Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science
6.111 - Introductory Digital Systems Laboratory

Problem Set 1 Solutions

Issued: February 13, 2004

Problem 1: Boolean Algebra Practice Problems *(Problem 1 was not graded.)*

1) $a + 0 = a$
2) $\bar{a} \cdot 0 = 0$
3) $a + \bar{a} = 1$
4) $a + a = a$
5) $a + ab = a(1 + b) = a$
6) $a + \bar{a}b = (a + \bar{a})(a + b) = a + b$
7) $a(\bar{a} + b) = a\bar{a} + ab = ab$
8) $ab + \bar{a}b = b(a + \bar{a}) = b$
9) $(\bar{a} + \bar{b})(\bar{a} + b) = \bar{a}\bar{a} + \bar{a}b + \bar{b}\bar{a} + \bar{b}b = \bar{a} + \bar{a}b + \bar{a}\bar{b} = \bar{a}(1 + b + \bar{b}) = \bar{a}$
10) $a(a + b + c + ...) = aa + ab + ac + ... = a + ab + ac + ... = a$
11) $f(a, b, ab) = a + b + ab = a + b$
12) $f(a, b, \bar{a} \cdot \bar{b}) = a + b + \bar{a}\bar{b} = a + b + \bar{a} = 1$
13) $f[a, b, \overline{(ab)}] = a + b + \overline{(ab)} = a + b + \bar{a} + \bar{b} = 1$
14) $y + y\bar{y} = y$
15) $xy + x\bar{y} = x(y + \bar{y}) = x$
16) $\bar{x} + y\bar{x} = \bar{x}(1 + y) = \bar{x}$
17) $(w + \bar{x} + y + \bar{z})y = y$
18) $(x + \bar{y})(x + y) = x$
19) $w + [w + (wx)] = w$
20) $x[x + (xy)] = x$
21) $\overline{(\bar{x} + \bar{x})} = x$
22) $\overline{(x + \bar{x})} = 0$
23) $w + (w\bar{x}yz) = w(1 + \bar{x}yz) = w$
24) $\bar{w} \cdot \overline{(wxyz)} = \bar{w}(\bar{w} + \bar{x} + \bar{y} + \bar{z}) = \bar{w}$
25) $xz + \bar{x}y + zy = xz + \bar{x}y$
26) $(x + z)(\bar{x} + y)(z + y) = (x + z)(\bar{x} + y)$
27) $\bar{x} + \bar{y} + xy\bar{z} = \bar{x} + \bar{y} + \bar{z}$

Problem 2: Karnaugh Maps and Minimal Expressions

It is best to approach each expression by first simplifying it using Boolean algebra learned Problem 1. If you can put the expression into a product-of-sums or sum-of-products form (not necessary minimal), then it is very easy to declare when the expression is a 0 or a 1, respectively.

1) $(a + b \cdot \overline{c}) + d \cdot (\overline{a} \cdot \overline{b} \cdot \overline{c} + a \cdot b)$

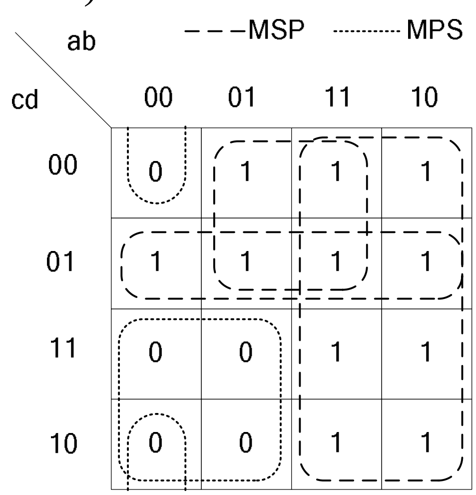
$= a + b \cdot \overline{c} + \overline{a} \cdot \overline{b} \cdot \overline{c} \cdot d + a \cdot b \cdot d$

$= a + b \cdot \overline{c} + \overline{a} \cdot \overline{b} \cdot \overline{c} \cdot d$

i)

a	b	c	d	**expr**
0	0	0	0	**0**
0	0	0	1	**1**
0	0	1	0	**0**
0	0	1	1	**0**
0	1	0	0	**1**
0	1	0	1	**1**
0	1	1	0	**0**
0	1	1	1	**0**
1	0	0	0	**1**
1	0	0	1	**1**
1	0	1	0	**1**
1	0	1	1	**1**
1	1	0	0	**1**
1	1	0	1	**1**
1	1	1	0	**1**
1	1	1	1	**1**

ii)



iii)

$MSP = a + b\overline{c} + \overline{c}d$

iv)

$MPS = (a + \overline{c}) \cdot (a + b + d)$

2) $(\overline{d}+b\cdot\overline{c})\cdot(c\cdot d+(\overline{a}+c)\cdot(\overline{c}+d))\cdot(b+\overline{c})$

$=(\overline{d}+\cdot b\cdot\overline{c})\cdot(c\cdot d+\overline{a}\cdot\overline{c}+\overline{a}\cdot d+\overline{c}\cdot c+c\cdot d)\cdot(b+\overline{c})$

$=(\overline{d}+b\cdot\overline{c})\cdot(b+\overline{c})\cdot(c\cdot d+\overline{a}\cdot\overline{c}+\overline{a}\cdot d)$

$=(b\cdot\overline{d}+b\cdot\overline{c}+b\cdot\overline{c}+\overline{c}\cdot\overline{d})\cdot(c\cdot d+\overline{a}\cdot\overline{c}+\overline{a}\cdot d)$

$=b\cdot\overline{d}\cdot c\cdot d+\overline{a}\cdot b\cdot\overline{c}\cdot\overline{d}+a\cdot b\cdot d\cdot\overline{d}+b\cdot\overline{c}\cdot c\cdot d+\overline{a}\cdot b\cdot\overline{c}+\overline{a}\cdot\overline{c}\cdot\overline{d}$

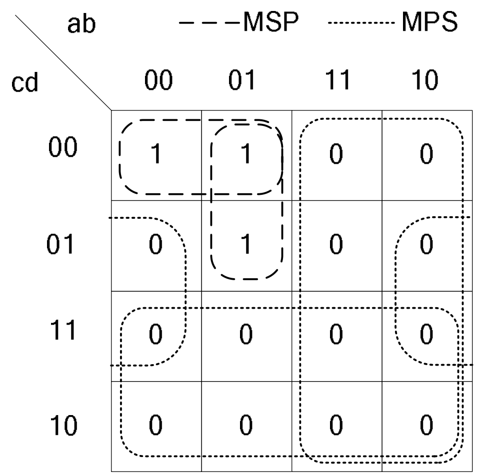
$=\overline{a}\cdot b\cdot\overline{c}\cdot\overline{d}+\overline{a}\cdot b\cdot\overline{c}+\overline{a}\cdot\overline{c}\cdot\overline{d}$

$=\overline{a}\cdot b\cdot\overline{c}+\overline{a}\cdot\overline{c}\cdot\overline{d}$

i)

a	b	c	d	**expr**
0	0	0	0	**1**
0	0	0	1	**0**
0	0	1	0	**0**
0	0	1	1	**0**
0	1	0	0	**1**
0	1	0	1	**1**
0	1	1	0	**0**
0	1	1	1	**0**
1	0	0	0	**0**
1	0	0	1	**0**
1	0	1	0	**0**
1	0	1	1	**0**
1	1	0	0	**0**
1	1	0	1	**0**
1	1	1	0	**0**
1	1	1	1	**0**

ii)



iii)

$MSP=\overline{a}\cdot\overline{c}\cdot\overline{d}+\overline{a}\cdot b\cdot\overline{c}$

iv)

$MPS=\overline{a}\cdot\overline{c}\cdot(b+\overline{d})$

3) $\overline{(w \cdot y)} \cdot (\overline{w} + \overline{y} + z) \cdot (w + x + \overline{y})$

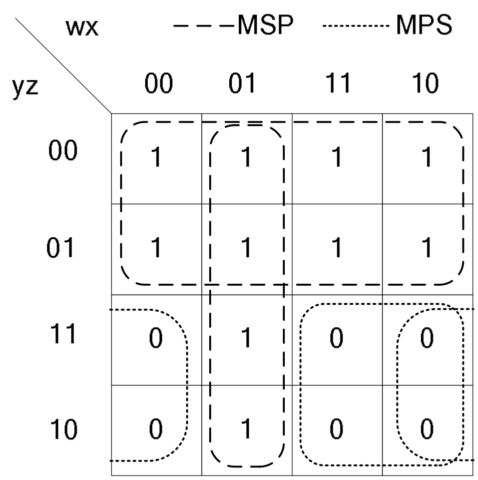
$= (\overline{w} + \overline{y}) \cdot (\overline{w} + \overline{y} + z) \cdot (w + x + \overline{y})$

$= (\overline{w} + \overline{y}) \cdot (w + x + \overline{y})$

i)

w	x	y	z	**expr**
0	0	0	0	**1**
0	0	0	1	**1**
0	0	1	0	**0**
0	0	1	1	**0**
0	1	0	0	**1**
0	1	0	1	**1**
0	1	1	0	**1**
0	1	1	1	**1**
1	0	0	0	**1**
1	0	0	1	**1**
1	0	1	0	**0**
1	0	1	1	**0**
1	1	0	0	**1**
1	1	0	1	**1**
1	1	1	0	**0**
1	1	1	1	**0**

ii)



iii)

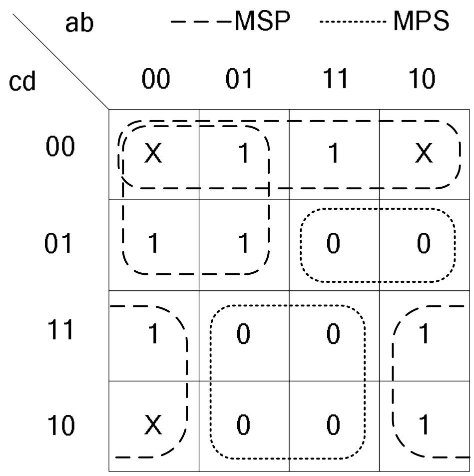
$MSP = \overline{y} + \overline{w} \cdot x$

iv)

$MPS = (\overline{w} + \overline{y}) \cdot (x + \overline{y})$

Problem 3: Karnaugh Maps with "Don't Cares"

1)



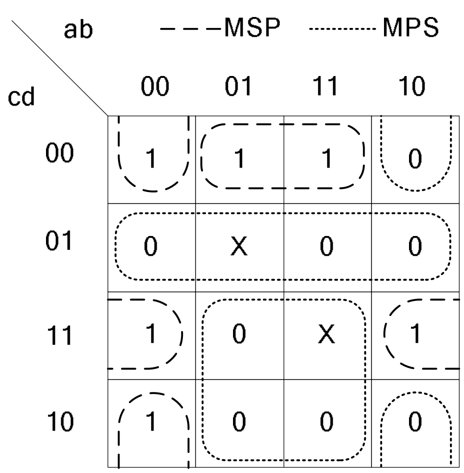
i) $MSP = \overline{c} \cdot \overline{d} + \overline{a} \cdot \overline{c} + \overline{b} \cdot c$

ii) $MPS = (\overline{b} + \overline{c}) \cdot (\overline{a} + c + \overline{d})$

iii) Yes, the solutions are unique.

iv) Yes. We can tell that the MSP=MPS, without algebra, if their groupings on the Karnaugh map do not overlap and collectively cover the map.

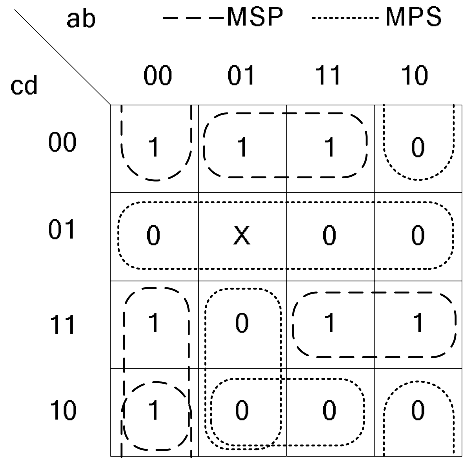
2)



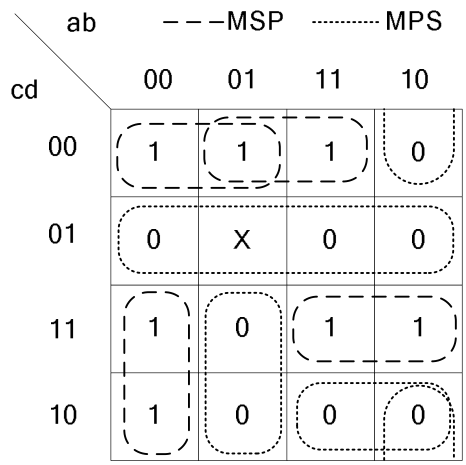
i) $MSP = b \cdot \overline{c} \cdot \overline{d} + \overline{a} \cdot \overline{b} \cdot \overline{d} + \overline{b} \cdot c \cdot d$

ii) $MPS = (c + \overline{d}) \cdot (\overline{b} + \overline{c}) \cdot (\overline{a} + b + d)$

iii) Yes, the solutions are unique. But, consider a similar K-map with the following groupings:



In this case, neither the MSP nor the MPS are unique since the following groupings are also valid. But in both groupings, the MSP=MPS.



iv) Yes, the MSP equals the MPS.

Problem 4: DeMorgan's Theorem

1) $\overline{\overline{(\overline{a}+d)}\cdot\overline{(\overline{a}+\overline{c})}}$

$=\overline{\overline{(\overline{a}+d)}}+\overline{\overline{(\overline{a}+\overline{c})}}$

$=(\overline{a}+d)+(\overline{a}+\overline{c})$

$=\overline{a}+\overline{c}+d$

2) $\overline{a\cdot b\cdot c}$ (already simplified)

3) $\overline{a+d}\cdot\overline{\overline{a}+\overline{c}}\cdot\overline{c+\overline{d}}$

$=(\overline{a}\cdot\overline{d})\cdot(\overline{\overline{a}}\cdot\overline{\overline{c}})\cdot(\overline{c}\cdot\overline{\overline{d}})$

$=(\overline{a}\cdot\overline{d})\cdot(a\cdot c)\cdot(\overline{c}\cdot d)$

$=0$

Problem 5: Setup and Hold Times for D Flip-Flop

1) The setup time is twice the delay of the inverter. The hold time is zero.
2) The new memory element is a <u>negative</u>-edge triggered flip flop.
3) The setup time is $2t_{inv}$, the hold time is zero, and the clock to Q delay is $2t_{inv}$.