# Class Notes for
# Modern Optics Project Laboratory - 6.161
# Optical Signals, Devices and Systems - 6.637
# **Geometric Optics**

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# Contents

# Chapter 2

# Geometric Optics

## 2.1 Lenses and Mirrors

Lenses fall into several classes as illustrated below in Figure 2.1

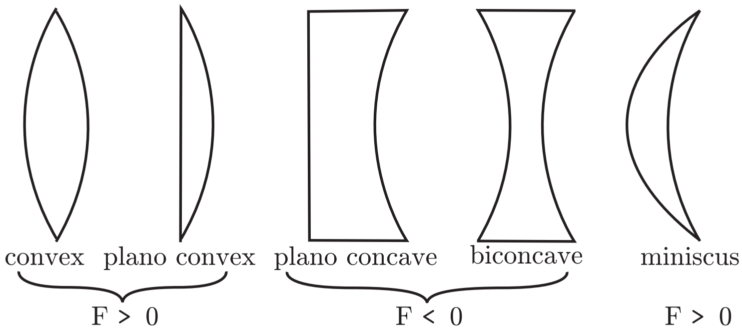


Figure 2.1: Types of lenses

Spherical lenses are lenses whose surfaces are part of spheres. The radii of curvature ($R_1$ and $R_2$) of the two surfaces may be different as illustrated below in Figure 2.2. In the figure, $R_1$ is convex and positive (+), and $R_2$ is concave and negative (−). The convention is that for a ray traveling in the $+z$ direction (to the right) if it encounters a convex surface, that surface will have a positive radius of curvature and vice-versa for a concave surface.

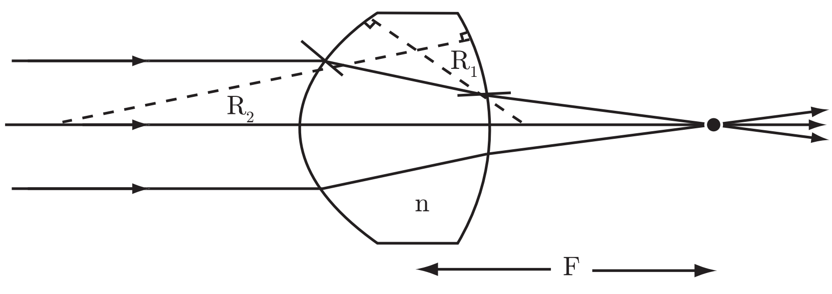


Figure 2.2: Ray propagation through a spherical lens

Figures 2.2 and 2.3 also show ray propagation through a biconvex and a biconcave lens, respectively. The relation between the focal length, refractive index and the radii of curvature, $R_1$ and $R_2$, of the lens surfaces is given by the lensmaker's formula:

$$\frac{1}{F} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) \tag{2.1}$$

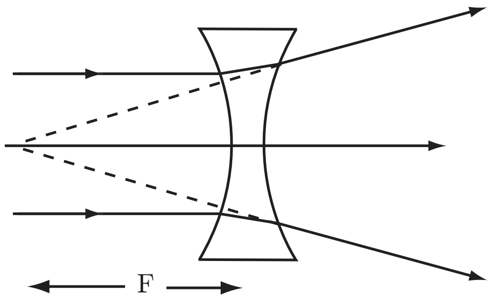


Figure 2.3: Ray propagation through a biconcave lens

### Imaging with Lenses and Mirrors

Figures 2.4 and 2.5 show imaging with a convex lens. Two planes are said to be an object and image pair if all rays from any point in the object plane pass through the corresponding point in the image plane. Given this definition, the two simplest rays to draw are those shown in Fig. 2.4 (one parallel to the principal axis, which passes through the back focus, and another which goes through the center of the lens (without deviation).The image is formed in the plane where these two rays intersect.

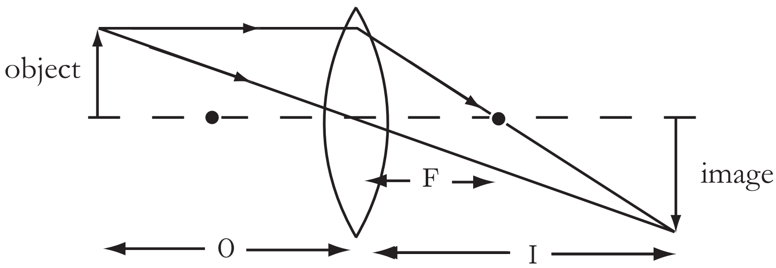


Figure 2.4: Imaging with a single convex lens: Real Image

In Figure 2.4 the object distance is larger than the focal length, and a real image is formed on the opposite side of the lens. In Figure 2.5 the object distance is smaller than the focal length, and a virtual image is formed on the same side of the lens. A *real image* is one that can be seen on a screen placed in the image plane. A *virtual image* requires an auxiliary lens, such as the lens of the eye, to view it. From Figures 2.4 and 2.5 it can be seen that the following conditions hold for a convex lens:

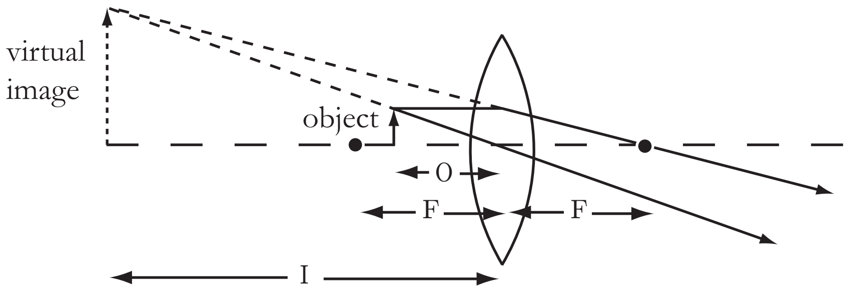


Figure 2.5: Imaging with a convex lens: Virtual Image

| | | |
|---|---|---|
| $O > 2F$ | $2F > I > F$ | Image is inverted and diminished |
| $O = 2F$ | $I = 2F$ | Image is inverted and has magnification of unity |
| $O < 2F$ | $I > 2F$ | Image is inverted and magnified |
| $O = F$ | $I = \infty$ | |
| $O < F$ | $I =$negative | Image is upright and virtual |

It can be shown by geometric considerations of Figure 2.4 or by the ABCD matrix approach (in a later section of this chapter) that the imaging condition is satisfied when

$$\frac{1}{O} + \frac{1}{I} = \frac{1}{F} \tag{2.2}$$

where $O$ is the objet distance, $I$ is the image distance and $F$ the focal length of the lens (all measured from the lens). It can also be shown that the magnification is given by $M = I/O$.

### Spherical Mirrors

Spherical mirrors are mirrors whose surfaces are parts of spheres. The ray tracing procedure for finding the location of the image is the same as that for lenses (see Figure 2.6). However, the relation between the focal length, $F$ of the mirror and the radius of curvature, $R$ of the mirror surface is much simpler:

$$F = \frac{R}{2} \tag{2.3}$$

for rays close to the principal axis.

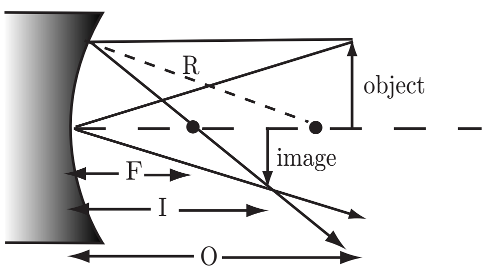


Figure 2.6: Image formation with a concave mirror

## 2.2 Ray Transfer Matrix Formalism (ABCD Matrix)

Here we present a matrix-based approach for the propagation of rays in simple optical systems that consist of lenses, mirrors and slabs of free space. In this approach, the position and direction of the ray incident on an optical element is denoted by the vector $(\rho_i,\ \rho_i')$, in cylindrical co-ordinates, where $\rho_i$ is the position of the ray on the element and $\rho_i'$ is the direction defined (as shown in Fig. 2.7, for a lens). Similarly $(\rho_0,\ \rho_0')$ define the output ray position and direction as shown.

We shall first develop matrix representations for a thin lens of focal length $F$, and for a slab of free space of thickness $d$. In general, the matrix representation will take the form

$$\begin{pmatrix} \rho_0 \\ \rho_0' \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \rho_i \\ \rho_i' \end{pmatrix} \tag{2.4}$$

First consider the lens shown in Fig. 2.7. A thin lens is defined as one for which a ray of light that enters at a height $\rho$ above the axis of the lens, exits the lens at the same height $\rho$. That is, there is no vertical displacement of the ray as it passes through the lens.

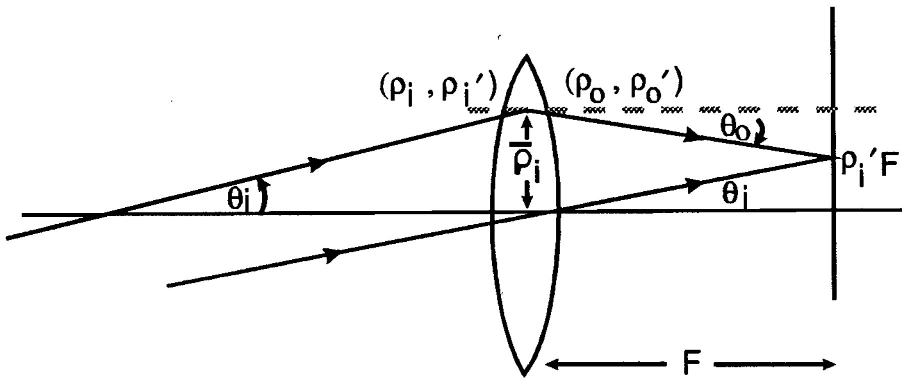


Figure 2.7: Ray propagation through a thin lens

We define $\rho_i'$ by

$$\rho_i' = \tan\theta_i \tag{2.5}$$

and we note that for a thin lens

$$\rho_0 = \rho_i \tag{2.6}$$

Further,

$$\rho_0' = \tan\theta_0 = -\frac{\rho_i - \rho_i' F}{F} \tag{2.7}$$

or,

$$\rho_0' = -\rho_i/F + \rho_i' \tag{2.8}$$

The latter two equations (Eqns. 2.6 and 2.8) can be rewritten in the desired matrix algebra form as

$$\begin{pmatrix} \rho_0 \\ \rho_0' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{F} & 1 \end{pmatrix} \begin{pmatrix} \rho_i \\ \rho_i' \end{pmatrix} \tag{2.9}$$

Thus $M_l$, the transformation matrix for a lens can be written as

$$M_l = \begin{pmatrix} 1 & 0 \\ -\frac{1}{F} & 1 \end{pmatrix} \tag{2.10}$$

### Slab of Free space

Similarly, for a slab of free space of length $d$, as shown in Fig. 2.8, it is clear that

$$\rho_0 = \rho_i + d\tan\theta_i \tag{2.11}$$

that is

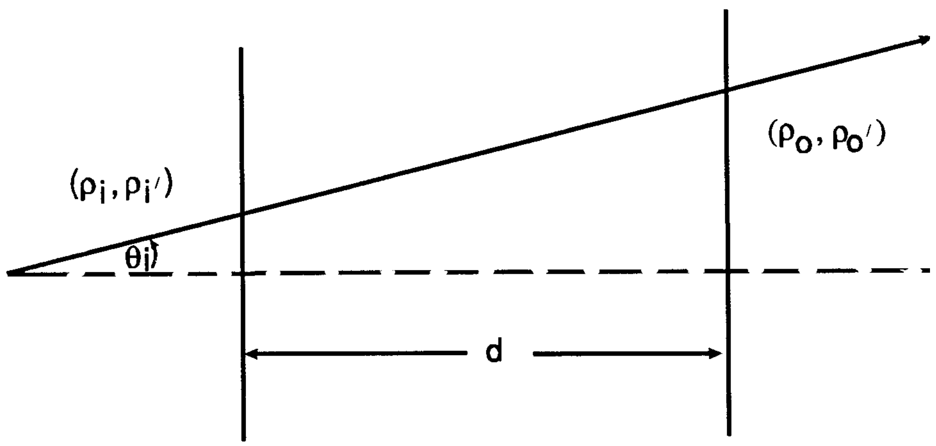


Figure 2.8: Propagation through a slab of free space

$$\rho_0 = \rho_i + d\rho_i' \tag{2.12}$$

and

$$\rho_0' = \rho_i' \tag{2.13}$$

Thus, the matrix representation for a slab of free space of length $d$ is

$$M_s = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \tag{2.14}$$

### Slab of Free Space followed by a Lens

Combining the above results, we find that propagation through a section of free space of thickness $d$ followed by a lens of focal length $F$(see Fig. 2.9) can be described by the matrix equation

$$\begin{pmatrix} \rho_0 \\ \rho_0' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{F} & 1 \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \rho_i \\ \rho_i' \end{pmatrix} \tag{2.15}$$

Thus, the ABCD matrix representation for free space followed by a lens is

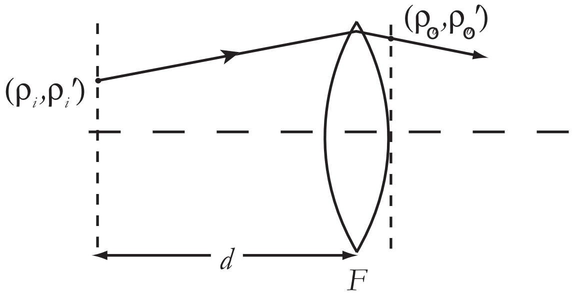


Figure 2.9: Slab of free space followed by a lens.

$$M_{sl} = \begin{pmatrix} 1 & d \\ -\frac{1}{F} & 1 - \frac{d}{F} \end{pmatrix} \tag{2.16}$$

.

### Lens Followed by a Slab of free space

Similarly for a lens followed by a slab of free space (see Fig. 2.10) we have

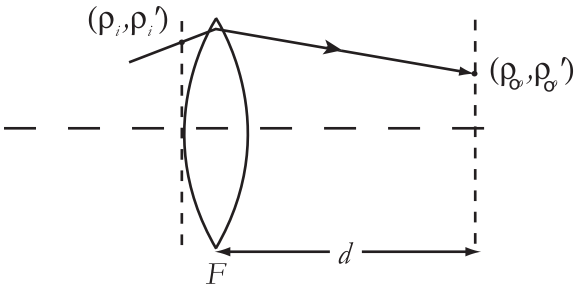


Figure 2.10: Lens followed by a slab of free space.

$$\begin{pmatrix} \rho_0 \\ \rho_0' \end{pmatrix} = \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -\frac{1}{F} & 1 \end{pmatrix} \begin{pmatrix} \rho_i \\ \rho_i' \end{pmatrix} \tag{2.17}$$

which implies that

$$M_{ls} = \begin{pmatrix} 1-\frac{d}{F} & d \\ -\frac{1}{F} & 1 \end{pmatrix} \tag{2.18}$$

.

From Eqns. 2.43 and 2.18 it should be clear that the matrices do not commute and so the order of multiplication is important. Note that for rays propagating left to right, the matrix elements are written down right to left. And, if the matrices are multiplied in the incorrect order, the effect is an interchange of the elements on the main diagonal. By multiplying successive free-space and lens ABCD matrices, one can see how a complex, multi-element wave-guiding system, as shown later in Figure 2.15, can be quickly and easily analyzed. Similarly, the ABCD approach can be used to analyze imaging systems for ray tracing in general.

### 2.2.1 Multiple Lens Systems

First let us again consider the single lens optical system shown in Fig. 2.11 where $d_1$ is the distance of the input object plane $P_1$ from the lens and $d_2$ is the distance of the output plane $P_2$ (not necessarily object and image planes, respectively).

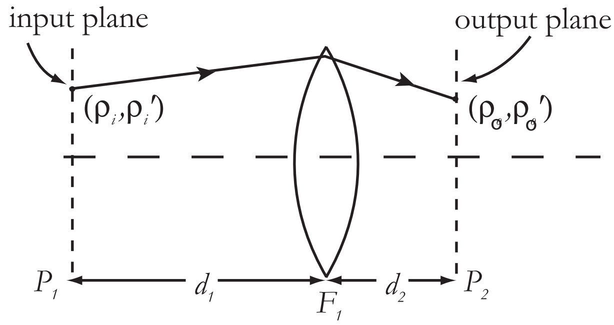


Figure 2.11: Single-lens optical system

This system can be thought of as two cascaded components: a slab of thickness $d_1$ followed by a lens and slab of free space of thickness $d_2$. Thus we have

$$\begin{pmatrix} \rho_0 \\ \rho_0' \end{pmatrix} = \begin{pmatrix} 1 & d_2 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & d_1 \\ -\frac{1}{F} & 1-\frac{d_1}{F} \end{pmatrix} \begin{pmatrix} \rho_i \\ \rho_i' \end{pmatrix} \tag{2.19}$$

and so

$$M_{\text{system}} = \begin{pmatrix} 1-\frac{d_2}{F} & d_1+d_2-\frac{d_1 d_2}{F} \\ -\frac{1}{F} & 1-\frac{d_1}{F} \end{pmatrix} \tag{2.20}$$

In the special case where $P_1$ and $P_2$ are object and image planes respectively, $\rho_o$ must be independent of $\rho_i'$. That is, all rays leaving a specific point $\rho_i$ in the input plane $P_1$ must pass through a single specific point in the $P_2$ (output) plane irrespective of their direction,

$\rho_i'$, with which they left the $P_1$ plane. Thus, by inspection of the $ABCD$ matrix, it is clear that when the imaging condition is satisfied, $B = 0$. This means that

$$d_1 + d_2 - \frac{d_1 d_2}{F} = 0 \tag{2.21}$$

$$\frac{d_1}{d_1 d_2} + \frac{d_2}{d_1 d_2} - \frac{1}{F} = 0 \tag{2.22}$$

hence

$$\frac{1}{d_2} + \frac{1}{d_1} = \frac{1}{F} \tag{2.23}$$

as expected.

The lateral magnification, with $B = 0$, is thus equal to A. That is,

$$M = \frac{\rho_o}{\rho_i} = A = (1 - \frac{d_2}{F}) \tag{2.24}$$

$$= (1 - [1 + \frac{d_2}{d_1}]) \tag{2.25}$$

$$= -\frac{d_2}{d_1} \tag{2.26}$$

as expected. Negative magnification means that the image is inverted relative to the object. In this single-lens case, a negative value for $d_2$ would mean that the image is virtual and is on the same side of the lens as the object.

### 2.2.2 Two-Lens Optical System

The two-lens optical system shown in Fig. 2.12 is very popular in many projector systems. The telescope and microscope are special cases. It is clear that this system can be thought of as the single-lens system followed by a lens-slab combination.

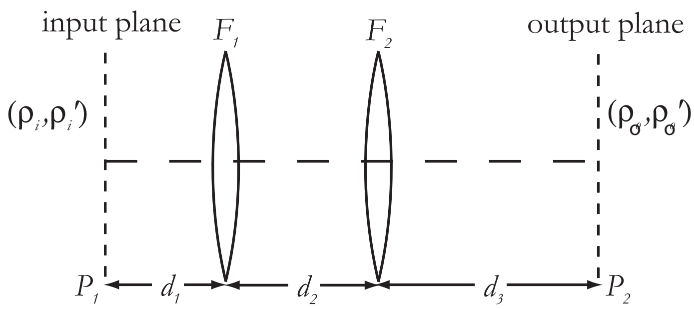


Figure 2.12: Two-lens optical system

It therefore follows that

$$\begin{pmatrix} \rho_0 \\ \rho_0' \end{pmatrix} = \begin{pmatrix} 1-\frac{d_3}{F_2} & d_3 \\ -\frac{1}{F_2} & 1 \end{pmatrix} \begin{pmatrix} 1-\frac{d_2}{F_1} & d_1+d_2-\frac{d_1 d_2}{F_1} \\ -\frac{1}{F_1} & 1-\frac{d_1}{F_1} \end{pmatrix} \begin{pmatrix} \rho_i \\ \rho_i' \end{pmatrix} \tag{2.27}$$

hence,

$$M_{system} = \begin{pmatrix} (1-\frac{d_3}{F_2})(1-\frac{d_2}{F_1})-\frac{d_3}{F_1} & (1-\frac{d_3}{F_2})(d_1+d_2-\frac{d_1 d_2}{F_1})+d_3(1-\frac{d_1}{F_1}) \\ -\frac{1}{F_2}(1-\frac{d_2}{F_1})-\frac{1}{F_1} & -\frac{1}{F_2}(d_1+d_2-\frac{d_1 d_2}{F_1})+(1-\frac{d_1}{F_1}) \end{pmatrix} \tag{2.28}$$

When this system is setup for imaging, $\rho_o$ is independent of $\rho_i'$ ($B = 0$). In this case, we have

$$d_1 + d_2 - \frac{d_1 d_2}{F_1} - \frac{d_1 d_3}{F_2} - \frac{d_2 d_3}{F_2} - \frac{d_1 d_2 d_3}{F_1 F_2} - \frac{d_1 d_3}{F_1} + d_3 = 0 \tag{2.29}$$

or

$$\frac{1}{d_3} = \frac{\frac{1}{F_2}(\frac{1}{d_1}+\frac{1}{d_2}-\frac{1}{F_1}) - \frac{1}{d_2}(\frac{1}{d_1}-\frac{1}{F_1})}{(\frac{1}{d_1}+\frac{1}{d_2}-\frac{1}{F_1})} \tag{2.30}$$

or

$$d_3 = \left[\frac{1}{F_2} - \frac{1}{d_2}\frac{(\frac{1}{d_1}-\frac{1}{F_1})}{(\frac{1}{d_1}+\frac{1}{d_2}+\frac{1}{F_1})}\right]^{-1} \tag{2.31}$$

The magnification $M_{2l}$ is equal to $A$ when $B = 0$.

$$M_{2l} = (1-\frac{d_3}{F_2})(1-\frac{d_2}{F_1}) - \frac{d_3}{F_1} \tag{2.32}$$

$$= d_3(\frac{d_2}{F_1 F_2} - \frac{1}{F_1} - \frac{1}{F_2}) + 1 - \frac{d_2}{F_1} \tag{2.33}$$

Even though $d_1$ does not appear explicitly in Eqn. 2.33, $M$ is not independent of $d_1$ since $d_1$ determines $d_3$. To find the dependence of $M$ on $d_1$ we can express $d_3$ in terms of $d_1$, as previously shown (2.30) and we find that

$$M_{2l} = 1 - \frac{d_2}{F_1} + \frac{(\frac{1}{d_1}+\frac{1}{d_2}-\frac{1}{F_1})(\frac{d_2}{F_1 F_2}-\frac{1}{F_1}-\frac{1}{F_2})}{\frac{1}{F_2}(\frac{1}{d_1}+\frac{1}{d_2}-\frac{1}{F_1}) - \frac{1}{d_2}(\frac{1}{d_1}-\frac{1}{F_1})} \tag{2.34}$$

In the special imaging case where $F_1 = F_2 = F$ and $d_1 = d_3$ we see that $M = -1$ as expected. Returning to the special case of a telescope (which is not a $P_1$ to $P_2$ imaging system) so we must go back to Eqn. 2.28, we note that since

$$\rho' = C\rho_i + D\rho_i' \tag{2.35}$$

the angular magnification $M_{tel}$ will be $D$ when $C = 0$. Indeed for the telescope since $d_2 = F_1 + F_2$, we see from Eqn. 2.28 that

$$C = -\frac{1}{F_2}(1 - \frac{d_2}{F_1}) - \frac{1}{F_1} = 0 \tag{2.36}$$

This implies that the angular magnification $M_{tel}$ is

$$M_{tele} = D = -\frac{1}{F_2}(d_1 + d_2 - \frac{d_1 d_2}{F_1}) + (1 - \frac{d_1}{F_1}) \tag{2.37}$$

$$= -\frac{F_1}{F_2} \tag{2.38}$$

when $d_2 = F_1 + F_2$ and is also independent of $d_1$ as expected.

## 2.3 Telescopes

In the refractive telescope shown in Fig. 2.13, it is presumed that the object is very far away and that it subtends a half-angle $\alpha$ to the unaided eye. Note that Fig. 2.13 shows only the set of rays (from from the "bottom" of the distant object). In the standard telescope configuration the objective and eyepiece have a common focus, as shown. The focal length of the objective $F_1$ is always greater than the focal length of the eyepiece and it produces a virtual image at infinity.

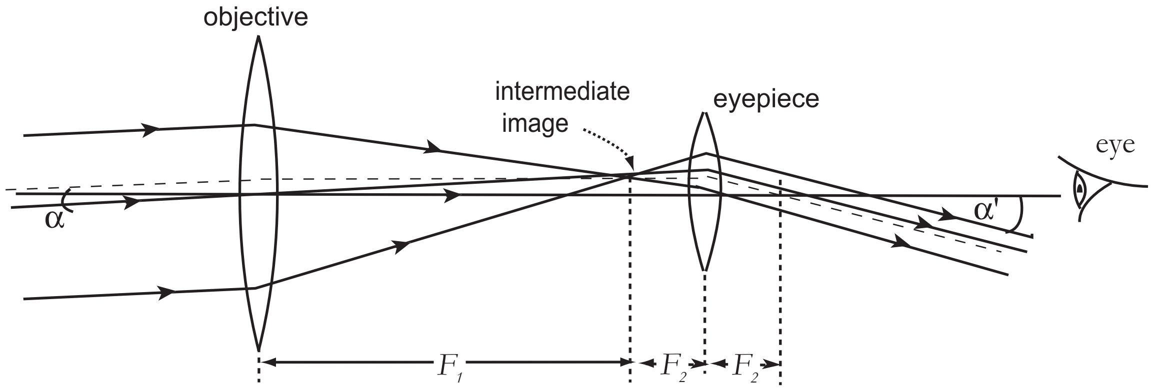


Figure 2.13: Ray diagram for a telescope.

The effect of the telescope is to increase the angle subtended at the eye from $\alpha$ to $\alpha'$. The magnification is defined by

$$M = \frac{\text{Angle image subtends at the eye with telescope}}{\text{Angle object subtends at the unaided eye}} = \frac{\alpha'}{\alpha} = -\frac{F_1}{F_2} \tag{2.39}$$

This can be seen from Fig. 2.13 by drawing in the special ray (dashed line) that passes through the intermediate image and is parallel to the principal axis in the region between the lenses. This ray must clearly pass through the back focus of the eyepiece and the front focus of the objective. It then follows that $tan\ \alpha' = h/F_2$ and $tan\ \alpha = h/F_1$, where $h$ is the height of the intermediate image, and with the small angle approximation it follows that

$$M = \frac{\alpha'}{\alpha} = -\frac{F_1}{F_2} \tag{2.40}$$

Many variants of the standard telescope exist. For example, the eyepiece may be a negative lens (Galilean telescope), mirrors may be used instead of lenses, or the objective may be a mirror and the eyepiece a lens.

## 2.4 Microscopes

With a microscope, it is presumed that the object is a distance $F_1 + \epsilon$ in front of the objective (of focal length $F_1$) and that $\epsilon$ is small. That is, the system is set up so that the intermediate image is formed near the focal plane of the eyepiece whose focal length is $F_2$ (see Fig. 2.14). The separation $g$ ($g >> 0, F_1$) between the focus of the objective and that of the eyepiece is called the barrel length and is typically 160 mm. Also, the focal length of the objective is much smaller than that of the eyepiece ($F_1 < F_2$).

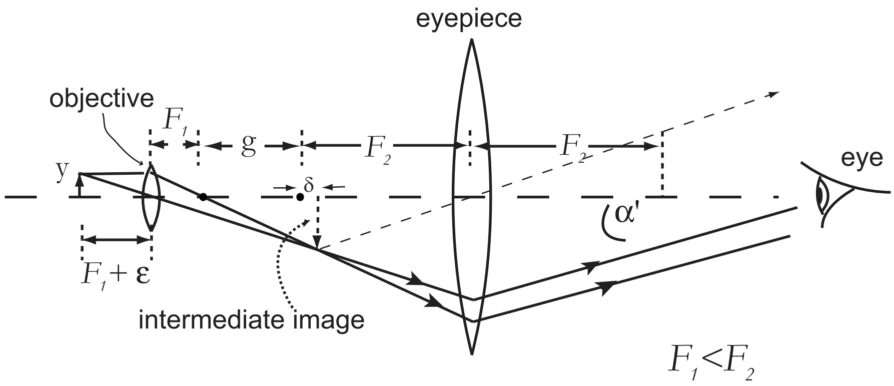


Figure 2.14: Ray diagram for a microscope

In this case, we define $\alpha$, the angle subtended by the object at the unaided eye to be $\alpha \approx \frac{y}{d_{min}}$ where $y$ is object size, and $d_{min}$ is the nearest distance for sharp seeing with the unaided eye (typically about 25 cm for a young person). By drawing in the ray from the intermediate image that passes undeviated through the center of the eyepiece, we can show that the angular magnification, $M$, is given by

$$M = \frac{\alpha'}{\alpha} \approx -\frac{g d_{min}}{F_1 F_2} \tag{2.41}$$

Alternatively, you can derive the magnification by first showing that the effective focal length of the combination of lenses is $\frac{F_1 F_2}{g}$ and then assuming the image is at $d_{min}$. In practice the eyepiece is adjusted for comfortable viewing, so the image is not at infinity.

## 2.5 Lens Waveguide

The lens waveguide (illustrated in Fig. 2.15) is a convenient means of studying how light propagates between two mirrors separated by a distance $d$. This is because reflection at a mirror with radius of curvature $R$ is equivalent, except for the folded path, to passage through a lens of focal length $R/2$.

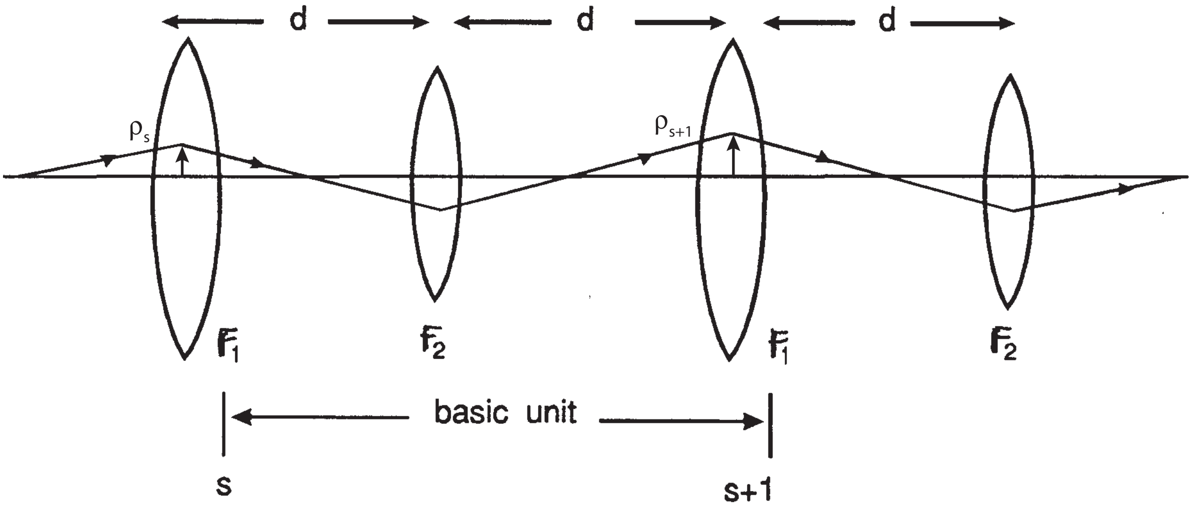


Figure 2.15: Propagation through a lens waveguide

We note that the lens waveguide consists of lenses and slabs of free-space. Thus the ABCD ray-matrix approach can be used to analyze this system. From the previous results we know that propagation through a section of free space of thickness $d$ followed by a lens of focal length $F$ (see Eqn. 2.15) is described by the matrix equation.

$$\begin{pmatrix} \rho_0 \\ \rho_0' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -\frac{1}{F} & 1 \end{pmatrix} \begin{pmatrix} 1 & d \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \rho_i \\ \rho_i' \end{pmatrix} \tag{2.42}$$

That is, the matrix for free space followed by a lens is

$$M_{sl} = \begin{pmatrix} 1 & d \\ -1/F & 1-d/F \end{pmatrix} \tag{2.43}$$

Returning to the lens waveguide of Fig. 2.15, notice that the system is spatially periodic in the $z$ direction with the unit cell described as shown. Our goal now is to find the relationship between $d$, $F_1$ and $F_2$ such that any ray passing through this system is confined (i.e., it does not escape). That is, $\rho_s$ remains finite.

So, for propagation over the basic unit shown in Fig. 2.15, we have using Eqn. 2.43

$$\begin{pmatrix} \rho_{s+1} \\ \rho_{s+1}' \end{pmatrix} = \begin{pmatrix} 1 & d \\ -1/F_1 & 1-d/F_1 \end{pmatrix} \begin{pmatrix} 1 & d \\ -1/F_2 & 1-d/F_2 \end{pmatrix} \begin{pmatrix} \rho_s \\ \rho_s' \end{pmatrix} \tag{2.44}$$

This is of the form

$$\begin{pmatrix} \rho_{s+1} \\ \rho'_{s+1} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} \rho_s \\ \rho'_s \end{pmatrix} \tag{2.45}$$

where

$$A = 1 - d/F_2 \tag{2.46}$$

$$B = d(2 - d/F_2) \tag{2.47}$$

$$C = -\left[\frac{1}{F_1} + \frac{1}{F_2}(1 - \frac{d}{F_1})\right] \tag{2.48}$$

$$D = -\frac{d}{F_1} + (1 - \frac{d}{F_1})(1 - \frac{d}{F_2}) \tag{2.49}$$

$$= 1 - \frac{2d}{F_1} - \frac{d}{F_2} + \frac{d^2}{F_1 F_2} \tag{2.50}$$

From Eqn. 2.45 it follows that

$$\rho_{s+1} = A\rho_s + B\rho'_s \tag{2.51}$$

or

$$\rho'_s = \frac{1}{B}(\rho_{s+1} - A\rho_s) \tag{2.52}$$

Therefore, by induction,

$$\rho'_{s+1} = \frac{1}{B}(\rho_{s+2} - A\rho_{s+1}) \tag{2.53}$$

but from Eqn. 2.45, we also have

$$\rho'_{s+1} = C\rho_s + D\rho'_s \tag{2.54}$$

Substituting for $\rho'_s$ and $\rho'_{s+1}$ using Eqns. 2.53 and 2.54 we find that

$$\frac{1}{B}\rho_{s+2} - \frac{A}{B}\rho_{s+1} - C\rho_s - \frac{D}{B}\rho_{s+1} + \frac{DA}{B}\rho_s = 0 \tag{2.55}$$

and noting that the determinant of the $ABCD$ matrix is unity; that is,

$$AD - BC = 1 \tag{2.56}$$

we find that the evolution of a ray through the guide obeys an equation of the form

$$\rho_{s+2} - 2b\rho_{s+1} + \rho_s = 0 \tag{2.57}$$

where

$$b = \frac{1}{2}(A + D) = (1 - \frac{d}{F_1} - \frac{d}{F_2} + \frac{d^2}{2F_1 F_2}) \tag{2.58}$$

A solution to the difference equation (Eqn. 2.58) is of the form

$$\rho_s = \rho_0 e^{js\phi} \tag{2.59}$$

Plugging this solution back into Eqn. 2.58, we get

$$e^{js\phi}[e^{j2\phi} - 2be^{j\phi} + 1] = 0 \tag{2.60}$$

And solving the quadratic equation within the parentheses of the above equation, we find

$$e^{j\phi} = b \pm j(1 - b^2)^{1/2} \tag{2.61}$$

Hence, $b$ is of the form

$$b = \cos\phi \tag{2.62}$$

The condition for confinement is that $\phi$ be real, so that $\rho_s$ oscillates ( see Eqn. 2.59) rather than grows or diminishes. That is, we want

$$-1 \leq \cos\phi \leq 1 \tag{2.63}$$

or

$$-1 \leq b \leq 1 \tag{2.64}$$

or

$$-1 \leq (1 - \frac{d}{F_1} - \frac{d}{F_2} + \frac{d^2}{2F_1F_2}) \leq 1 \tag{2.65}$$

Adding 1 to all sections of this relationship and dividing by 2 yields

$$0 \leq (1 - \frac{d}{2F_1})(1 - \frac{d}{2F_2}) \leq 1 \tag{2.66}$$

Because of the similarity between a cavity of two mirrors and a lens waveguide, Eqn. 2.66 is the condition that must be satisfied to confine a beam inside a laser cavity. Since the relationship between a lens of focal length $F$ and a mirror of focal length $F$ is that the radius of curvature of the equivalent mirror is $2F$, we can transform the lens waveguide confinement condition above into the laser resonator stability condition below.

$$0 \leq (1 - \frac{d}{R_1})(1 - \frac{d}{R_2}) \leq 1 \tag{2.67}$$

Figure 2.16 is a graphical plot of the boundaries of Eqn. 2.67 depicting the various regions of stability and the points that correspond to a few of the standard resonator geometries. Also, given a laser cavity which satisfies the stability condition (Eqn. 2.67) it can be shown that if $\phi$ satisfies the condition

$$2n\phi = 2m\pi \tag{2.68}$$

where $n$ and $m$ are integers, the ray will return to its starting point after n round trips and thus will constantly retrace the same pattern on the mirrors.

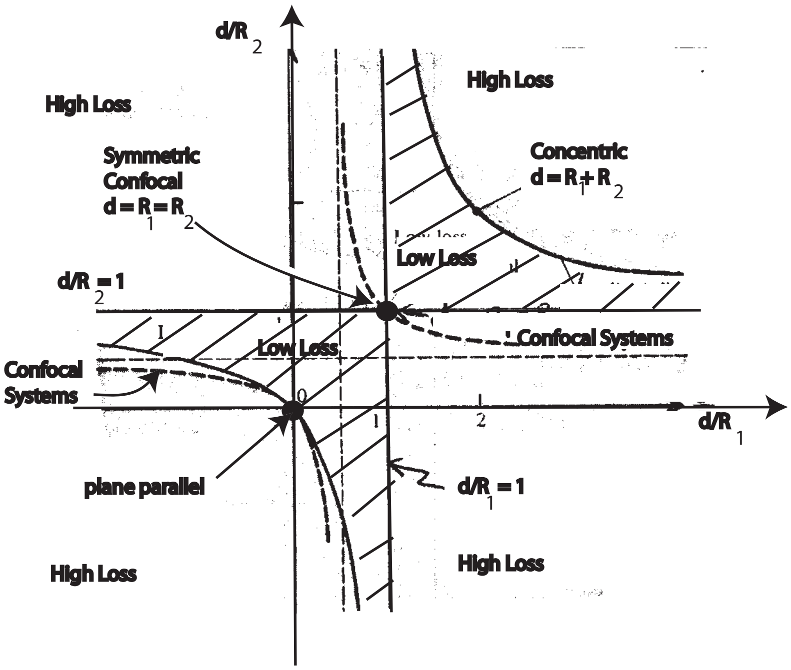


Figure 2.16: Regions of Stability for Laser Cavities - Adapted from A. Yariv, *Optical Electronics in Modern Communications*, 5th Ed. New York: Oxford University Press, 1997