

Massachusetts Institute of Technology
 Department of Electrical Engineering and Computer Science
 6.111 - Introductory Digital Systems Laboratory

Problem Set 1

Issued: February 8, 2006

Due: February 21, 2006

Boolean Algebra Practice Problems (do not turn in):

Simplify each expression by algebraic manipulation. Try to recognize when it is appropriate to transform to the dual, simplify, and re-transform (e.g. no. 6). Try doing the problems before looking at the solutions which are at the end of this problem set.

1) $a + 0 =$

2) $\bar{a} \cdot 0 =$

3) $a + \bar{a} =$

4) $a + a =$

5) $a + ab =$

6) $a + \bar{a}b =$

7) $a(\bar{a} + b) =$

8) $ab + \bar{a}b =$

9) $(\bar{a} + \bar{b})(\bar{a} + b) =$

10) $a(a + b + c + \dots) =$

For (11), (12), (13), $f(a, b, c) = a + b + c$

11) $f(a, b, ab) =$

12) $f(a, b, \bar{a} \cdot \bar{b}) =$

13) $f[a, b, (\bar{a}b)] =$

14) $y + y\bar{y} =$

15) $xy + x\bar{y} =$

16) $\bar{x} + y\bar{x} =$

17) $(w + \bar{x} + y + \bar{z})y =$

18) $(x + \bar{y})(x + y) =$

19) $w + [w + (wx)] =$

20) $x[x + (xy)] =$

21) $\overline{(x + \bar{x})} =$

22) $(x + \bar{x}) =$

23) $w + (w\bar{x}yz) =$

24) $\bar{w} \cdot \overline{(wxyz)} =$

25) $xz + \bar{x}y + zy =$

26) $(x + z)(\bar{x} + y)(z + y) =$

27) $\bar{x} + \bar{y} + xy\bar{z} =$

Problem 1: Karnaugh Maps and Minimal Expressions

For each of the following Boolean expressions, give:

- i) The truth table,
- ii) The Karnaugh map,
- iii) The MSP expression, (Show groupings)
- iv) The MPS expression. (Show groupings)

1) $(\bar{a} + b \cdot \bar{d}) \cdot (c \cdot b \cdot a + \bar{c} \cdot d)$

2) $\overline{(w \cdot x + y \cdot z + y \cdot w \cdot x)}$

Problem 2: Karnaugh Maps with “Don’t Cares”

Karnaugh Maps are useful for finding minimal implementations of Boolean expressions with only a few variables. However, they can be a little tricky when “don't cares” (X) are involved. Using the following K-Maps:

		ab			
		00	01	11	10
cd	00	X	0	0	1
	01	1	0	0	X
	11	0	X	0	1
	10	0	0	0	1

(1)

		ab			
		00	01	11	10
cd	00	1	0	0	1
	01	0	1	X	1
	11	X	1	0	0
	10	1	1	0	X

(2)

- i) Find the minimal sum of products expression. Show your groupings.
- ii) Find the minimal product of sums expression. Show your groupings.
- iii) Are your solutions unique? If not, list and show the other minimal expressions.
- iv) Does the MPS = MSP?

Problem 3: DeMorgan’s Theorem

Use DeMorgan's Theorems to simplify the following expressions:

$$1) \overline{\overline{(a+d)} \cdot \overline{(b+c)}}$$

$$2) \overline{(a \cdot b \cdot c) + (c \cdot d)}$$

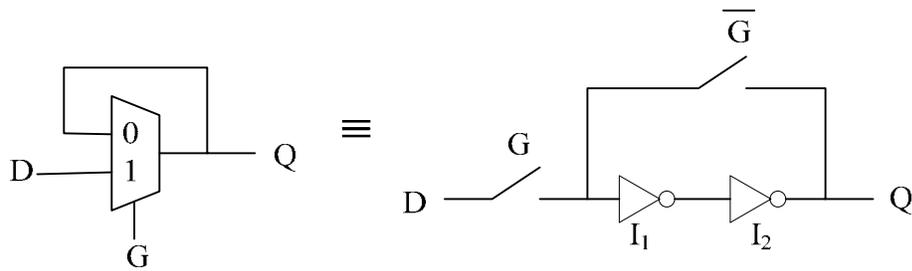
$$3) \overline{a+d} \cdot \overline{b+c} \cdot \overline{c+d}$$

Problem 4: Transistor/Gate Level Synthesis

- 1) Construct a transistor level circuit of the following function using NMOS and PMOS devices: $F = \overline{A \cdot (B + C)}$
- 2) Construct a gate level circuit of the same function only using NAND gates.

Problem 5: Setup and Hold Times for D Flip-Flop (*Flip-flops will be covered in lecture 4*)

- 1) Let a D latch be implemented using a mux and realized as follows:

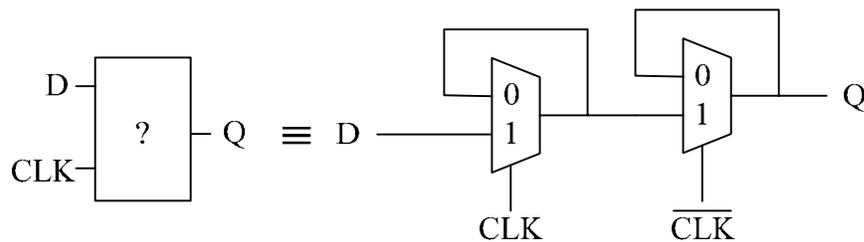


You may assume the following:

- G and \overline{G} are complements and have zero skew, i.e. when G is 1, \overline{G} is exactly 0, and vice versa.
- Assume the switches are ideal, with no delay. E.g. when G is 0, the switch is open.
- The propagation delay of the inverters is t_{inv} (assume that the contamination delay or minimum delay is equal to the propagation delay).

What is the setup and hold time of this latch?

- What memory element is created when two muxes are cascaded as in the figure below? Assume that CLK and \overline{CLK} are complements with zero skew.



- What is the setup time, hold time, and clock to Q delay of the above memory element?

Solutions to the Boolean Algebra Practice Problems

- 1) $a + 0 = a$
- 2) $\bar{a} \cdot 0 = 0$
- 3) $a + \bar{a} = 1$
- 4) $a + a = a$
- 5) $a + ab = a(1 + b) = a$
- 6) $a + \bar{a}b = (a + \bar{a})(a + b) = a + b$
- 7) $a(\bar{a} + b) = a\bar{a} + ab = ab$
- 8) $ab + \bar{a}b = b(a + \bar{a}) = b$
- 9) $(\bar{a} + \bar{b})(\bar{a} + b) = \bar{a}\bar{a} + \bar{a}b + \bar{b}\bar{a} + \bar{b}b = \bar{a} + \bar{a}b + \bar{a}\bar{b} = \bar{a}(1 + b + \bar{b}) = \bar{a}$
- 10) $a(a + b + c + \dots) = aa + ab + ac + \dots = a + ab + ac + \dots = a$
- 11) $f(a, b, ab) = a + b + ab = a + b$
- 12) $f(a, b, \bar{a} \cdot \bar{b}) = a + b + \overline{ab} = a + b + \bar{a} = 1$
- 13) $f[a, b, \overline{(ab)}] = a + b + \overline{(ab)} = a + b + \bar{a} + \bar{b} = 1$
- 14) $y + y\bar{y} = y$
- 15) $xy + x\bar{y} = x(y + \bar{y}) = x$
- 16) $\bar{x} + y\bar{x} = \bar{x}(1 + y) = \bar{x}$
- 17) $(w + \bar{x} + y + \bar{z})y = y$
- 18) $(x + \bar{y})(x + y) = x$
- 19) $w + [w + (wx)] = w$
- 20) $x[x + (xy)] = x$
- 21) $\overline{(\bar{x} + \bar{x})} = x$
- 22) $\overline{(x + \bar{x})} = 0$
- 23) $w + (\overline{wxyz}) = w(1 + \overline{xyz}) = w$
- 24) $\bar{w} \cdot \overline{(wxyz)} = \bar{w}(\bar{w} + \bar{x} + \bar{y} + \bar{z}) = \bar{w}$
- 25) $xz + \bar{x}y + zy = xz + \bar{x}y$
- 26) $(x + z)(\bar{x} + y)(z + y) = (x + z)(\bar{x} + y)$
- 27) $\bar{x} + \bar{y} + xy\bar{z} = \bar{x} + \bar{y} + \bar{z}$