



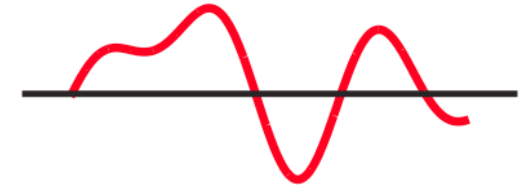
Analog Building Blocks

- Sampling theorem
- Undersampling, antialiasing
- FIR digital filters
- Quantization noise, oversampling
- OpAmps, DACs, ADCs

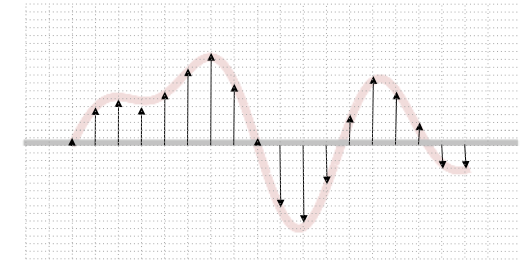
Thu: Lab 4 Checkoff

Digital Representations of Analog Waveforms

Continuous time
Continuous values



Discrete time
Discrete values



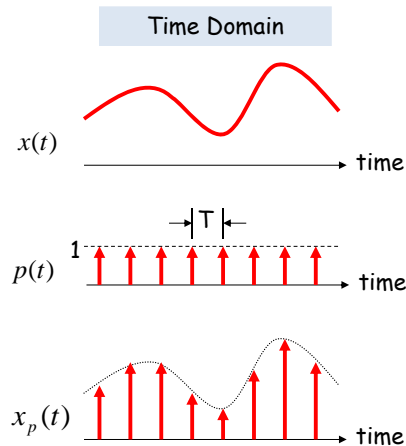
Discrete Time

Let's use an **impulse train** to sample a continuous-time function at a regular interval T :

$\delta(x)$ is a narrow impulse at $x=0$,
where $\int_{-\infty}^{\infty} f(t)\delta(t-a)dt = f(a)$

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$$

$$x(t) \rightarrow \text{[multiplier]} \rightarrow x_p(t)$$

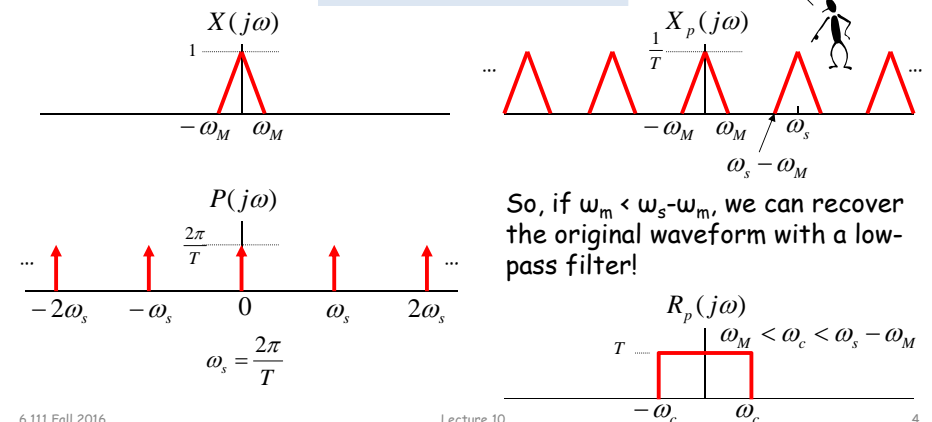


Reconstruction

Is it possible to reconstruct the original waveform using only the discrete time samples?

$$x_p(t) \rightarrow \text{[Reconstruction Block } R_p \text{]} \rightarrow x(t)$$

Frequency Domain



So, if $\omega_m < \omega_s - \omega_m$, we can recover the original waveform with a low-pass filter!

Sampling Theorem

Let $x(t)$ be a band-limited signal, ie, $X(j\omega)=0$ for $|\omega| > \omega_M$. Then $x(t)$ is uniquely determined by its samples $x(nT)$, $n = 0, \pm 1, \pm 2, \dots$, if

$$\omega_s > 2\omega_M$$

$2\omega_M$ is called the "Nyquist rate" and $\omega_s/2$ the "Nyquist frequency"

where

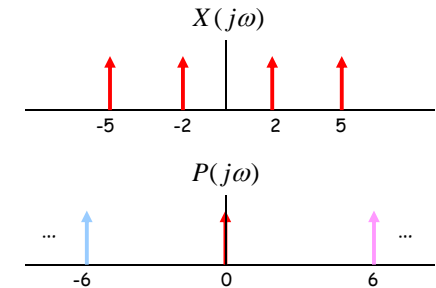
$$\omega_s = \frac{2\pi}{T}$$

Given these samples, we can reconstruct $x(t)$ by generating a periodic impulse train in which successive impulses have amplitudes that are successive sample values, then passing the train through an ideal LPF with gain T and a cutoff frequency greater than ω_M and less than $\omega_s - \omega_M$.

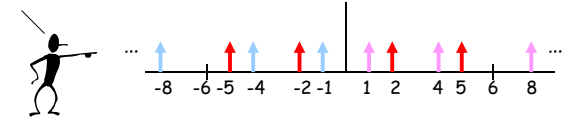
Undersampling → Aliasing

If $\omega_s \leq 2\omega_M$ there's an overlap of frequencies between one image and its neighbors and we discover that those overlaps introduce additional frequency content in the sampled signal, a phenomenon called **aliasing**.

$$\omega_M = 5, \omega_s = 6$$



There are now tones at 1 (= 6 - 5) and 4 (= 6 - 2) in addition to the original tones at 2 and 5.

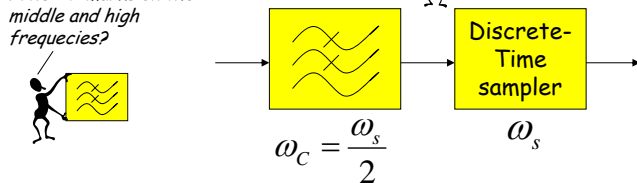


Antialias Filters

If we wish to create samples at some fixed frequency ω_s , then to avoid aliasing we need to use a low-pass filter on the original waveform to remove any frequency content $\geq \omega_s/2$.

This is the symbol for a low-pass filter - see the little "x" marks on the middle and high frequencies?

We need this antialiasing filter - it has to have a reasonably sharp cutoff



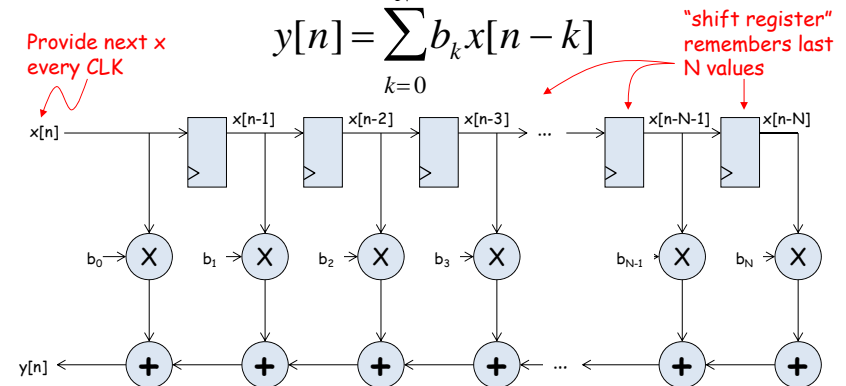
The frequency response of human ears essentially drops to zero above 20kHz. So the "Red Book" standard for CD Audio chose a 44.1kHz sampling rate, yielding a Nyquist frequency of 22.05kHz. The 2kHz of elbow room is needed because practical antialiasing filters have finite slope...
 $f_s = (3 \text{ samples/line})(490 \text{ lines/frame})(30 \text{ frames/s}) = 44.1 \text{ kHz}$

More info: <http://www.cs.columbia.edu/~hgs/audio/44.1.html>

Digital Filters

Equation for an N-tap finite impulse response (FIR) filter:

$$y[n] = \sum_{k=0}^N b_k x[n-k]$$



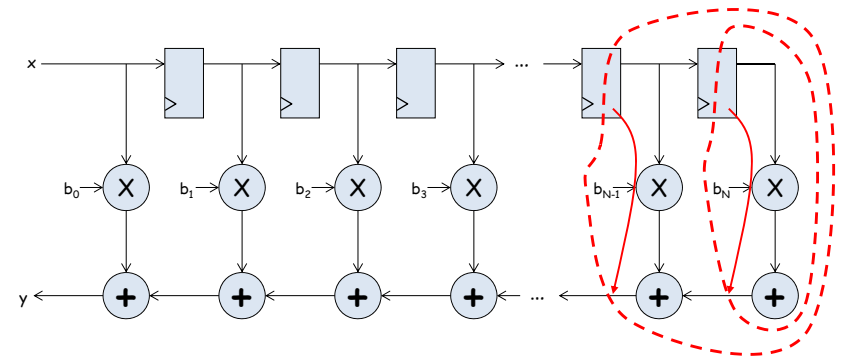
What components are part of the t_{pd} of this circuit?
 How does t_{pd} grow as N gets larger?

Filter coefficients

- Use Matlab command: $b = \text{fir1}(N, \omega_c / (\omega_s / 2))$
 - N is the number of taps (we'll get $N+1$ coefficients). Larger N gives sharper roll-off in filter response; usually want N to be as large as reasonably possible.
 - ω_c is the cutoff frequency (3kHz in Lab 5)
 - ω_s is the sample frequency (48kHz in Lab 5)
 - The second argument to the `fir1` command is the cutoff frequency as a fraction of the Nyquist frequency (i.e., half the sample rate).
 - By default you get a lowpass filter, but can also ask for a highpass, bandpass, bandstop.
- The b coefficients are real numbers between 0 and 1. But since we don't want to do floating point arithmetic, we usually scale them by some power of two and then round to integers.
 - Since coefficients are scaled by 2^5 , we'll have to re-scale the answer by dividing by 2^5 . But this is easy - just get rid of the bottom 5 bits!

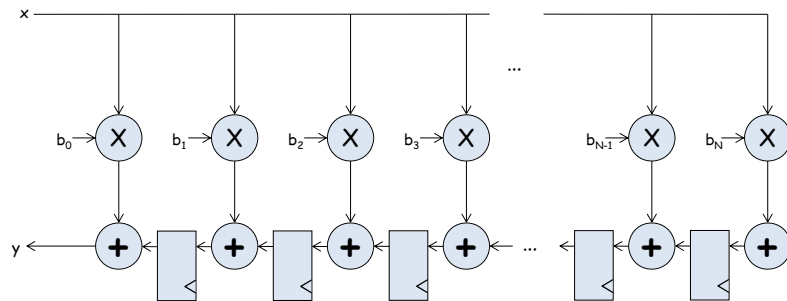
Retiming the FIR circuit

Apply the cut-set retiming transformation repeatedly...



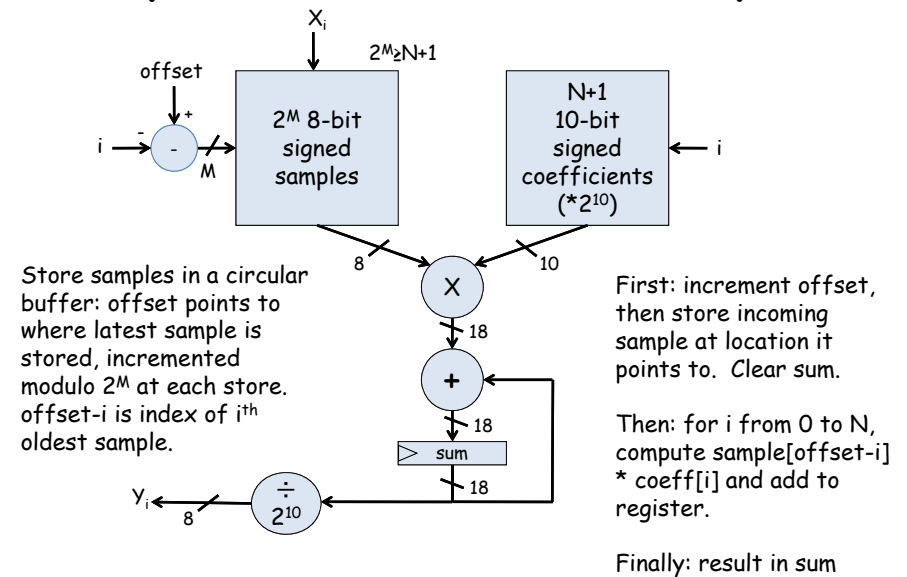
Retimed FIR filter circuit

"Transposed Form" of a FIR filter



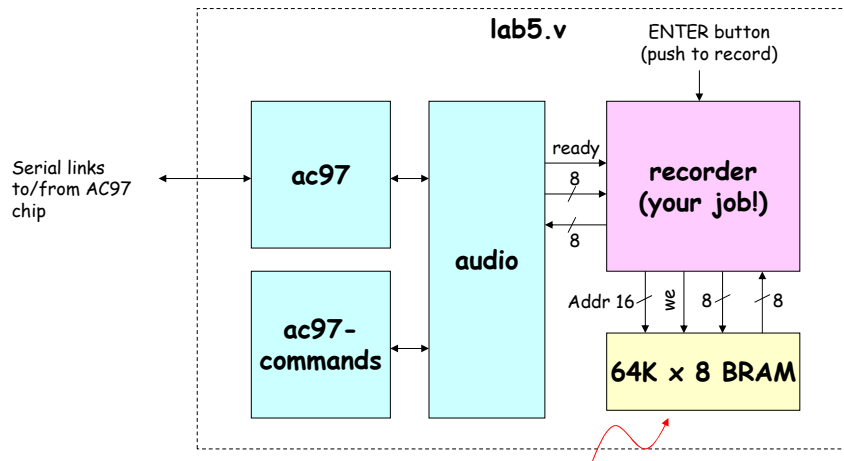
What components are part of the t_{PD} of this circuit?
How does t_{PD} grow as N gets larger?

N-tap FIR: less hardware, N+1 cycles...



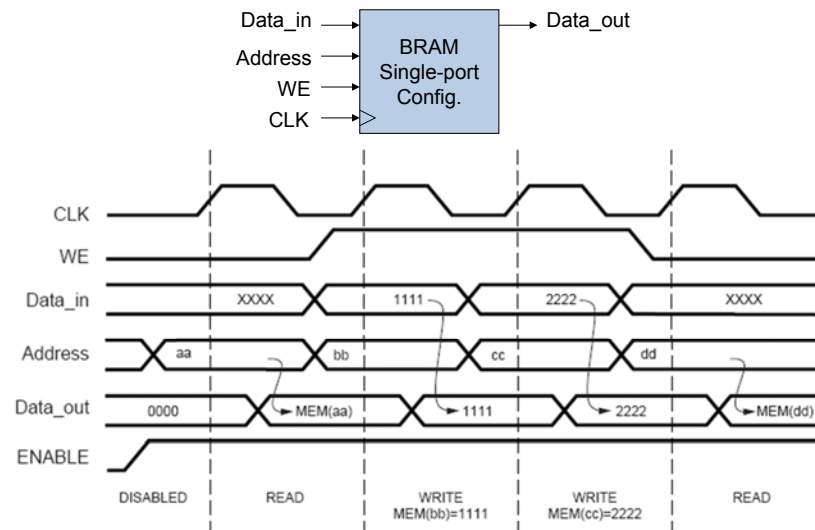
Lab 5A overview

Assignment: build a voice recorder that records and plays back 8-bit PCM data @ 6KHz



About 11 seconds of speech @ 6KHz

BRAM Operation

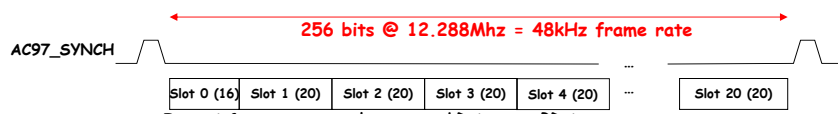
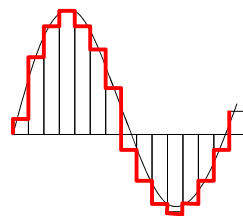


Source: Xilinx App Note 463

AC97: PCM data

PCM = pulse code modulation

Sample waveform at 48kHz, encode results as an N-bit signed number. For our AC97 chip, N = 18.



FPGA sends output frame to AC97 while AC97 sends input frame to FPGA

READY

ready selects a particular clock_27mhz clock edge when you should store input data from the AC97 (from_ac97_data) and provide new output to the AC97 (to_ac97_data).

Lab 5a* w/ FIR filter

- Since we're down-sampling by a factor of 8, to avoid aliasing (makes the recording sound "scratchy") we need to pass the incoming samples through a low-pass antialiasing filter to remove audio signal above 3kHz (Nyquist frequency of a 6kHz sample rate).



- We need a low-pass reconstruction filter (the same filter as for antialiasing!) when playing back the 6kHz samples. Actually we'll run it at 48kHz and achieve a 6kHz playback rate by feeding it a sample, 7 zeros, the next sample, 7 more zeros, etc.



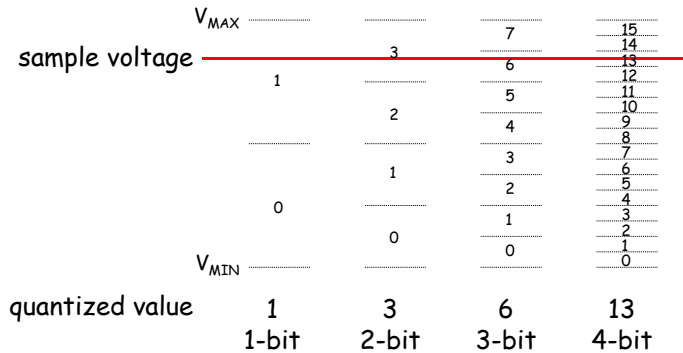
*Choose Lab5a or Lab5b

..., X_i, 0, 0, 0, 0, 0, 0, X_{i+1}, 0, 0, 0, 0, 0, 0, X_{i+2}, ...

Discrete Values

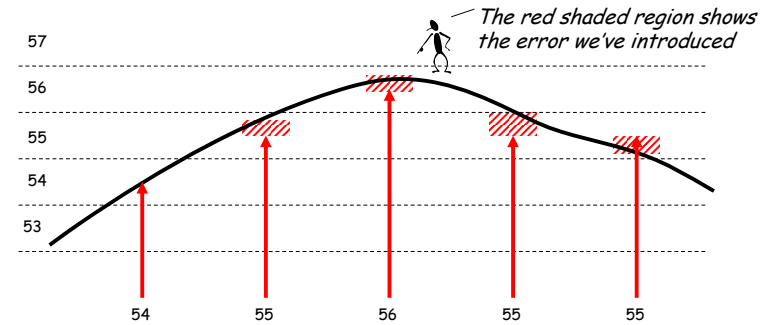
If we use N bits to encode the magnitude of one of the discrete-time samples, we can capture 2^N possible values.

So we'll divide up the range of possible sample values into 2^N intervals and choose the index of the enclosing interval as the encoding for the sample value.

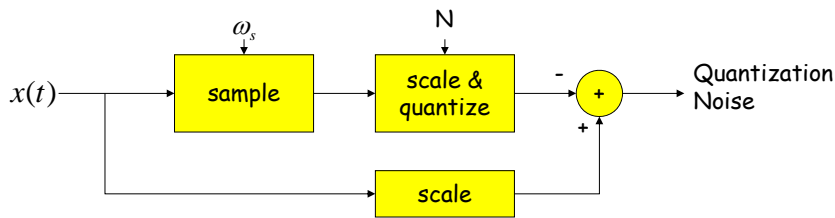


Quantization Error

Note that when we quantize the scaled sample values we may be off by up to $\pm \frac{1}{2}$ step from the true sampled values.

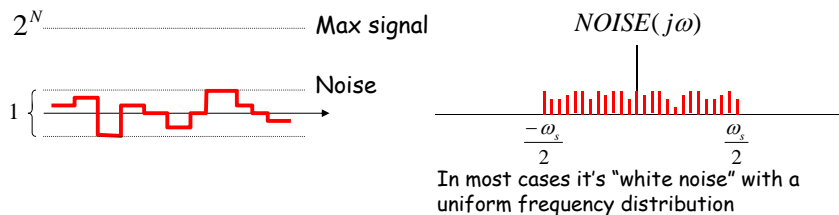


Quantization Noise



Time Domain

Freq. Domain



SNR: Signal-to-Noise Ratio

$$SNR = 10 \log_{10} \left(\frac{P_{SIGNAL}}{P_{NOISE}} \right) = 10 \log_{10} \left(\frac{A_{SIGNAL}^2}{A_{NOISE}^2} \right) = 20 \log_{10} \left(\frac{A_{SIGNAL}}{A_{NOISE}} \right)$$

↙ RMS amplitude

SNR is measured in decibels (dB). Note that it's a logarithmic scale: if SNR increases by 3dB the ratio has increased by a factor 2. When applied to audible sounds: the ratio of normal speech levels to the faintest audible sound is 60-70 dB.

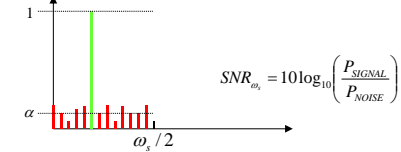
$$SNR = 20 \log_{10} \left(\frac{A_{signal}}{A_{noise}} \right) \approx 20 \log_{10} (2^N)$$

$$\approx N \cdot 6.02 \text{ dB}$$

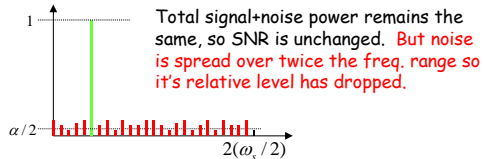
Oversampling

To avoid aliasing we know that ω_s must be at least $2\omega_M$. Is there any advantage to oversampling, i.e., $\omega_s = K \cdot 2\omega_M$?

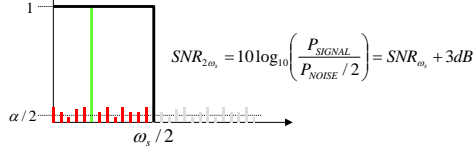
Suppose we look at the frequency spectrum of quantized samples of a sine wave: (sample freq. = ω_s)



Let's double the sample frequency to $2\omega_s$.



Now let's use a low pass filter to eliminate half the noise! Note that we're not affecting the signal at all...



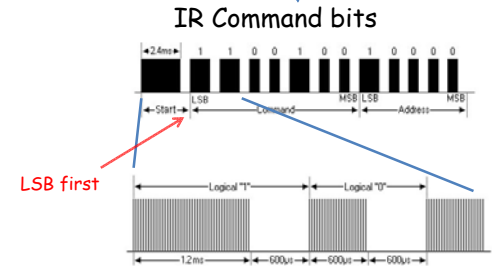
Oversampling+LPF reduces noise by 3dB/octave

Lab 5b Overview

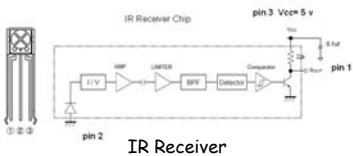
Assignment: build a digital system to "learn" four Sony Infrared Command (SIRC) and use it to control a Sony television.

- Data sent via 950nm IR modulated at 40kHz.
- Data width: 12, 15 or 20 bit protocol (use 12 bit).
- Start bit: 2400us
High: 1200us
Low: 600us
- Transmit FSM provided
- Learn/store remote commands

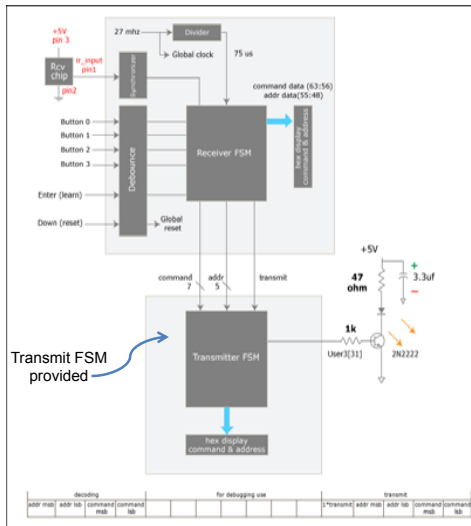
Volume down command
Address: 0000_1 (TV 1)
Command: 001_0011 (19)



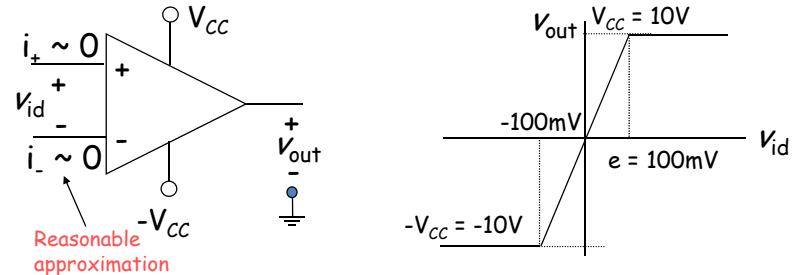
Lab 5b Block Diagram



- IR receiver demodulates signal and provides input into labkit - powered by 5V from labkit.
- 2N2222 BJT used to power IR transmitter (note bypass caps) - power from labkit
- Command code and channel displayed on hex display



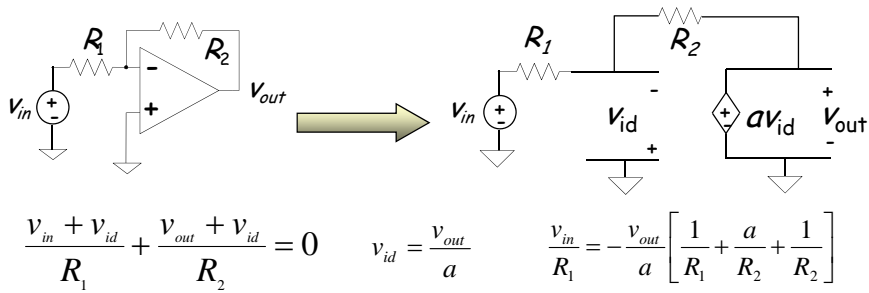
Our Analog Building Block: OpAmp



Linear Mode	Negative Saturation	Positive Saturation
$V_{id} \approx 0$	$V_{id} < -e$	$V_{id} > e$
$V_{out} = aV_{id}$		
If $-V_{CC} < V_{out} < V_{CC}$		

Very small input range for "open loop" configuration

The Power of (Negative) Feedback



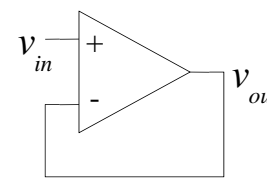
$$\frac{v_{in} + v_{id}}{R_1} + \frac{v_{out} + v_{id}}{R_2} = 0 \quad v_{id} = \frac{v_{out}}{a} \quad \frac{v_{in}}{R_1} = -\frac{v_{out}}{a} \left[\frac{1}{R_1} + \frac{a}{R_2} + \frac{1}{R_2} \right]$$

$$\frac{v_{out}}{v_{in}} = -\frac{R_2 a}{(1+a)R_1 + R_2} \approx -\frac{R_2}{R_1} \text{ (if } a \gg 1)$$

- Overall (closed loop) gain does not depend on open loop gain
- Trade gain for robustness
- Easier analysis approach: "virtual short circuit approach"
 - $v_+ = v_- = 0$ if OpAmp is linear

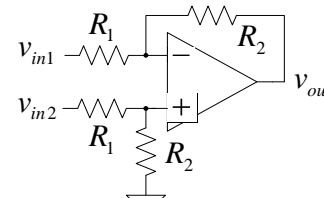
Basic OpAmp Circuits

Voltage Follower (buffer)



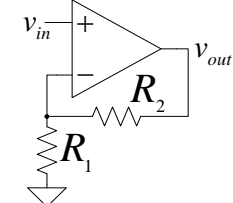
$$v_{out} \approx v_{in}$$

Differential Input



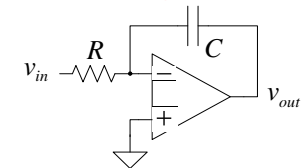
$$v_{out} \approx \frac{R_2}{R_1} (v_{in2} - v_{in1})$$

Non-inverting



$$v_{out} \approx \frac{R_1 + R_2}{R_1} v_{in}$$

Integrator



$$v_{out} \approx -\frac{1}{RC} \int_{-\infty}^t v_{in} dt$$

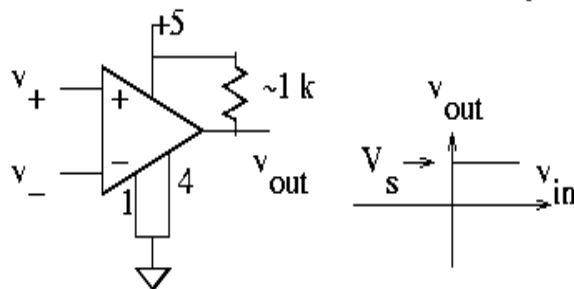
OpAmp as a Comparator

Analog Comparator:

Is $V_+ > V_-$? The Output is a DIGITAL signal

Analog Comparator: Analog to TTL

LM 311 Needs Pull-Up



LM311 uses a single supply voltage

Digital to Analog

Common metrics:

- Conversion rate - DC to ~500 MHz (video)
- # bits - up to ~24
- Voltage reference source (internal / external; stability)
- Output drive (unipolar / bipolar / current) & settling time
- Interface - parallel / serial
- Power dissipation

Common applications:

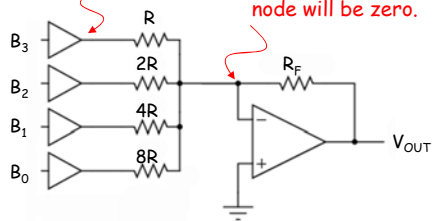
- Real world control (motors, lights)
- Video signal generation
- Audio / RF "direct digital synthesis"
- Telecommunications (light modulation)
- Scientific & Medical (ultrasound, ...)

DAC: digital to analog converter

How can we convert a N-bit binary number to a voltage?

$V_i = 0$ volts if $B_i = 0$
 $V_i = V$ volts if $B_i = 1$

OPAMP will vary V_{OUT} to maintain this node at 0V, i.e., the sum of the currents flowing into this node will be zero.



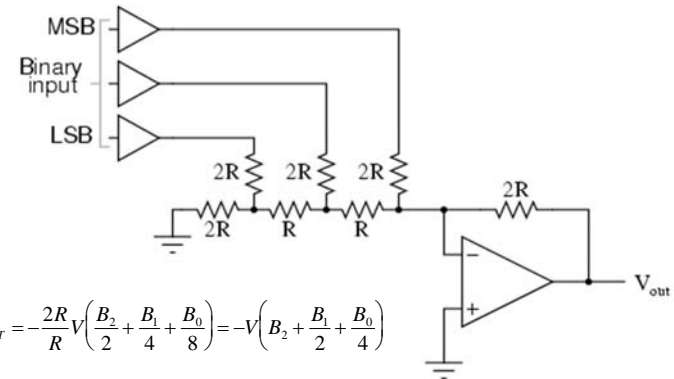
OKAY, this'll work, but the voltages produced by the drivers and various R's must be carefully matched in order to get equal steps.



$$\frac{V_{OUT}}{R_F} + \frac{B_3 V}{R} + \frac{B_2 V}{2R} + \frac{B_1 V}{4R} + \frac{B_0 V}{8R} = 0$$

$$V_{OUT} = -\frac{R_F}{R} V \left(B_3 + \frac{B_2}{2} + \frac{B_1}{4} + \frac{B_0}{8} \right)$$

R-2R Ladder DAC Architecture



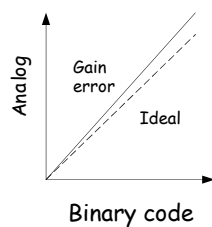
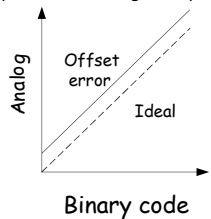
$$V_{OUT} = -\frac{2R}{R} V \left(\frac{B_2}{2} + \frac{B_1}{4} + \frac{B_0}{8} \right) = -V \left(B_2 + \frac{B_1}{2} + \frac{B_0}{4} \right)$$

R-2R Ladder achieves large current division ratios with only two resistor values

Non-idealities in Data Conversion

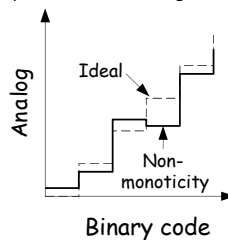
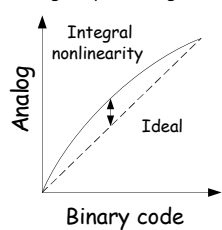
Offset - a constant voltage offset that appears at the output when the digital input is 0

Gain error - deviation of slope from ideal value of 1

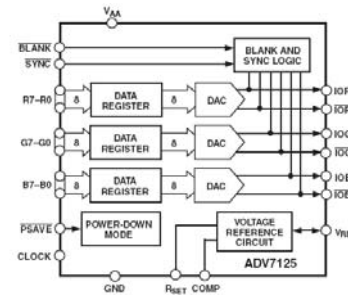


Integral Nonlinearity - maximum deviation from the ideal analog output voltage

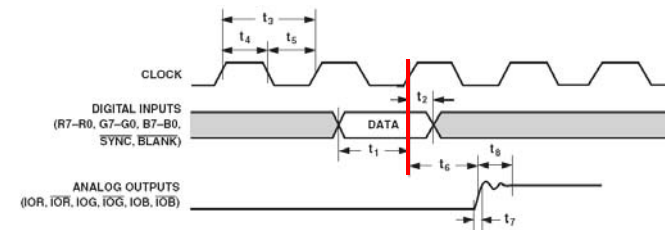
Differential nonlinearity - the largest increment in analog output for a 1-bit change



Labkit: ADV7125 Triple Out Video DAC



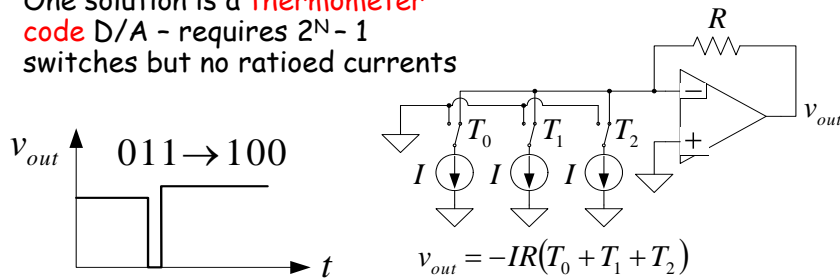
- Three 8-bit DACs
- Single Supply Op.: 3.3 to 5V
- Internal bandgap voltage ref
- Output: 2-26 mA
- 330 MSPS (million samples per second)
- Simple edge-triggered register-based interface



Glitching and Thermometer D/A

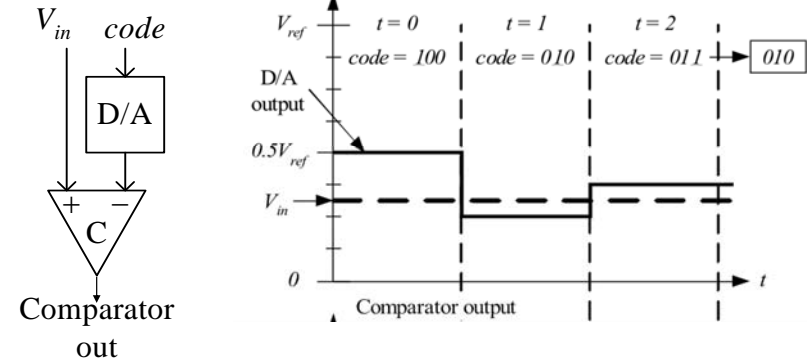
- **Glitching** is caused when switching times in a D/A are not synchronized
- **Example:** Output changes from 011 to 100 - MSB switch is delayed
- **Filtering** reduces glitch but increases the D/A settling time
- One solution is a **thermometer code D/A** - requires $2^N - 1$ switches but no ratioed currents

Binary	Thermometer
0 0	0 0 0
0 1	0 0 1
1 0	0 1 1
1 1	1 1 1



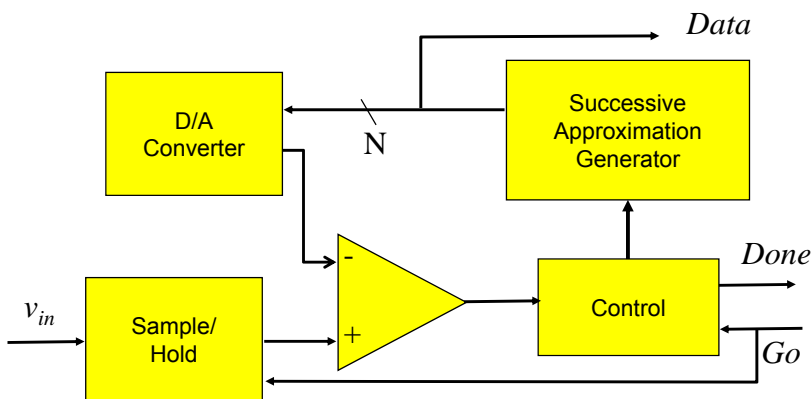
Successive-Approximation A/D

- D/A converters are typically compact and easier to design. Why not A/D convert using a D/A converter and a comparator?
- DAC generates analog voltage which is compared to the input voltage
- If DAC voltage > input voltage then set that bit; otherwise, reset that bit
- This type of ADC takes a fixed amount of time proportional to the bit length



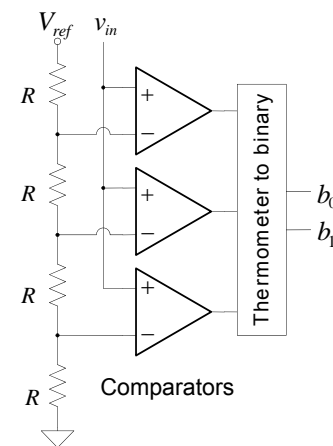
Example: 3-bit A/D conversion, $2 \text{ LSB} < V_{in} < 3 \text{ LSB}$

Successive-Approximation A/D



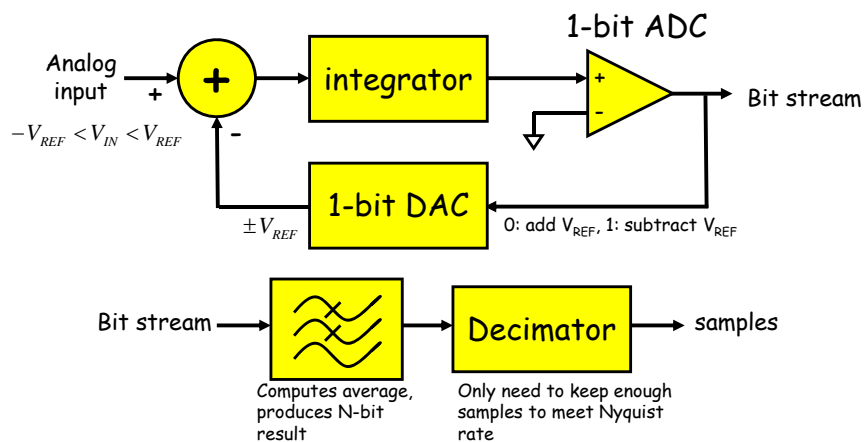
Serial conversion takes a time equal to $N(t_{D/A} + t_{comp})$

Flash A/D Converter



- Brute-force A/D conversion
- Simultaneously compare the analog value with every possible reference value
- Fastest method of A/D conversion
- **Size scales exponentially with precision** (requires 2^N comparators)

Sigma Delta ADC



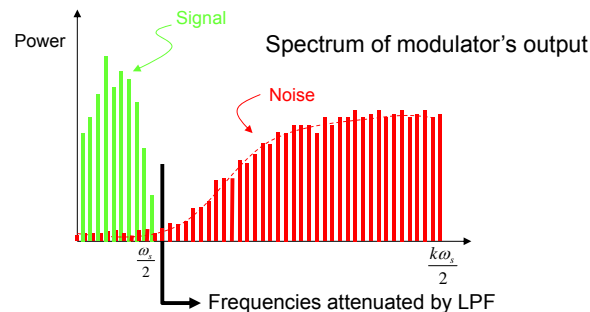
Average of bit stream ($1=V_{REF}$, $0=-V_{REF}$) gives voltage

With $V_{REF}=1V$: $V_{IN}=0.5$: 1110..., $V_{IN}=-0.25$: 00100101..., $V_{IN}=0.6$: 11110

<http://designtools.analog.com/dt/sdtutorial/sdtutorial.html#instructions>

So, what's the big deal?

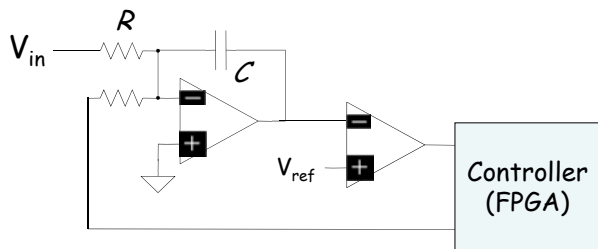
- Can be run at high sampling rates, oversampling by, say, 8 or 9 octaves for audio applications; low power implementations
- Feedback path through the integrator changes how the noise is spread across the sampling spectrum.



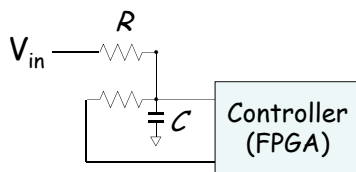
- Pushing noise power to higher frequencies means more noise is eliminated by LPF: N^{th} order $\Sigma\Delta$ SNR = $(3+N*6)$ dB/octave

Sigma Delta ADC

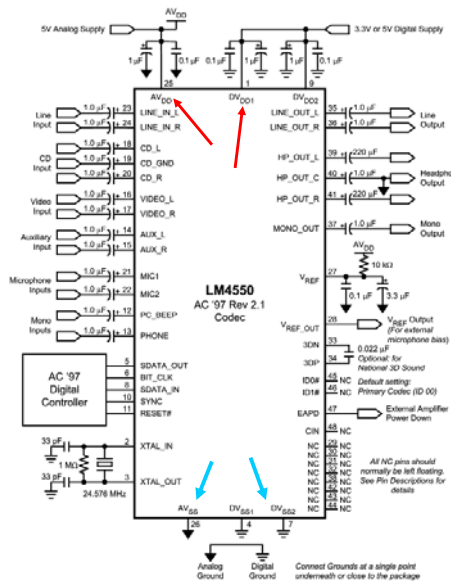
- A simple ADC:



- Poor Man's ADC:



AD Supply Voltages Consideration

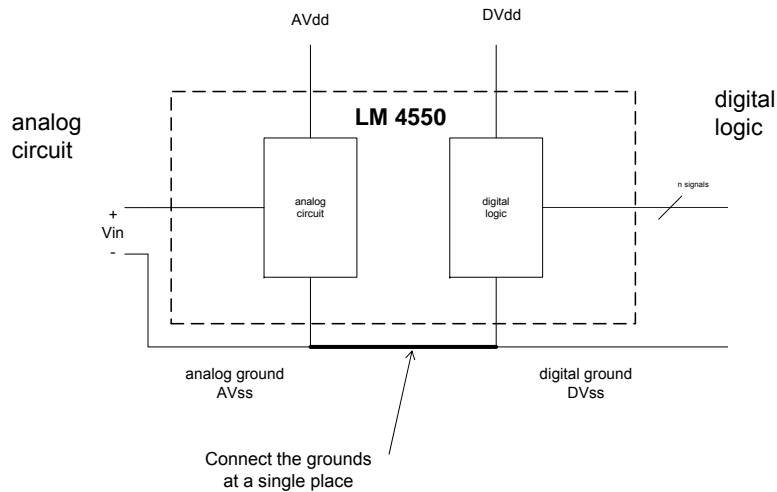


Noise caused by current spikes in fast switching digital circuits:

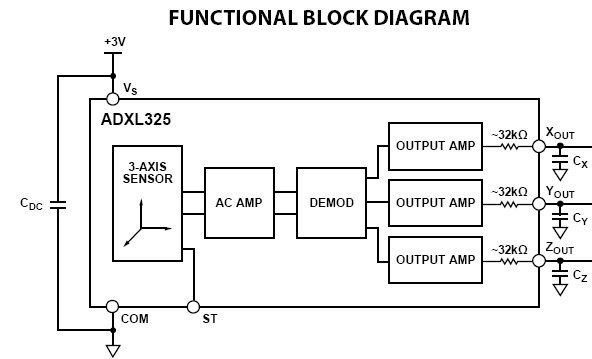
$$i_c = c \frac{dv}{dt}$$

- AV_{DD} Positive Analog Supply Voltage
- AV_{SS} Analog Ground
- DV_{DD} Positive Digital Supply Voltage
- DV_{SS} Digital Ground

Digital/Analog Grounds



Sensors

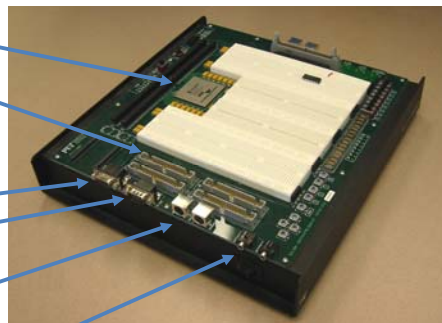


3 Axis 5G accelerometer

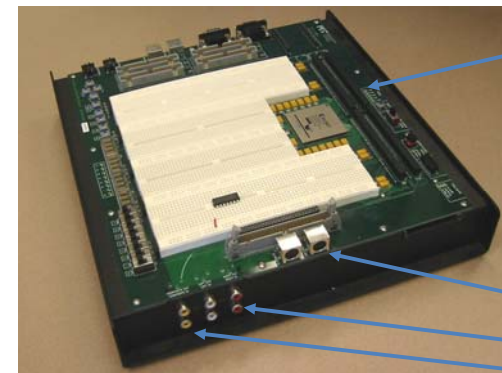
- Many sensors have native analog outputs: thermocouples, accelerometers, pressure gauge, ..
- 3-axis accelerometer now used in cell phones, games, iPods, laptops, 6.111 projects

Labkit Hardware

- Xilinx FPGA
- Logic analyzer pods
 - 4 banks/pods of 16 data lines
 - (analyzerN_clock) and a 16-bit data bus (analyzerN_data[15:0]) N=1,2,3,4
- VGA video output
- RS-232 Serial IO
- PS/2 keyboard and mouse input
- AC97 audio input/output
 - Intel standard for PC audio systems
 - codec's ADCs and DACs operate at a 48kHz sample rate, with 18 bits of precision
- 128Mbits Flash memory, (2) 512k x 36 ZBT SRAM



Labkit Hardware



- Bidirectional user
 - general purpose I/O, such as connecting to devices on the breadboards
 - bidirectional (inout) signals user1[31:0] through user4[31:0]
- TV Video
 - S video input/output
 - Audio input/output
 - Composite video input/output

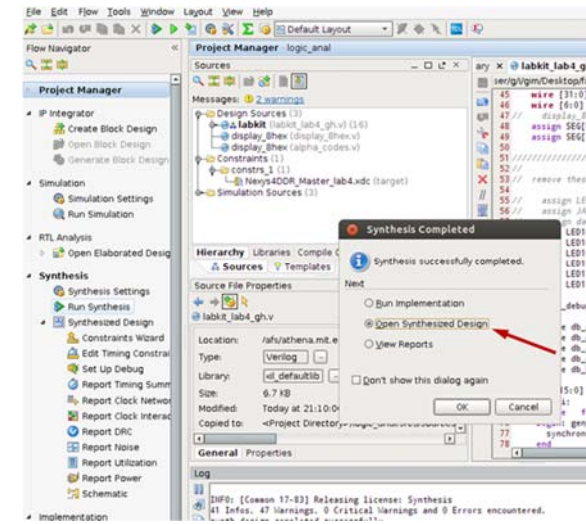
Integrated Logic Analyzer ILA

- Allows user to view actual signals in design with a virtual logic analyzer.
- Useful for debugging designs that seem to work in simulation but not implementation.
- Be sure change the divider clock from 25_000_000 to 3 or 4 clock cycles. (count == (SW[14] ? 3 : 24_999_999)) count <= 0;
- To view the signals, additional signals are placed and routed but used internally to display the waveforms.

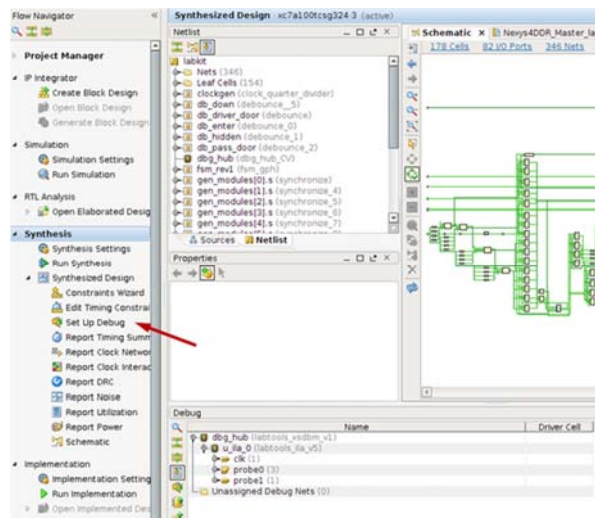
```
(* mark_debug = "true" *) wire [2:0] state; // virtual test probes
(* mark_debug = "true" *) wire driver_door; // virtual test

// more Verilog, etc...
```

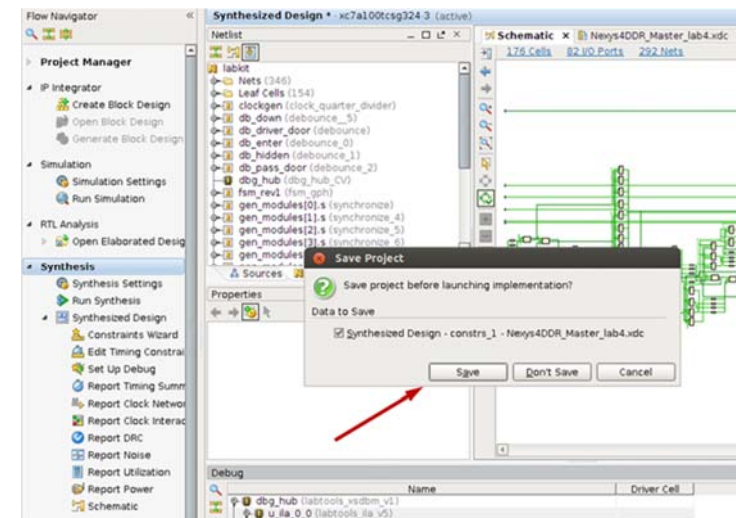
Run Synthesized Design



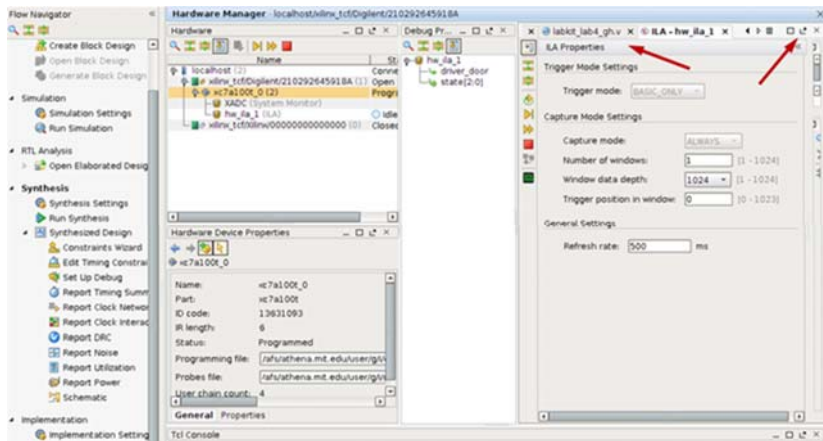
Setup Debug



Save Constraints

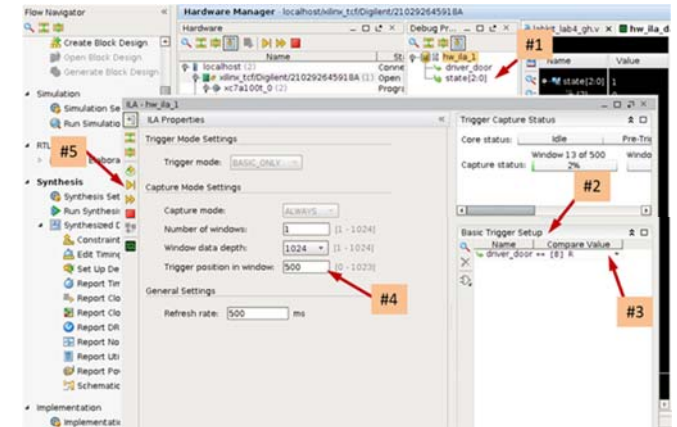


Implementation



Load the bit file to the FPGA. Undock ILA window .
Set up the trigger for the ILA.

Set Trigger



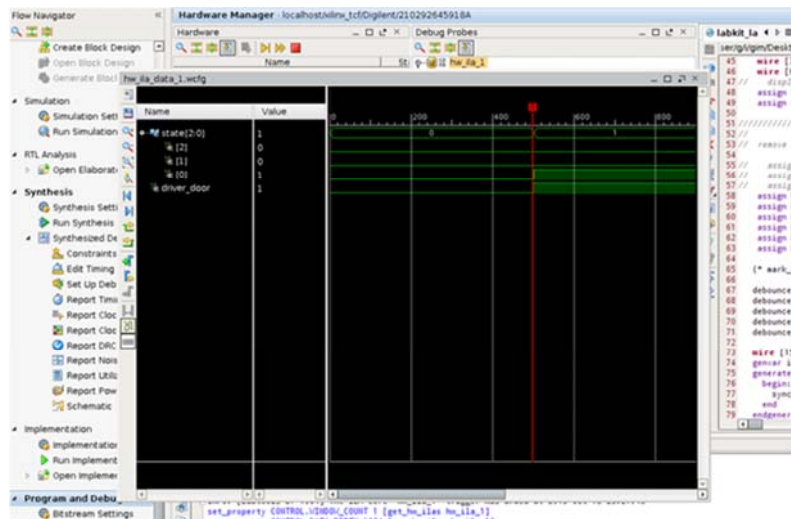
Select and drag trigger (1,2)

Set trigger value (3)

Select trigger position (4)

Run (5)

Waveform Display



Upload Lab 4 Verilog

- Submit by Monday
- Grading
 - Proper use of blocking and non-blocking assignments
 - **Readable Code (reformatted)** with comments and consistent indenting [use emacs or VIM]
 - Use of default in case statement
 - Use of parameter statements for symbolic name and constants (state==5 vs state==DATA_READY)
 - Parameterized modules when appropriate
 - Readable logical flow, properly formatted (see "Verilog Editors")
 - No long nested if statements.
 - Score 1 to 3 (3 perfect); 1/2 point off for each occurrence.