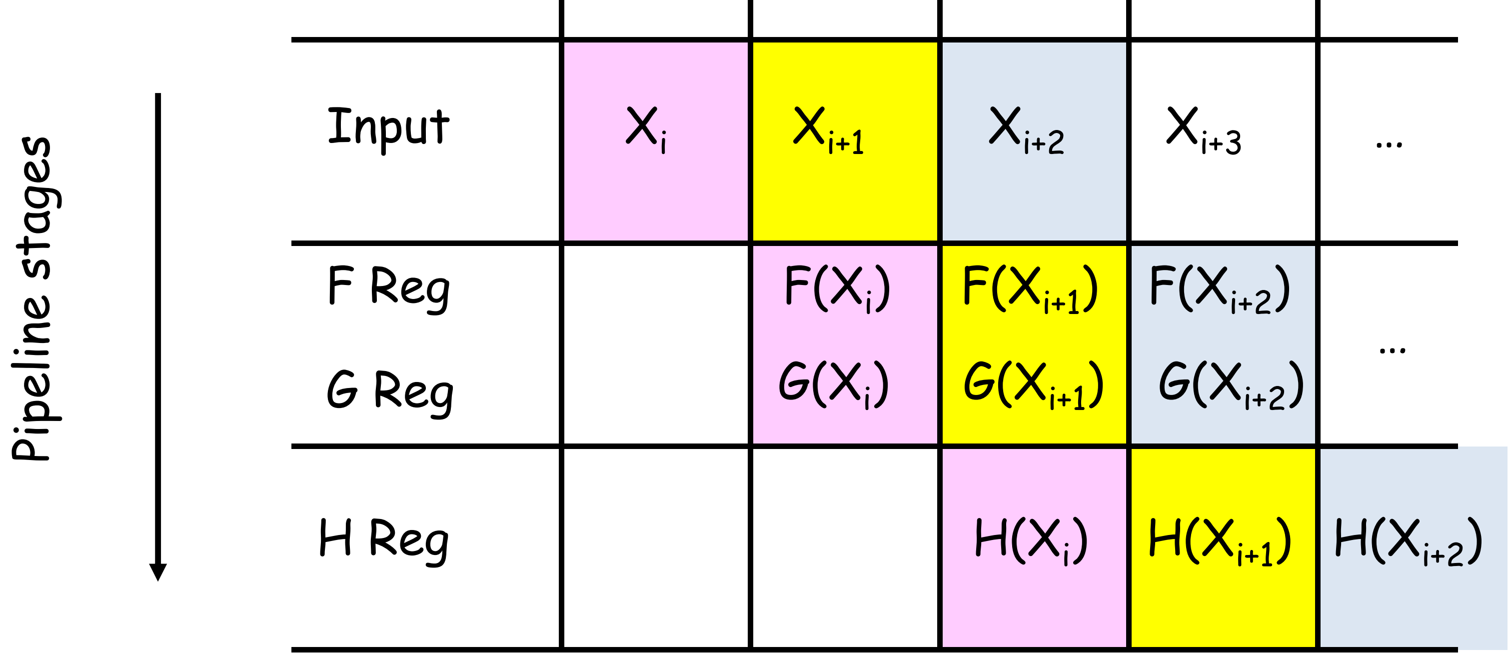
$L = 45$
$T = 1/45$

Pipeline diagrams



Clock cycle →

Pipeline stages ↓



	i	i+1	i+2	i+3	
Input	X_i	X_{i+1}	X_{i+2}	X_{i+3}	…
F Reg G Reg		$F(X_i)$ $G(X_i)$	$F(X_{i+1})$ $G(X_{i+1})$	$F(X_{i+2})$ $G(X_{i+2})$	…
H Reg			$H(X_i)$	$H(X_{i+1})$	$H(X_{i+2})$

The results associated with a particular set of input data moves *diagonally* through the diagram, progressing through one pipeline stage each clock cycle.

Pipeline Conventions

DEFINITION:

a *K-Stage Pipeline* ("K-pipeline") is an acyclic circuit having exactly K registers on *every* path from an input to an output.

a COMBINATIONAL CIRCUIT is thus an 0-stage pipeline.

CONVENTION:

Every pipeline stage, hence every K-Stage pipeline, has a register on its *OUTPUT* (not on its input).

ALWAYS:

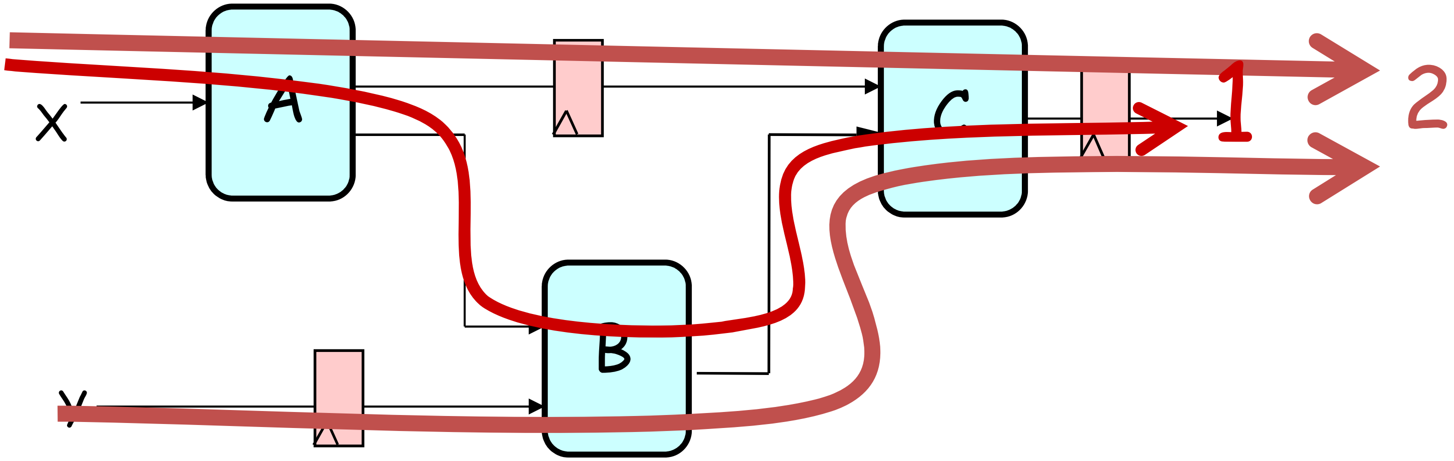
The CLOCK common to all registers must have a period sufficient to cover propagation over combinational paths PLUS (input) register t_{PD} PLUS (output) register t_{SETUP}.

The LATENCY of a K-pipeline is K times the period of the clock common to all registers.

The THROUGHPUT of a K-pipeline is the frequency of the clock.

Ill-formed pipelines

Consider a BAD job of pipelining:



For what value of K is the following circuit a K-Pipeline? __________ none

Problem:

Successive inputs get mixed: e.g., B(A(X_{i+1}), Y_i). This happened because some paths from inputs to outputs have 2 registers, and some have only 1!

This CAN'T HAPPEN on a well-formed K pipeline!

A pipelining methodology

Step 1:

Add a register on each output.

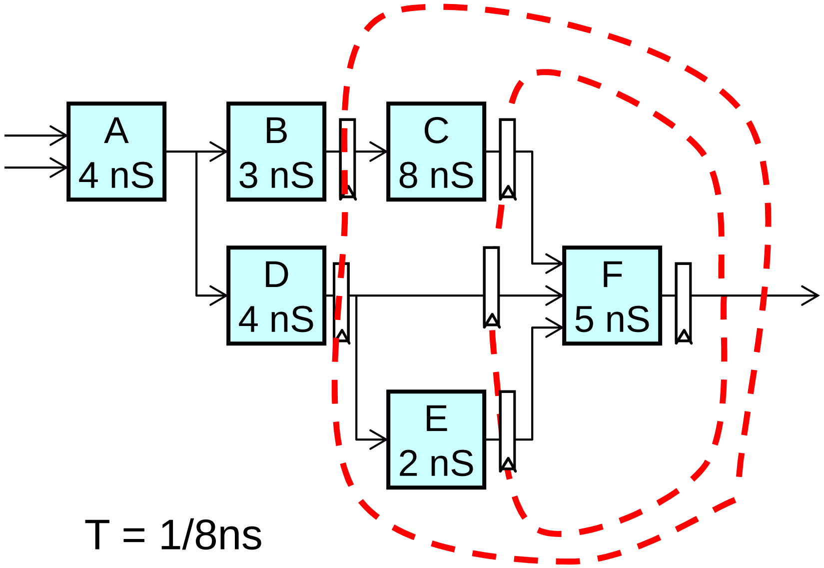
Step 2:

Add another register on each output. Draw a cut-set contour that includes all the new registers and some part of the circuit. Retime by moving regs from all outputs to all inputs of cut-set.

Repeat until satisfied with T.

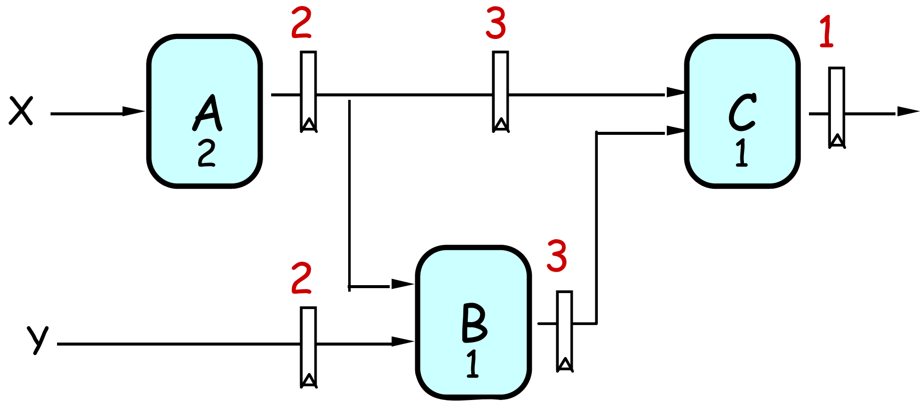
STRATEGY:

Focus your attention on placing pipelining registers around the slowest circuit elements (BOTTLENECKS).



T = 1/8ns
L = 24ns

Pipeline Example

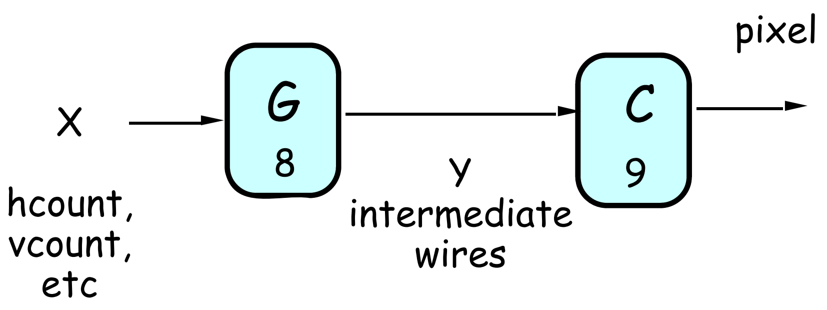


	LATENCY	THROUGHPUT
0-pipe:	4	1/4
1-pipe:	4	1/4
2-pipe:	4	1/2
3-pipe:	6	1/2

OBSERVATIONS:

- 1-pipeline improves neither L or T.
- T improved by breaking long combinational paths, allowing faster clock.
- Too many stages cost L, don't improve T.
- Back-to-back registers are often required to keep pipeline well-formed.

Pipeline Example - Verilog



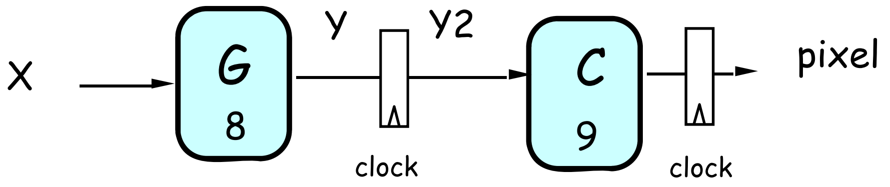
Lab 3 Pong

- G = game logic 8ns tpd
- C = draw round puck, use multiply with 9ns tpd
- System clock 65mhz = 15ns period – opps

```
No pipeline
   assign y = G(x);      // logic for y
   assign pixel = C(y)   // logic for pixel
```

⇔

```
reg [N:0] x,y;
reg [23:0] pixel
always @ *  begin
   y=G(x);
   pixel = C(y);
end
```

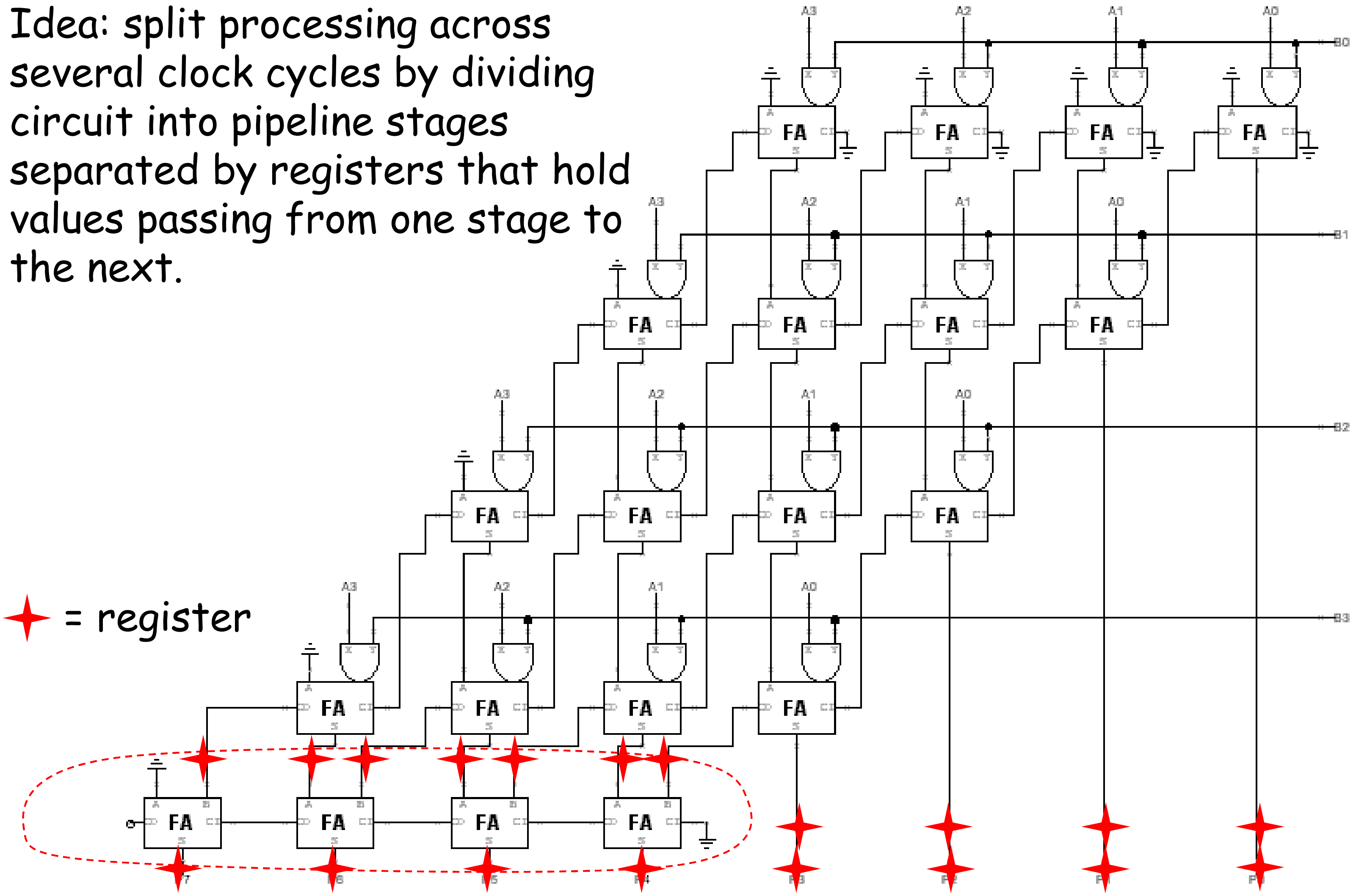


```
Pipeline
   always @(posedge clock)  begin
     ...
     y2 <= G(x);        // pipeline y
     pixel <= C(y2)     // pipeline pixel
end
```

Latency = 2 clock cyles!
Implications?

Increasing Throughput: Pipelining

Idea: split processing across several clock cycles by dividing circuit into pipeline stages separated by registers that hold values passing from one stage to the next.

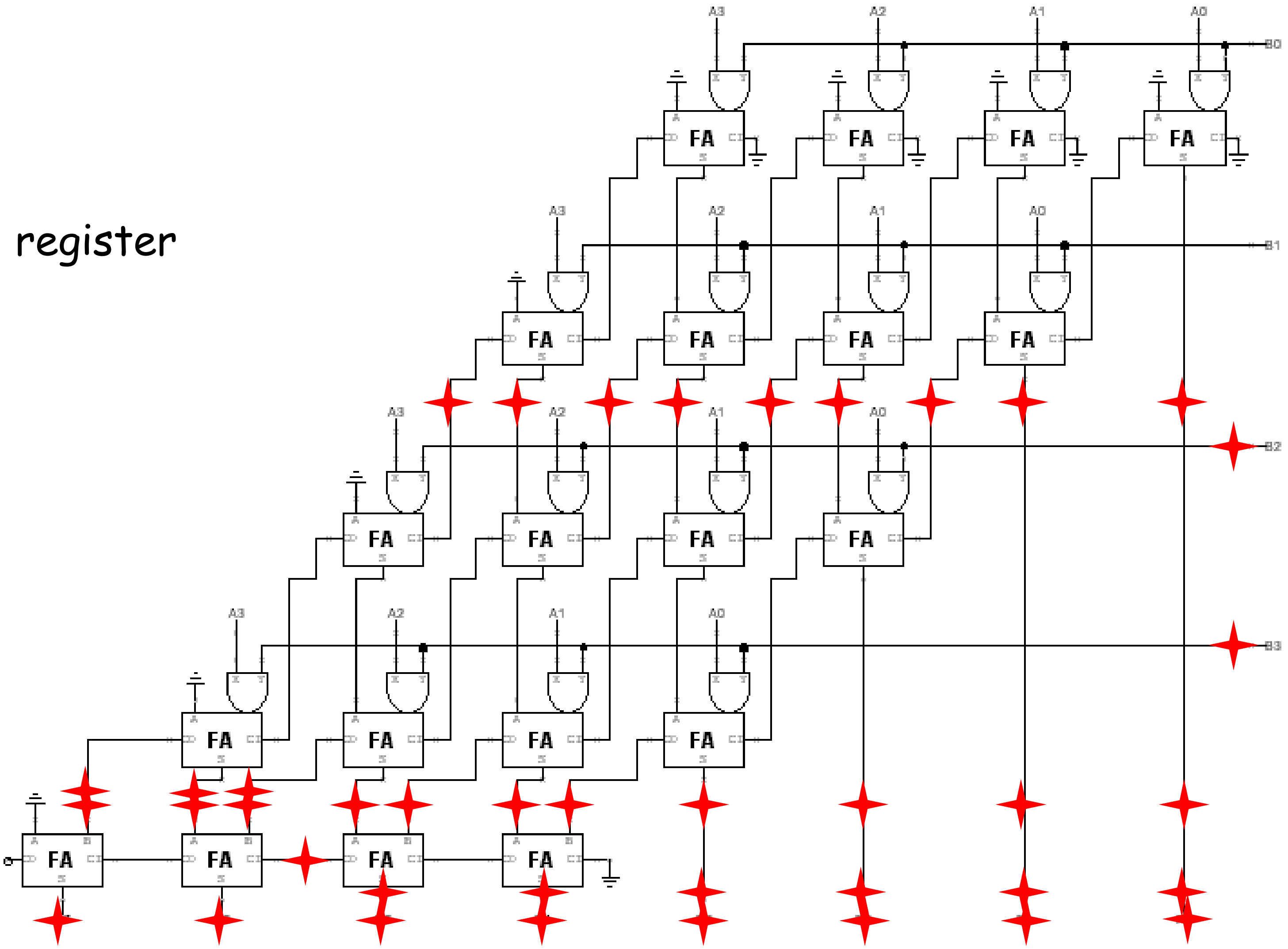


✦ = register

Throughput = $1/4t_{PD,FA}$ instead of $1/8t_{PD,FA}$)

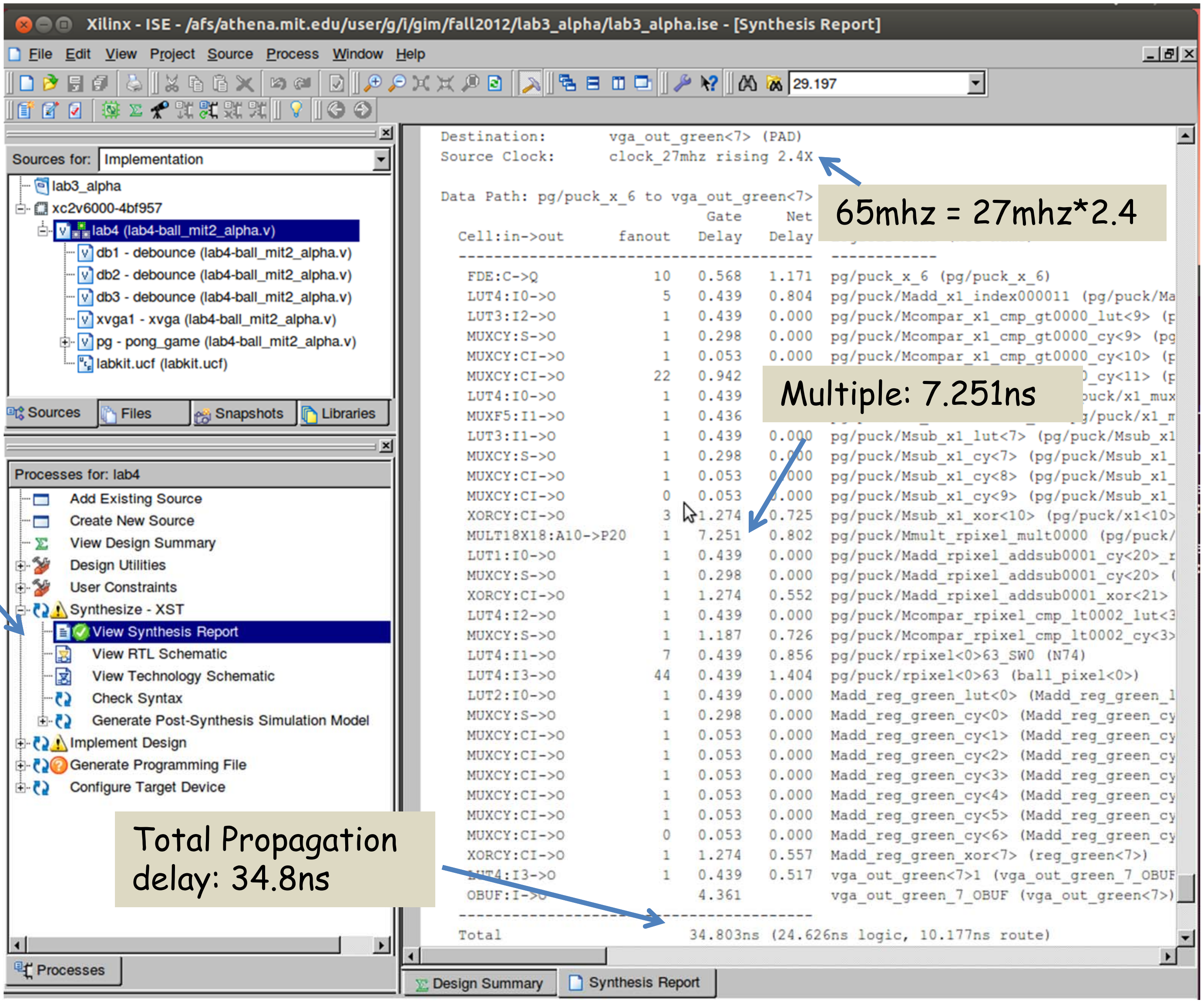
How about $t_{PD} = 1/2t_{PD,FA}$?

✦ = register



Timing Reports

Synthesis report



```
Destination:       vga_out_green<7> (PAD)
Source Clock:      clock_27mhz rising 2.4X

Data Path: pg/puck_x_6 to vga_out_green<7>
                                Gate     Net
  Cell:in->out      fanout   Delay   Delay
  ----------------------------------------  ------------
   FDE:C->Q             10   0.568   1.171  pg/puck_x_6 (pg/puck_x_6)
   LUT4:I0->O            5   0.439   0.804  pg/puck/Madd_x1_index000011 (pg/puck/Ma
   LUT3:I2->O            1   0.439   0.000  pg/puck/Mcompar_x1_cmp_gt0000_lut<9> (p
   MUXCY:S->O            1   0.298   0.000  pg/puck/Mcompar_x1_cmp_gt0000_cy<9> (pg
   MUXCY:CI->O           1   0.053   0.000  pg/puck/Mcompar_x1_cmp_gt0000_cy<10> (p
   MUXCY:CI->O          22   0.942                                    0_cy<11> (p
   LUT4:I0->O            1   0.439                                    puck/x1_mux
   MUXF5:I1->O           1   0.436                                   g/puck/x1_m
   LUT3:I1->O            1   0.439   0.000  pg/puck/Msub_x1_lut<7> (pg/puck/Msub_x1
   MUXCY:S->O            1   0.298   0.000  pg/puck/Msub_x1_cy<7> (pg/puck/Msub_x1_
   MUXCY:CI->O           1   0.053   0.000  pg/puck/Msub_x1_cy<8> (pg/puck/Msub_x1_
   MUXCY:CI->O           0   0.053   0.000  pg/puck/Msub_x1_cy<9> (pg/puck/Msub_x1_
   XORCY:CI->O           3   1.274   0.725  pg/puck/Msub_x1_xor<10> (pg/puck/x1<10>
   MULT18X18:A10->P20    1   7.251   0.802  pg/puck/Mmult_rpixel_mult0000 (pg/puck/
   LUT1:I0->O            1   0.439   0.000  pg/puck/Madd_rpixel_addsub0001_cy<20>_r
   MUXCY:S->O            1   0.298   0.000  pg/puck/Madd_rpixel_addsub0001_cy<20> (
   XORCY:CI->O           1   1.274   0.552  pg/puck/Madd_rpixel_addsub0001_xor<21>
   LUT4:I2->O            1   0.439   0.000  pg/puck/Mcompar_rpixel_cmp_lt0002_lut<3
   MUXCY:S->O            1   1.187   0.726  pg/puck/Mcompar_rpixel_cmp_lt0002_cy<3>
   LUT4:I1->O            7   0.439   0.856  pg/puck/rpixel<0>63_SW0 (N74)
   LUT4:I3->O           44   0.439   1.404  pg/puck/rpixel<0>63 (ball_pixel<0>)
   LUT2:I0->O            1   0.439   0.000  Madd_reg_green_lut<0> (Madd_reg_green_l
   MUXCY:S->O            1   0.298   0.000  Madd_reg_green_cy<0> (Madd_reg_green_cy
   MUXCY:CI->O           1   0.053   0.000  Madd_reg_green_cy<1> (Madd_reg_green_cy
   MUXCY:CI->O           1   0.053   0.000  Madd_reg_green_cy<2> (Madd_reg_green_cy
   MUXCY:CI->O           1   0.053   0.000  Madd_reg_green_cy<3> (Madd_reg_green_cy
   MUXCY:CI->O           1   0.053   0.000  Madd_reg_green_cy<4> (Madd_reg_green_cy
   MUXCY:CI->O           1   0.053   0.000  Madd_reg_green_cy<5> (Madd_reg_green_cy
   MUXCY:CI->O           0   0.053   0.000  Madd_reg_green_cy<6> (Madd_reg_green_cy
   XORCY:CI->O           1   1.274   0.557  Madd_reg_green_xor<7> (reg_green<7>)
   LUT4:I3->O            1   0.439   0.517  vga_out_green<7>1 (vga_out_green_7_OBUF
   OBUF:I->O                 4.361          vga_out_green_7_OBUF (vga_out_green<7>)
  ----------------------------------------
  Total                     34.803ns (24.626ns logic, 10.177ns route)
```

65mhz = 27mhz*2.4

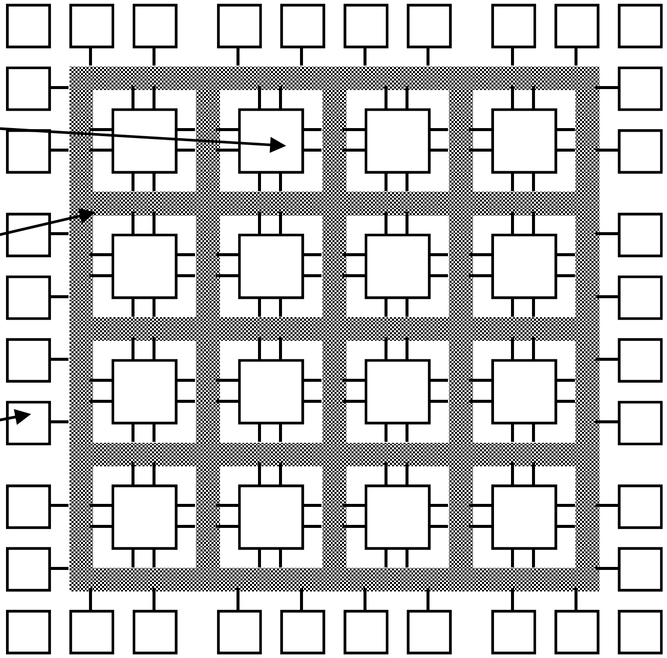
Multiple: 7.251ns

Total Propagation delay: 34.8ns

History of Computational Fabrics

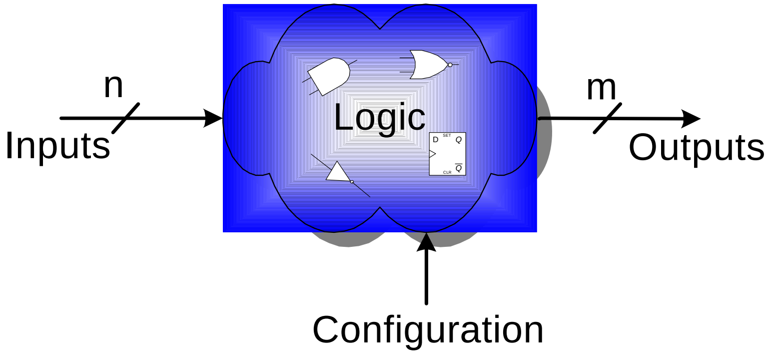
- Discrete devices: relays, transistors (1940s-50s)
- Discrete logic gates (1950s-60s)
- Integrated circuits (1960s-70s)
  - e.g. TTL packages: Data Book for 100's of different parts
- Gate Arrays (IBM 1970s)
  - Transistors are pre-placed on the chip & Place and Route software puts the chip together automatically – only program the interconnect (mask programming)
- Software Based Schemes (1970's- present)
  - Run instructions on a general purpose core
- Programmable Logic (1980's to present)
  - A chip that be reprogrammed after it has been fabricated
  - Examples: PALs, EPROM, EEPROM, PLDs, FPGAs
  - Excellent support for mapping from Verilog
- ASIC Design (1980's to present)
  - Turn Verilog directly into layout using a library of standard cells
  - Effective for high-volume and efficient use of silicon area

Reconfigurable Logic



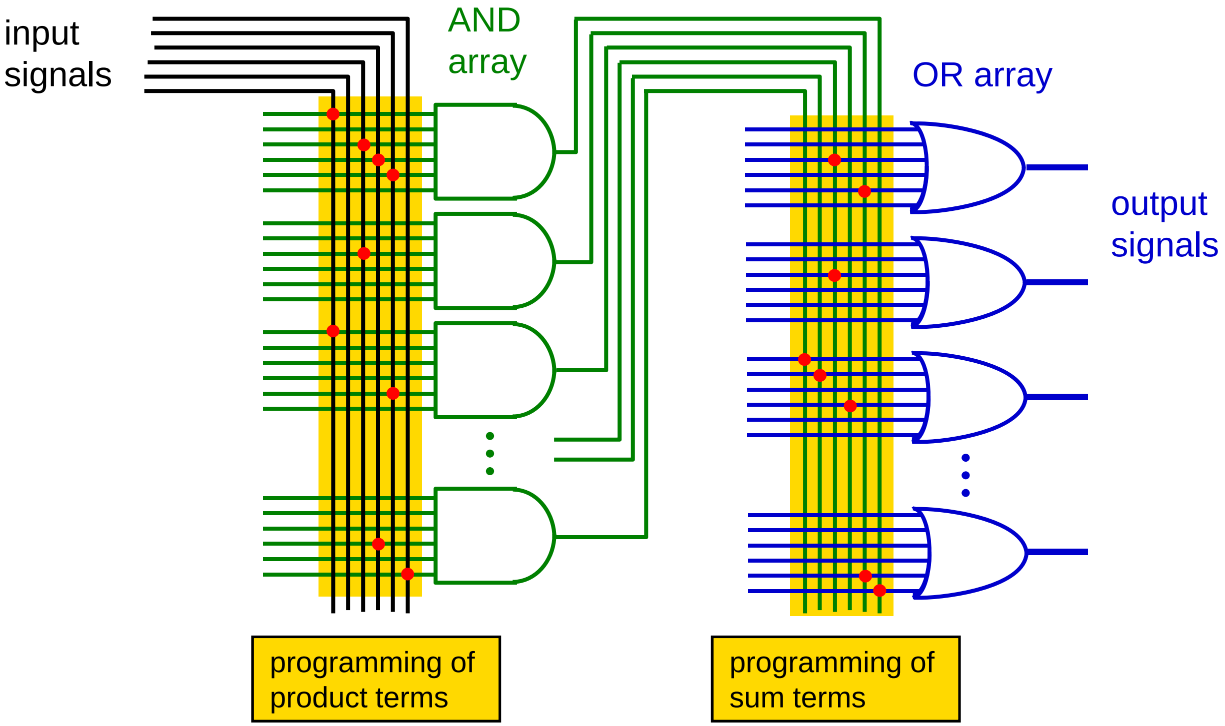
- Logic blocks
  - To implement combinational and sequential logic
- Interconnect
  - Wires to connect inputs and outputs to logic blocks
- I/O blocks
  - Special logic blocks at periphery of device for external connections

- Key questions:
  - How to make logic blocks programmable? (after chip has been fabbed!)
  - What should the logic granularity be?
  - How to make the wires programmable? (after chip has been fabbed!)
  - Specialized wiring structures for local vs. long distance routes?
  - How many wires per logic block?

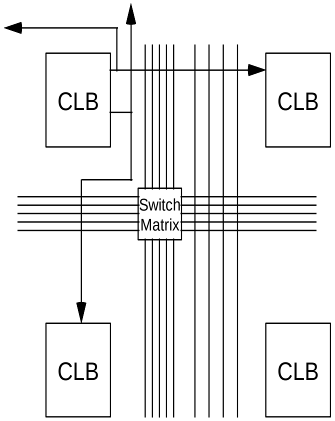


Programmable Array Logic (PAL)

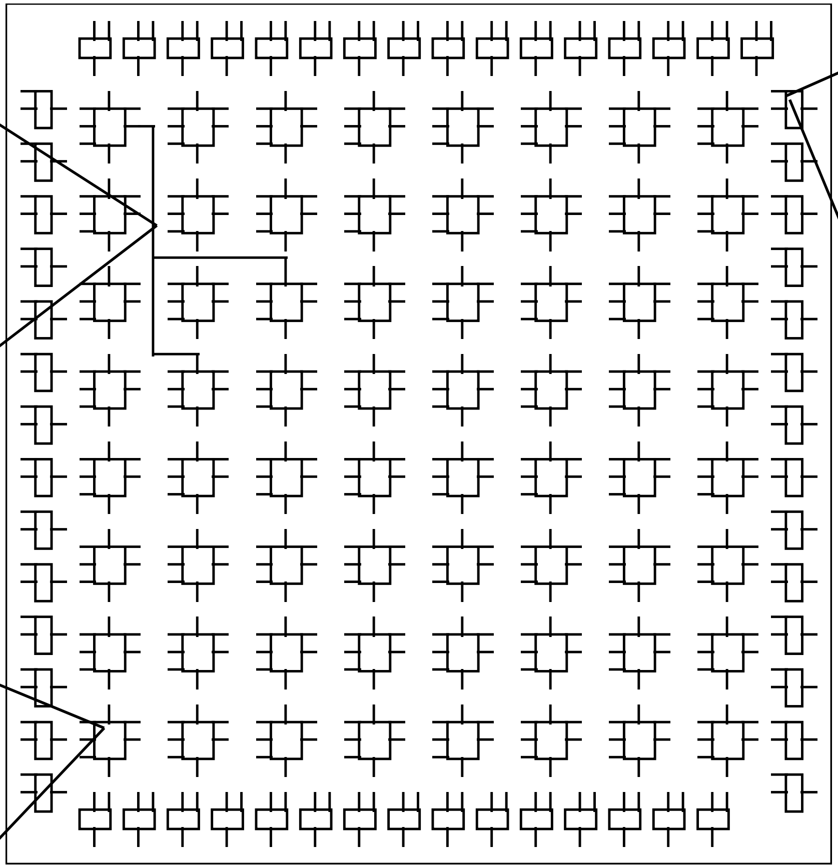
- Based on the fact that any combinational logic can be realized as a sum-of-products
- PALs feature an array of AND-OR gates with programmable interconnect

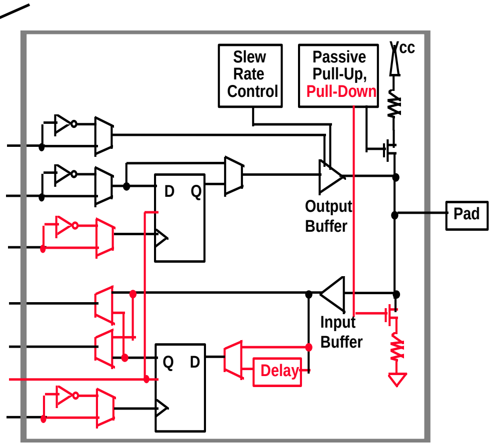


RAM Based Field Programmable Logic - Xilinx

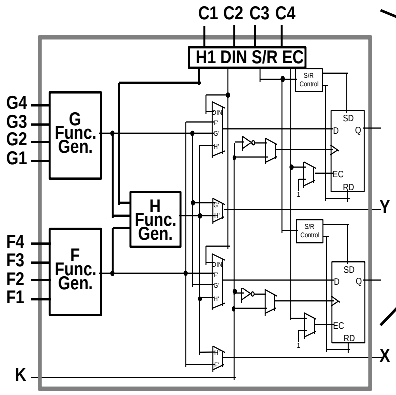


Programmable Interconnect





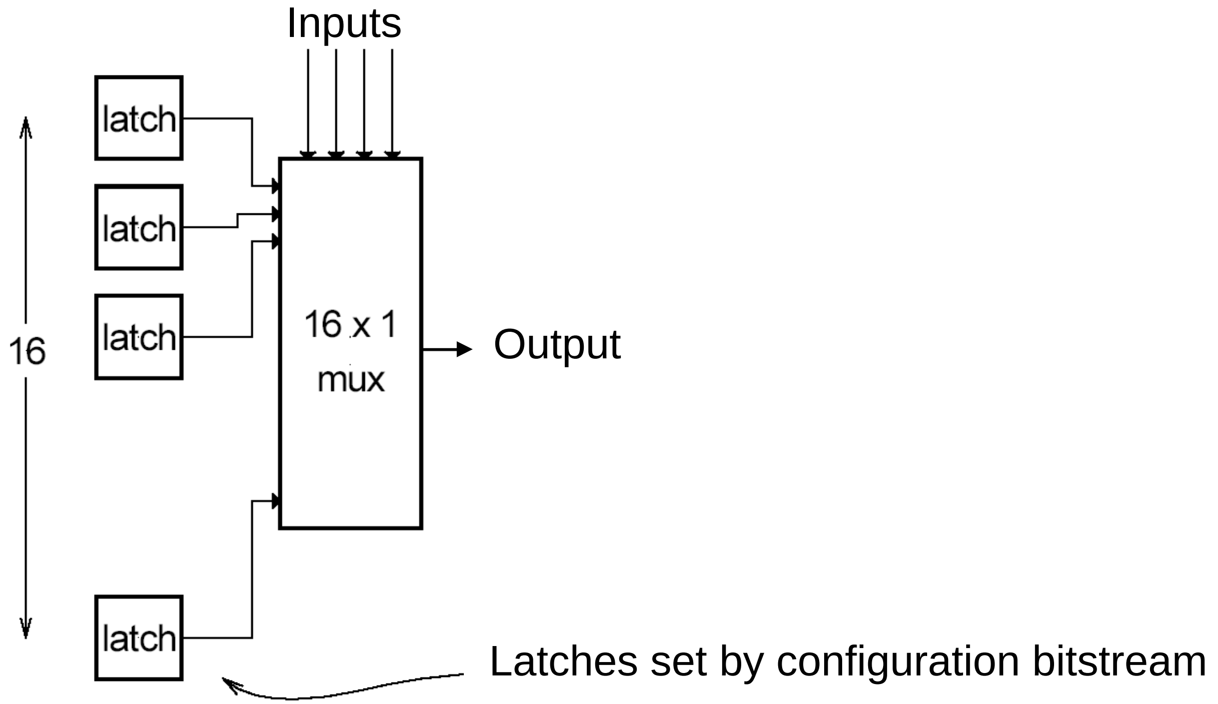
I/O Blocks (IOBs)



Configurable Logic Blocks (CLBs)

LUT Mapping

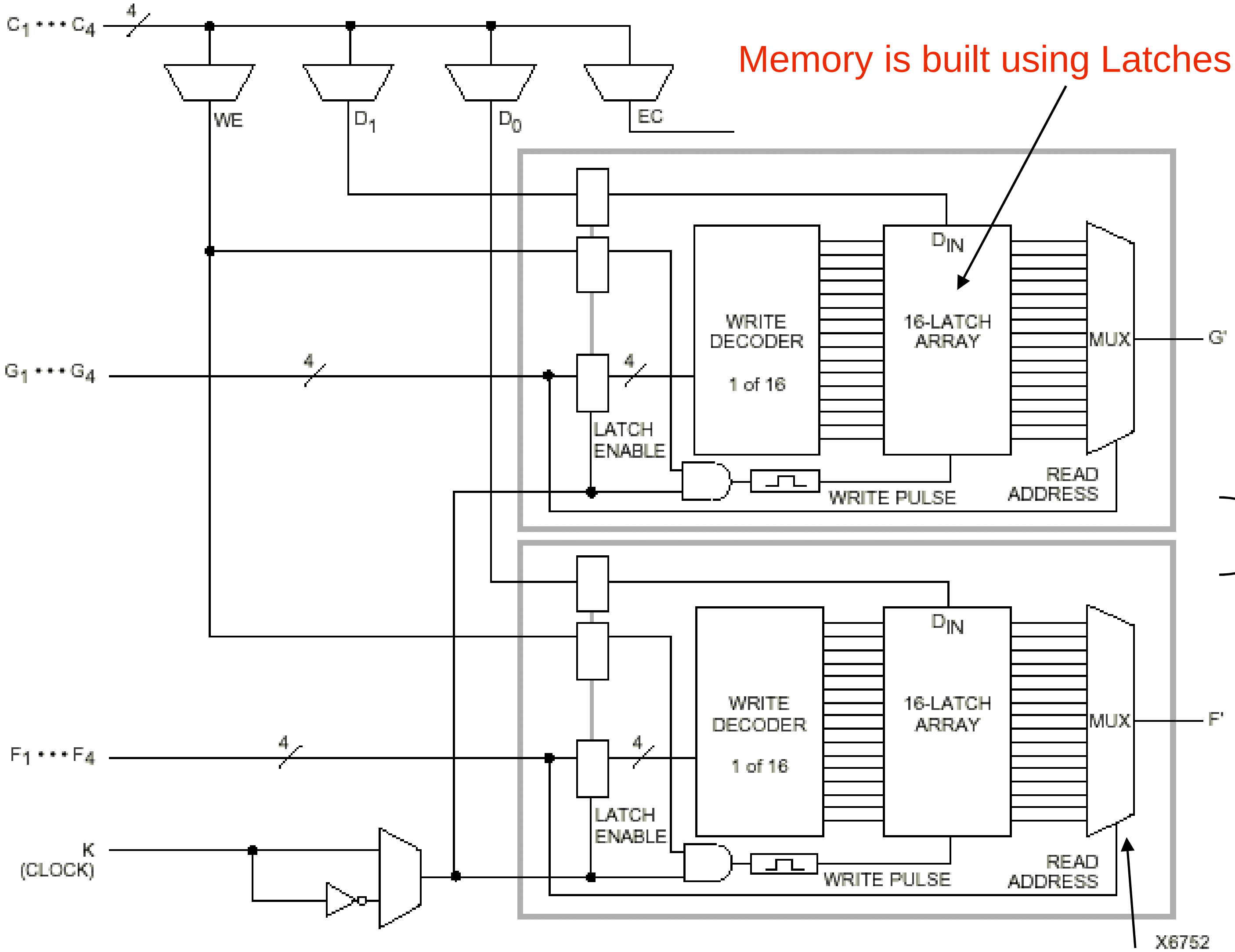
- N-LUT direct implementation of a truth table: any function of n-inputs.
- N-LUT requires 2^N storage elements (latches)
- N-inputs select one latch location (like a memory)



Latches set by configuration bitstream

4LUT example

Configuring the CLB as a RAM



Memory is built using Latches not FFs

16x2

gure 4: 16x2 (or 16x1) Edge-Triggered Single-Port RAM

Read is same a LUT Function!

Xilinx 4000 Interconnect

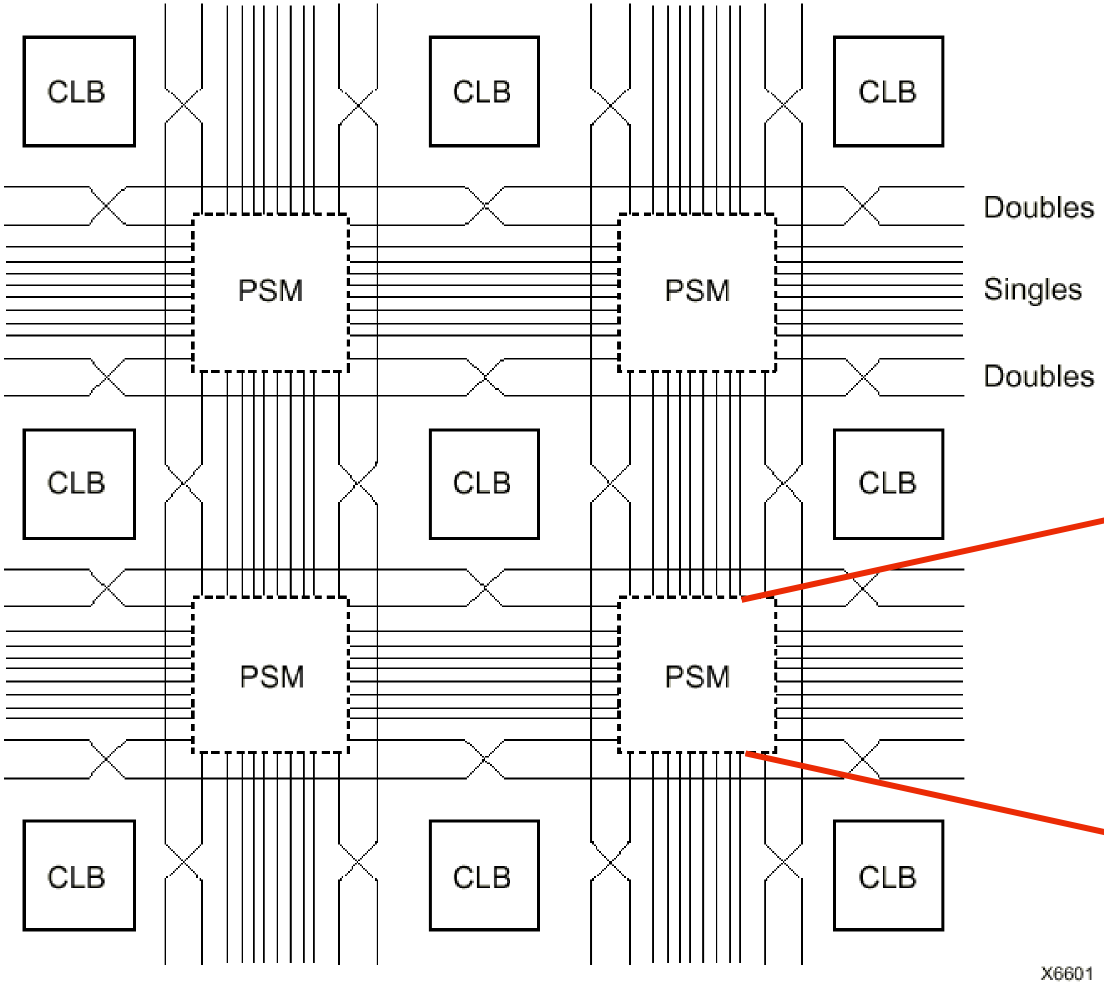
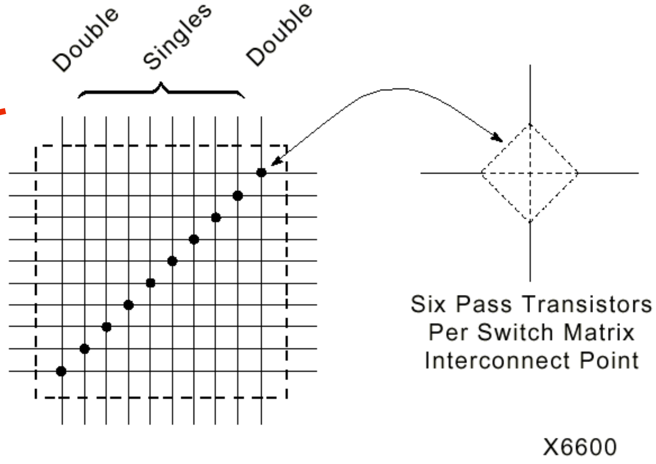
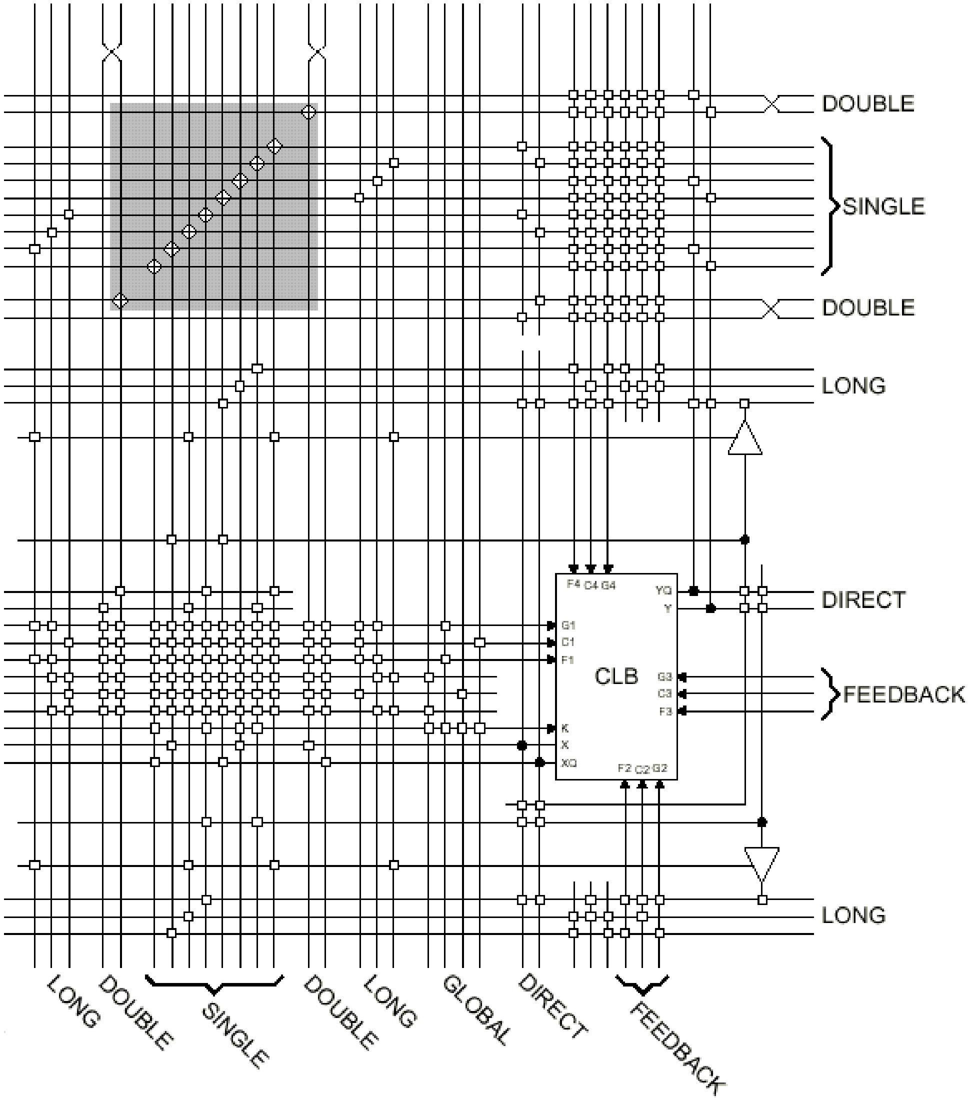


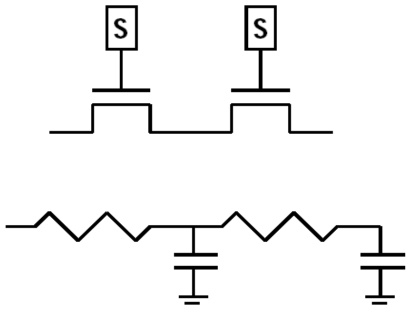
Figure 28: Single- and Double-Length Lines, with Programmable Switch Matrices (PSMs)



: Programmable Switch Matrix (PSM)

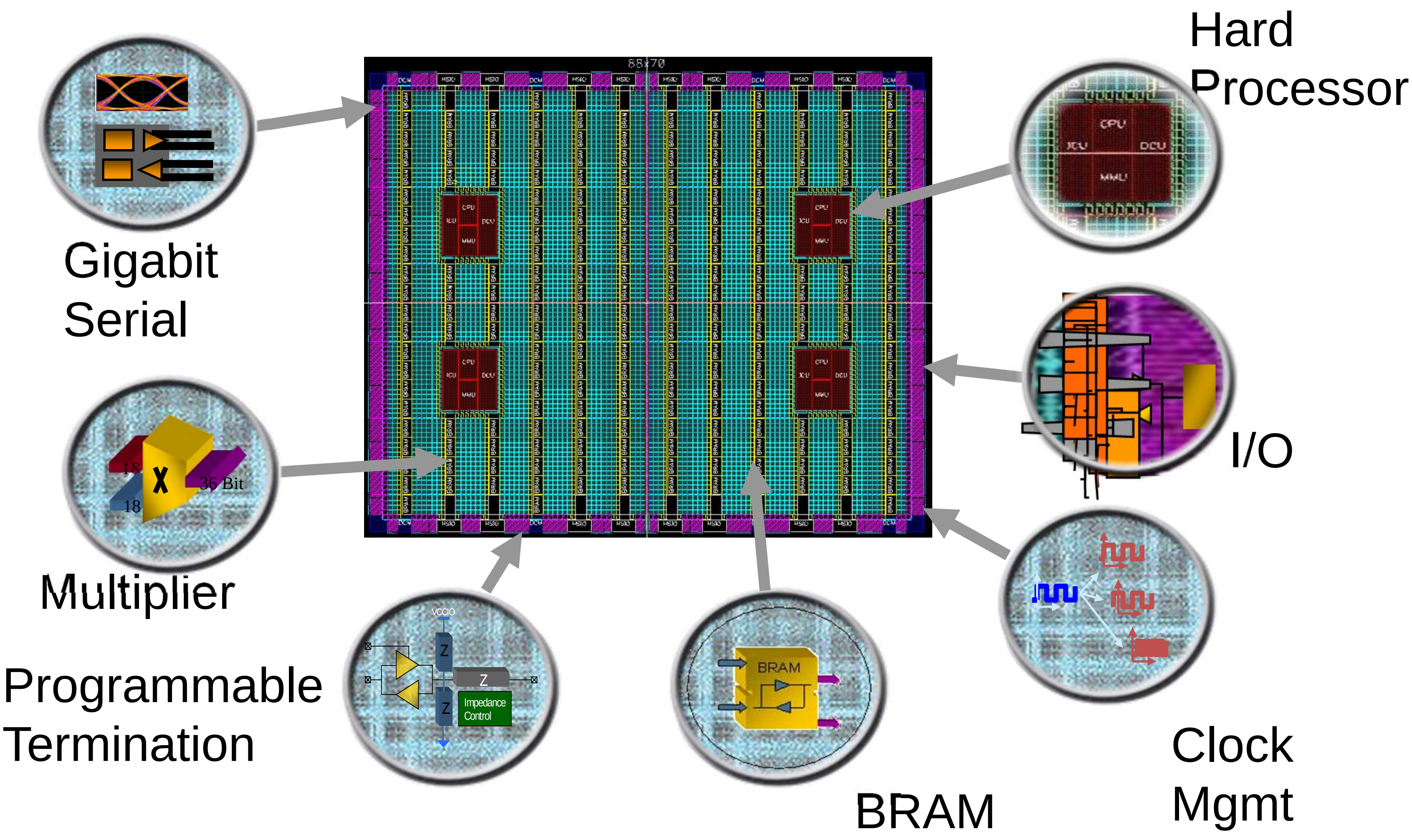
Xilinx 4000 Interconnect Details





Wires are not ideal!

Add Bells & Whistles



Gigabit Serial

Hard Processor

I/O

Multiplier

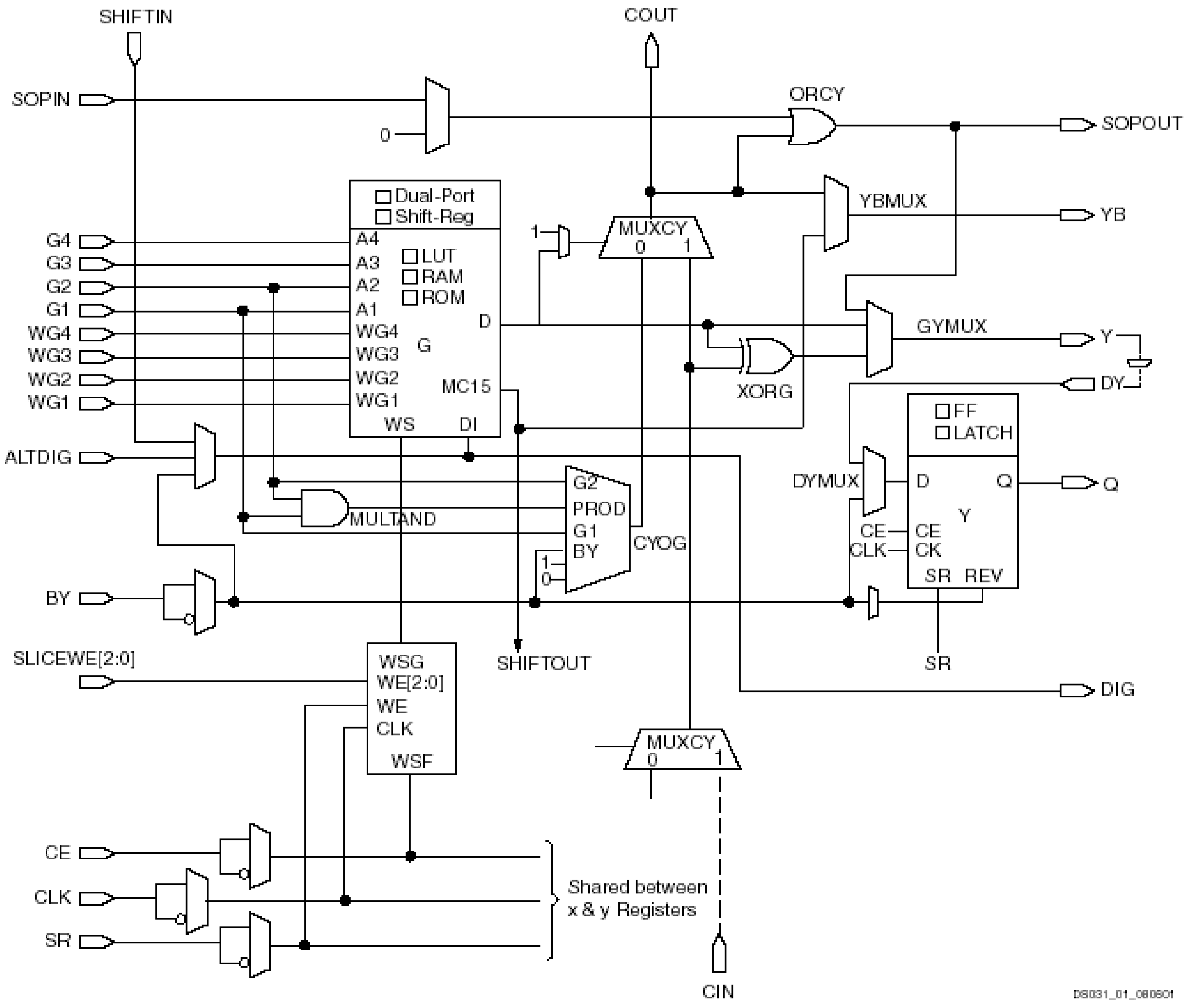
Programmable Termination

BRAM

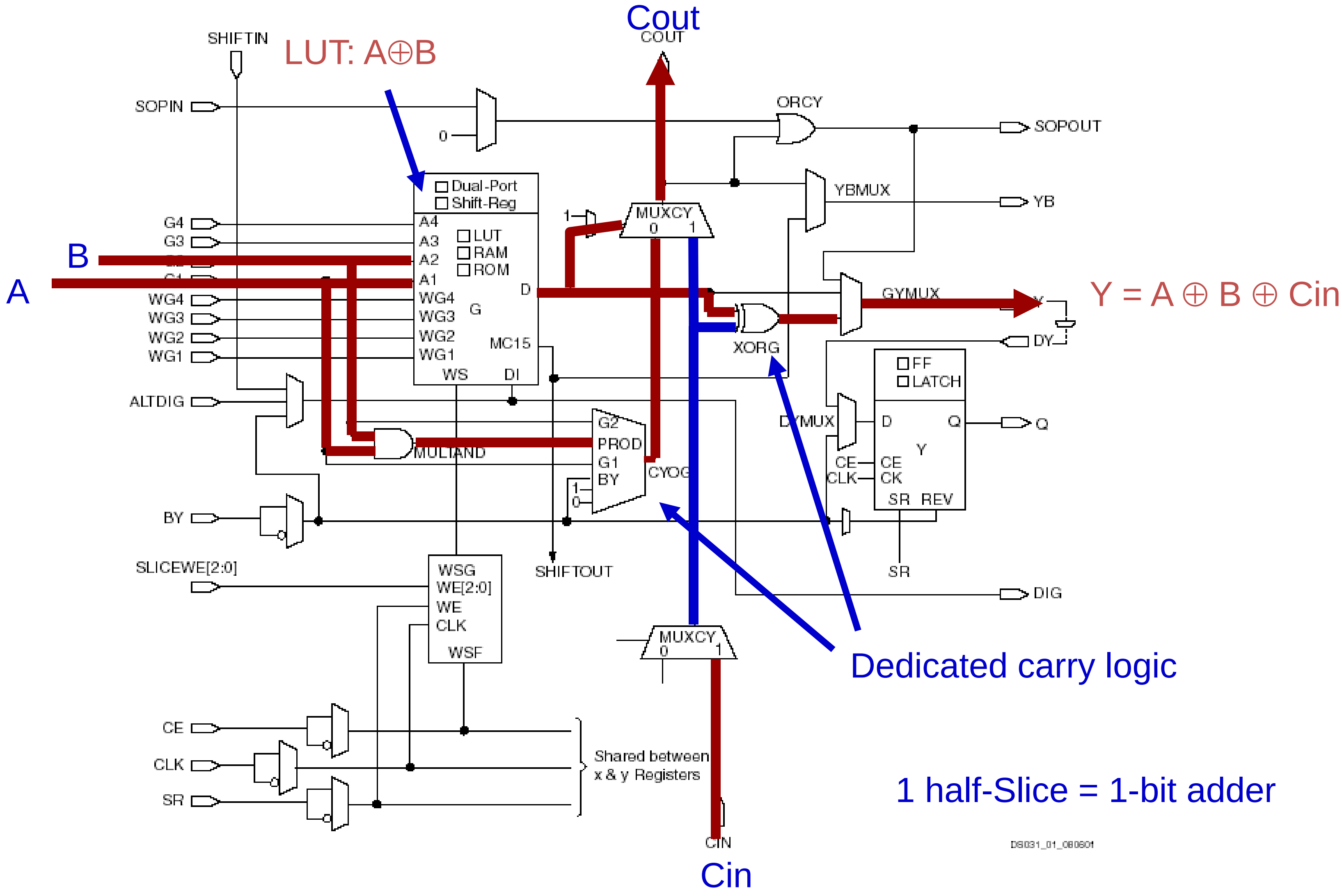
Clock Mgmt

Courtesy of David B. Parlour, ISSCC 2004 Tutorial,
“The Reality and Promise of Reconfigurable Computing in Digital Signal Processing”

The Virtex II CLB (Half Slice Shown)



Adder Implementation



Cout

LUT: A⊕B

B

A

Y = A ⊕ B ⊕ Cin

Dedicated carry logic

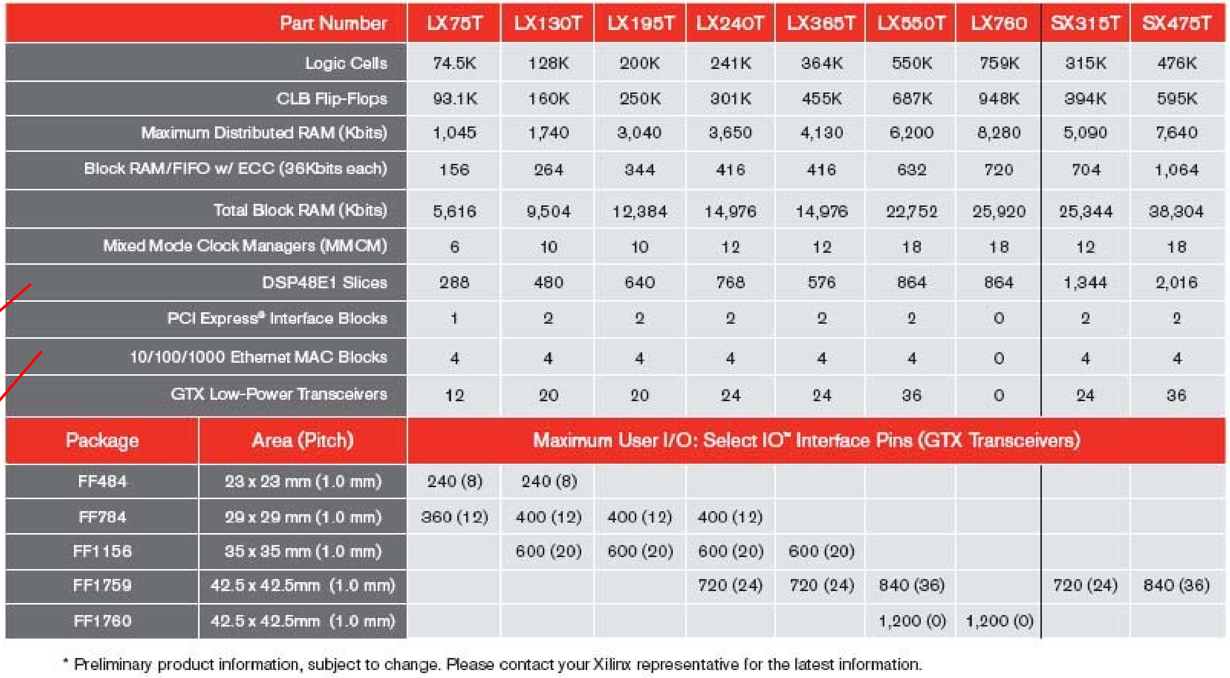
1 half-Slice = 1-bit adder

Cin

FPGA's

DSP with 25x18 multiplier

Gigabit ethernet support



Part Number	LX75T	LX130T	LX195T	LX240T	LX365T	LX550T	LX760	SX315T	SX475T
Logic Cells	74.5K	128K	200K	241K	364K	550K	759K	315K	476K
CLB Flip-Flops	93.1K	160K	250K	301K	455K	687K	948K	394K	595K
Maximum Distributed RAM (Kbits)	1,045	1,740	3,040	3,650	4,130	6,200	8,280	5,090	7,640
Block RAM/FIFO w/ ECC (36Kbits each)	156	264	344	416	416	632	720	704	1,064
Total Block RAM (Kbits)	5,616	9,504	12,384	14,976	14,976	22,752	25,920	25,344	38,304
Mixed Mode Clock Managers (MMCM)	6	10	10	12	12	18	18	12	18
DSP48E1 Slices	288	480	640	768	576	864	864	1,344	2,016
PCI Express® Interface Blocks	1	2	2	2	2	2	0	2	2
10/100/1000 Ethernet MAC Blocks	4	4	4	4	4	4	0	4	4
GTX Low-Power Transceivers	12	20	20	24	24	36	0	24	36

Package	Area (Pitch)	Maximum User I/O: Select IO™ Interface Pins (GTX Transceivers)								
FF484	23 x 23 mm (1.0 mm)	240 (8)	240 (8)							
FF784	29 x 29 mm (1.0 mm)	360 (12)	400 (12)	400 (12)	400 (12)					
FF1156	35 x 35 mm (1.0 mm)		600 (20)	600 (20)	600 (20)	600 (20)				
FF1759	42.5 x 42.5mm (1.0 mm)				720 (24)	720 (24)	840 (36)		720 (24)	840 (36)
FF1760	42.5 x 42.5mm (1.0 mm)						1,200 (0)	1,200 (0)		

* Preliminary product information, subject to change. Please contact your Xilinx representative for the latest information.

	CLB	Dist RAM	Block RAM	Multipliers
Virtex 2	8,448	1,056 kbit	2,592 kbit	144 (18x18)
Virtex 6	667,000	6,200 kbit	22,752 kbit	1,344 (25x18)
Spartan 3E	240	15 kbit	72 kbit	4 (18x18)
Artix-7 A100	7,925	1,188 kbit	4,860 kbit	240 (25x18)

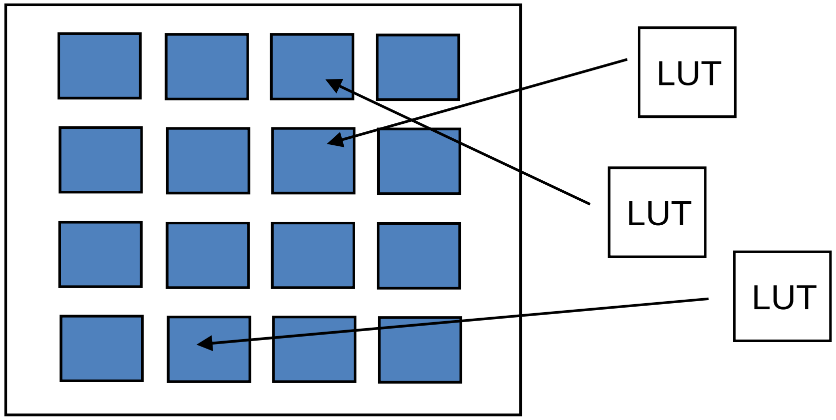
Design Flow - Mapping

- Technology Mapping: Schematic/HDL to Physical Logic units
- Compile functions into basic LUT-based groups (function of target architecture)

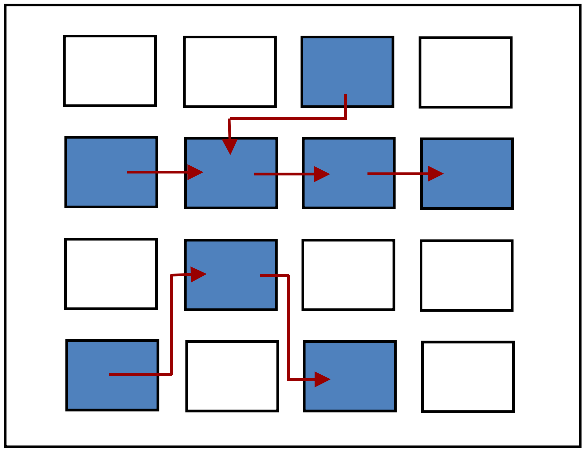
```
always @(posedge clock or negedge reset)
  begin
    if (! reset)
       q <= 0;
    else
       q <= (a&b&c)||(b&d);
  end
```

Design Flow – Placement & Route

- Placement – assign logic location on a particular device



- Routing – iterative process to connect CLB inputs/outputs and IOBs. Optimizes critical path delay – *can take hours or days for large, dense designs*



Iterate placement if timing not met

Satisfy timing? → Generate Bitstream to config device

Challenge! Cannot use full chip for reasonable speeds (wires are not ideal).

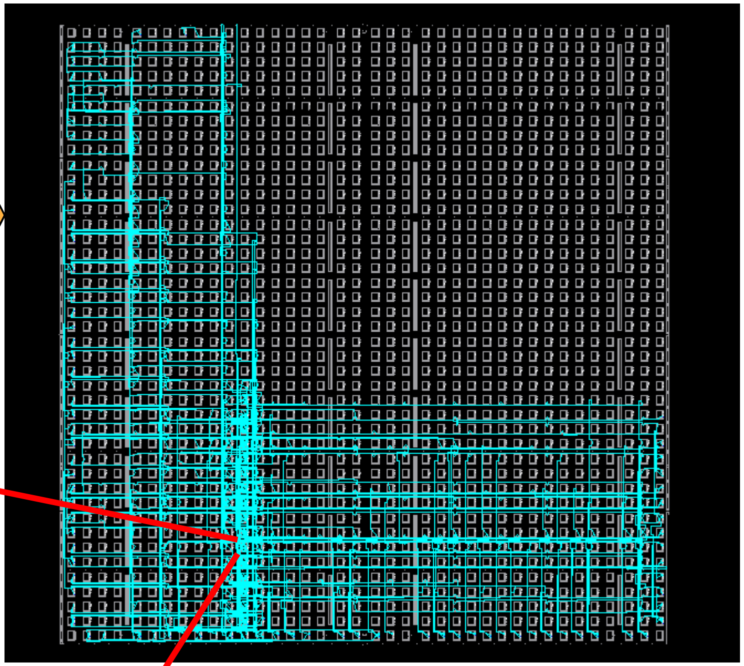
Typically no more than 50% utilization.

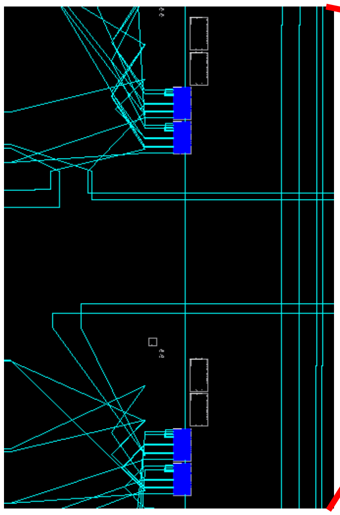
Example: Verilog to FPGA

```
module adder64 (
  input  [63:0] a, b;
  output [63:0] sum);

  assign sum = a + b;
endmodule
```

- Synthesis
- Tech Map
- Place&Route



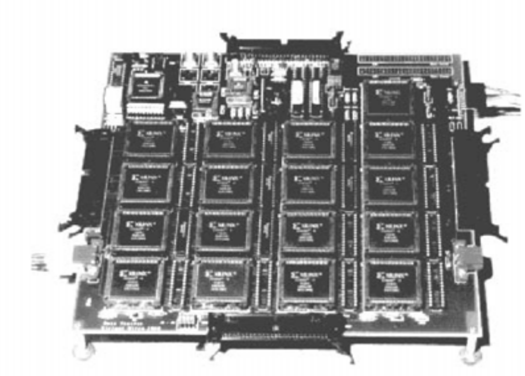


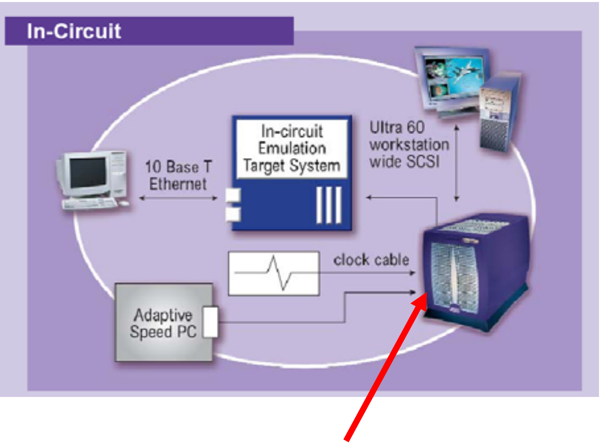
64-bit Adder Example

Virtex II – XC2V2000

How are FPGAs Used?

Logic Emulation





FPGA-based Emulator

(courtesy of IKOS)

- Prototyping
  - ☐ Ensemble of gate arrays used to emulate a circuit to be manufactured
  - ☐ Get more/better/faster debugging done than with simulation
- Reconfigurable hardware
  - ☐ One hardware block used to implement more than one function
- Special-purpose computation engines
  - ☐ Hardware dedicated to solving one problem (or class of problems)
  - ☐ Accelerators attached to general-purpose computers (e.g., in a cell phone!)

Summary

- FPGA provide a flexible platform for implementing digital computing
- A rich set of macros and I/Os supported (multipliers, block RAMS, ROMS, high-speed I/O)
- A wide range of applications from prototyping (to validate a design before ASIC mapping) to high-performance spatial computing
- Interconnects are a major bottleneck (physical design and locality are important considerations)

Test Bench

```
module sample_tf;
   // Inputs
   reg bit_in;
   reg [3:0] bus_in;

   // Outputs
   wire out_bit;
   wire [7:0] out_bus;

   // Instantiate the Unit Under Test (UUT)
   sample uut (
      .bit_in(bit_in),
      .bus_in(bus_in),
      .out_bit(out_bit),
      .out_bus(out_bus)
   );

   initial begin
      // Initialize Inputs
      bit_in = 0;
      bus_in = 0;

      // Wait 100 ns for global reset to finish
      #100;

      // Add stimulus here

   end

endmodule
```

```
module sample(
   {input bit_in,
    input [3:0] bus_in,

   {output out_bit,
    output [7:0] out_bus
    );
  . . . Verilog . . .

 endmodule
```