

Massachusetts Institute of Technology  
 Department of Electrical Engineering and Computer Science  
 6.111 - Introductory Digital Systems Laboratory

**Problem Set 1**

**Issued:** February 6, 2008

**Due:** February 19, 2008

**Boolean Algebra Practice Problems (do not turn in):**

Simplify each expression by algebraic manipulation. Try to recognize when it is appropriate to transform to the dual, simplify, and re-transform (e.g. no. 6). Try doing the problems before looking at the solutions which are at the end of this problem set.

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|--|--|
| 1) $a + 0 =$ _____<br>2) $\bar{a} \cdot 0 =$ _____<br>3) $a + \bar{a} =$ _____<br>4) $a + a =$ _____<br>5) $a + ab =$ _____<br>6) $a + \bar{a}b =$ _____<br>7) $a(\bar{a} + b) =$ _____<br>8) $ab + \bar{a}b =$ _____<br>9) $(\bar{a} + \bar{b})(\bar{a} + b) =$ _____<br>10) $a(a + b + c + \dots) =$ _____<br>For (11), (12), (13), $f(a, b, c) = a + b + c$<br>11) $f(a, b, ab) =$ _____<br>12) $f(a, b, \bar{a} \cdot \bar{b}) =$ _____<br>13) $f[a, b, (\bar{a}b)] =$ _____ | 14) $y + y\bar{y} =$ _____<br>15) $xy + x\bar{y} =$ _____<br>16) $\bar{x} + y\bar{x} =$ _____<br>17) $(w + \bar{x} + y + \bar{z})y =$ _____<br>18) $(x + \bar{y})(x + y) =$ _____<br>19) $w + [w + (wx)] =$ _____<br>20) $x[x + (xy)] =$ _____<br>21) $\overline{(\bar{x} + \bar{x})} =$ _____<br>22) $\overline{(x + \bar{x})} =$ _____<br>23) $w + (\overline{wxyz}) =$ _____<br>24) $\bar{w} \cdot (\overline{wxyz}) =$ _____<br>25) $xz + \bar{x}y + zy =$ _____<br>26) $(x + z)(\bar{x} + y)(z + y) =$ _____<br>27) $\bar{x} + \bar{y} + xy\bar{z} =$ _____ |
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**Problem 1: Karnaugh Maps and Minimal Expressions**

For each of the following Boolean expressions, give:

- i) The truth table,
- ii) The Karnaugh map,
- iii) The minimal sum of products expression. (Show groupings)
- iv) The minimal product of sums expression. (Show groupings)

- 1)  $(\bar{a} + b \cdot \bar{d}) \cdot (c \cdot b \cdot a + \bar{c} \cdot d)$
- 2)  $\overline{(w + \bar{x})(z\bar{y} + x)}$

## Problem 2: Karnaugh Maps with “Don’t Cares”

Karnaugh Maps are useful for finding minimal implementations of Boolean expressions with only a few variables. However, they can be a little tricky when “don't cares” (X) are involved. Using the following K-Maps:

		ab			
		00	01	11	10
cd	00	X	0	0	1
	01	1	0	0	X
	11	0	X	0	1
	10	0	0	0	1

(1)

		ab			
		00	01	11	10
cd	00	1	X	0	1
	01	1	1	1	0
	11	0	0	X	0
	10	X	0	1	1

(2)

- i) Find the minimal sum of products expression. Show your groupings.
- ii) Find the minimal product of sums expression. Show your groupings.
- iii) Are your solutions unique? If not, list and show the other minimal expressions.
- iv) Does the MPS = MSP?

## Problem 3: DeMorgan's Theorem

Use DeMorgan's Theorems to simplify the following expressions:

$$1) \overline{\overline{(a+d)} \cdot \overline{(b+c)}}$$

$$2) \overline{(a \cdot b \cdot \bar{c}) + (\bar{c} \cdot d)}$$

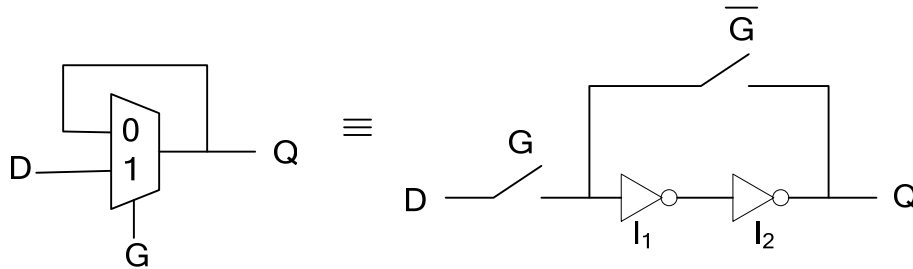
$$3) \overline{\overline{a+d} \cdot \overline{b+c} \cdot \overline{c+d}}$$

## Problem 4: Transistor/Gate Level Synthesis

- 1) Construct a transistor level circuit with inputs  $A$ ,  $B$ , and  $C$ , and output  $F$  of the following function using NMOS and PMOS devices:  $F = \overline{A + B \cdot C}$
- 2) Construct a gate level circuit of the same function with inputs  $A$ ,  $B$ , and  $C$ , and output  $F$  only using NAND gates.

**Problem 5: Setup and Hold Times for D Flip-Flop** (*Flip-flops will be covered in lecture 4*)

- 1) Let a D latch be implemented using a mux and realized as follows:

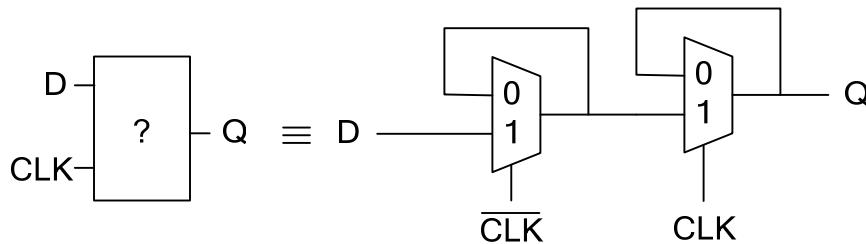


You may assume the following:

- $G$  and  $\overline{G}$  are complements and have zero skew, i.e. when  $G$  is 1,  $\overline{G}$  is exactly 0, and vice versa.
- Assume the switches are ideal, with no delay. E.g. when  $G$  is 0, the switch is open.
- The propagation delay of the inverters is  $t_{inv}$  (assume that the contamination delay or minimum delay is equal to the propagation delay).

What is the setup and hold time of this latch?

- 2) What memory element is created when two muxes are cascaded as in the figure below? Assume that  $CLK$  and  $\overline{CLK}$  are complements with zero skew.



- 3) What is the setup time, hold time, and clock to Q delay of the above memory element?

**Solutions to the Boolean Algebra Practice Problems**

1)  $a + 0 = a$

2)  $\bar{a} \cdot 0 = 0$

3)  $a + \bar{a} = 1$

4)  $a + a = a$

5)  $a + ab = a(1 + b) = a$

6)  $a + \bar{a}b = (a + \bar{a})(a + b) = a + b$

7)  $a(\bar{a} + b) = a\bar{a} + ab = ab$

8)  $ab + \bar{a}b = b(a + \bar{a}) = b$

9)  $(\bar{a} + \bar{b})(\bar{a} + b) = \bar{a}\bar{a} + \bar{a}b + \bar{b}\bar{a} + \bar{b}b = \bar{a} + \bar{a}b + \bar{a}\bar{b} = \bar{a}(1 + b + \bar{b}) = \bar{a}$

10)  $a(a + b + c + \dots) = aa + ab + ac + \dots = a + ab + ac + \dots = a$

11)  $f(a, b, ab) = a + b + ab = a + b$

12)  $f(a, b, \bar{a} \cdot \bar{b}) = a + b + \bar{a}\bar{b} = a + b + \bar{a} = 1$

13)  $f[a, b, \overline{(ab)}] = a + b + \overline{(ab)} = a + b + \bar{a} + \bar{b} = 1$

14)  $y + y\bar{y} = y$

15)  $xy + x\bar{y} = x(y + \bar{y}) = x$

16)  $\bar{x} + y\bar{x} = \bar{x}(1 + y) = \bar{x}$

17)  $(w + \bar{x} + y + \bar{z})y = y$

18)  $(x + \bar{y})(x + y) = x$

19)  $w + [w + (wx)] = w$

20)  $x[x + (xy)] = x$

21)  $\overline{(\bar{x} + \bar{x})} = x$

22)  $\overline{(x + \bar{x})} = 0$

23)  $w + (w\bar{x}yz) = w(1 + \bar{x}yz) = w$

24)  $\bar{w} \cdot \overline{(wxyz)} = \bar{w}(\bar{w} + \bar{x} + \bar{y} + \bar{z}) = \bar{w}$

25)  $xz + \bar{x}y + zy = xz + \bar{x}y$

26)  $(x + z)(\bar{x} + y)(z + y) = (x + z)(\bar{x} + y) = xy + \bar{x}z$

27)  $\bar{x} + \bar{y} + xy\bar{z} = \bar{x} + \bar{y} + \bar{z}$