

Problem Set No. 2  
Spring Term, 2023

Basic Electromagnetic  
Wave Phenomena

Issued Thursday 2/16/2023  
Due Thursday 2/23/2023

**Reading recommendation:** Class Notes, Chapter 2. Watch light polarization videos (will send over the weekend). Be neat in your work!

**6.161 STUDENTS: Do any four**

**6.637 STUDENTS Do all five**

### Problem 2.1

A typical laser consists of a gain medium located inside a resonator formed by two highly reflecting mirrors. Consider a gas laser with the design shown, in which Brewster windows are employed to seal the laser tube containing the gas (gain medium). Assume the light generated by the gain medium (when appropriately stimulated) is unpolarized, and that one of the mirrors is partially transmissive as shown. Describe, using Fig. 1, the polarization properties of the output beam from such a laser. Ignore multiple reflections from the Brewster window surfaces.

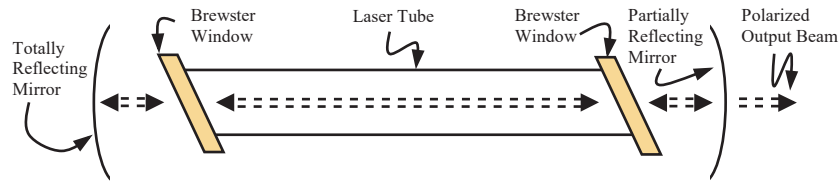


Figure 1: Diagram of laser with Brewster polarizing windows

### Problem 2.2

Using dipole radiation arguments in conjunction with the diagrams below, describe and explain the general state of polarization of sun light that is scattered from the clear overhead sky towards the observer in each of the following three cases: (i) at sunrise, (ii) noon, and (iii) sunset. That is, your answer will explain the mechanism which accounts for the partial polarization of sky light. Draw as many diagrams as you need for a clear explanation. Perform the experiment to verify your answers.

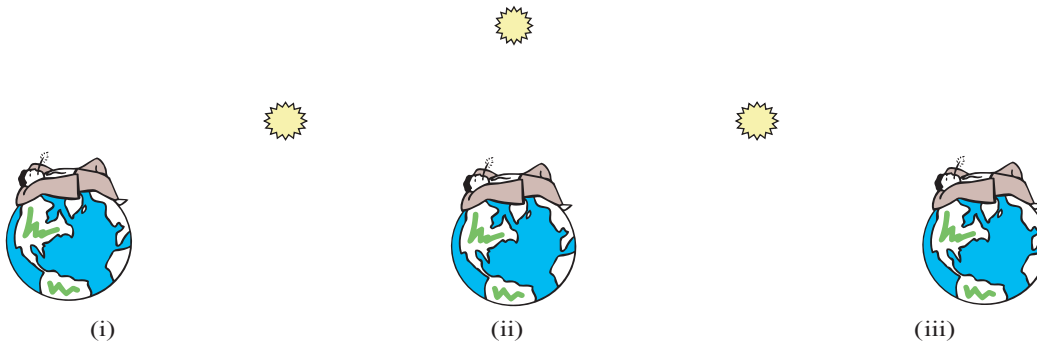


Figure 2: Observer looking at overhead sky at (i) dawn, (ii) noon and (iii) dusk

### Problem 2.3

Consider an electromagnetic plane wave that is described by

$$\overline{E}(\overline{r}, t) = \Re \left\{ (\hat{x} + 2j\hat{y}) E_0 e^{j(\frac{2\pi}{1 \times 10^{-5}} z - \frac{2\pi}{1 \times 10^{-13}} t)} \right\}$$

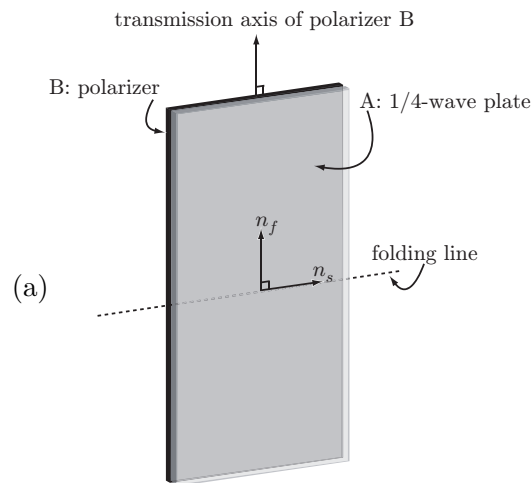
where all units are in MKS. For the following questions, be thorough with your derivations! No credit be given for answers without explanation or calculation.

- (a) What is the direction of propagation of the wave?
- (b) What is the polarization state of the wave?
- (c) What is the value of the wave-number,  $k$ ?
- (d) What is the temporal frequency,  $\nu$ , of the wave?
- (e) What is the index of refraction,  $n$ , of the medium?
- (f) What is the free-space wavelength,  $\lambda$ , of the wave?
- (g) Write the equation for the  $\overline{H}$ -field associated with this wave.
- (h) What is the complex Poynting vector for the wave?
- (i) Compute the intensity of the wave by time-averaging the square of  $\overline{E}(\overline{r}, t)$
- (j) Recompute the intensity from the complex Poynting vector

## Problem 2.4

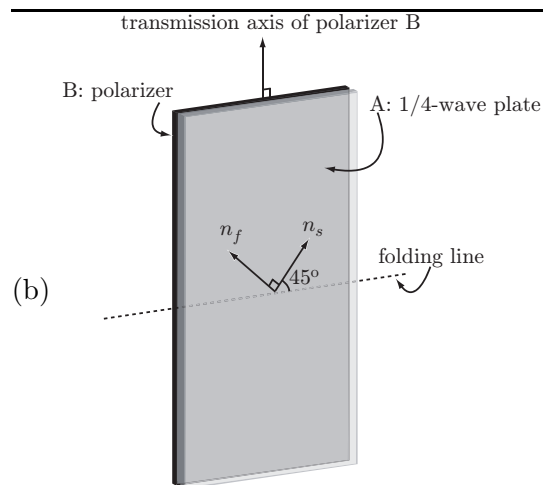
You are given 4 composite sheets, each consisting of a sandwich of waveplates and polarizers with their axes oriented as shown below. The sides of each sheet are labeled A and B. A sheet can be folded so that either AA or BB are on the inside of the folded sheet. Unpolarized light of intensity  $I_0$  is incident on the folded sheets at normal incidence. For each of the eight cases:

- Without resorting to the use of Jones matrix algebra, use your basic understanding to arrive at the transmitted intensity and polarization state of the output light in the boxes below each diagram.
- In each case, explain your reasoning as to how you arrived at your answers.
- Finally, verify your answers in each case by performing the Jones vector-matrix operations



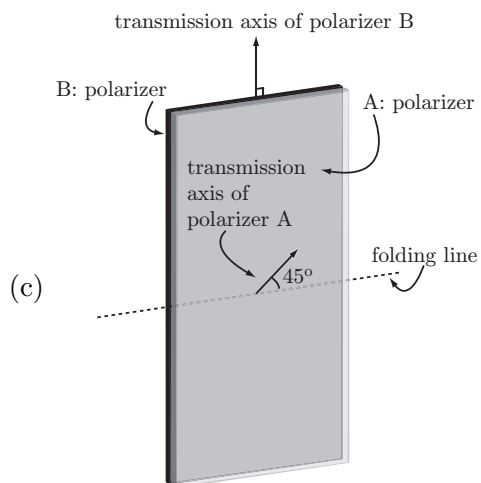
	Intensity	Polarization State
AA inside		

	Intensity	Polarization State
BB inside		



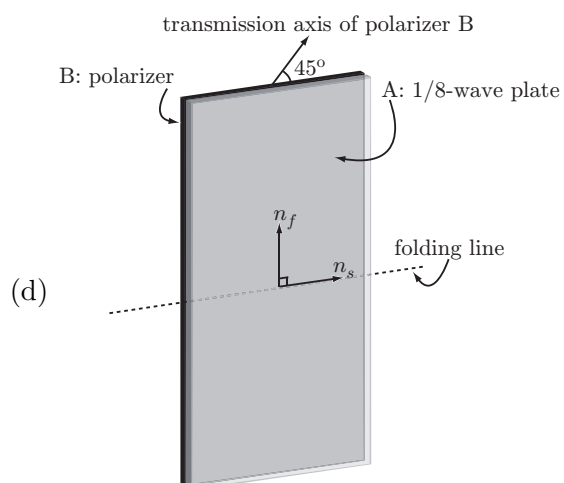
	Intensity	Polarization State
AA inside		

	Intensity	Polarization State
BB inside		



AA inside	Intensity	Polarization State

BB inside	Intensity	Polarization State

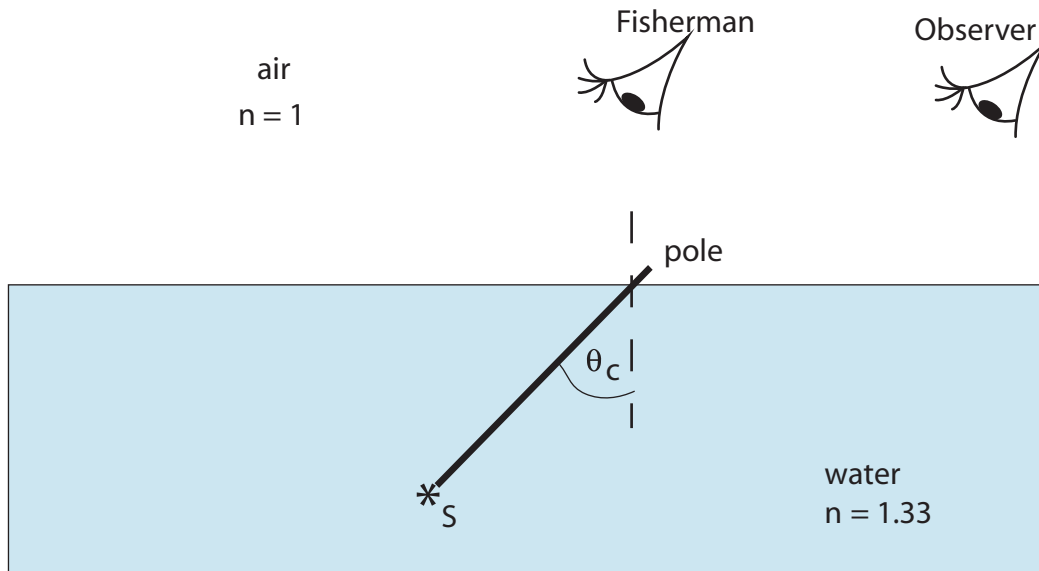


AA inside	Intensity	Polarization State

BB inside	Intensity	Polarization State

### Problem 2.5 - Fisherman's Paradox

A fisherman sitting just beyond on the bow (front) of a boat places a fishing pole into a lake of refractive index 1.33 at the critical angle. The eye of the fisherman is located directly above the point where the pole enters the water as shown below.



- (1) What is the value of the critical angle?  
(2) What is the value of the Brewster angle?
- To the fisherman, does the pole appear to bend upwards, bend downwards, or neither? Draw a ray diagram on the illustration above, to help clearly explain your answer.
- If the fisherman turns on the light bulb  $S$  at the end of the pole (so fish can see at night):
  - Will the fisherman see the light? Draw rays on the diagram below to support your answer.
  - A cool curious observer, still wearing her sunglasses (with the polarizer axis vertical) comes up behind the fisherman (as shown).
    - She naturally looks into the water underneath the pole to get a view of the light at the end of the pole. Will the observer see the light? Draw rays on the diagram below to support your answer.
    - Next the observer looks into the water above the pole. Will the observer see the light? Draw rays on the diagram below to support your answer.
- If the light source  $S$  happens to be wrapped in a cylindrical sheet of linear polarizer (cylinder axis horizontal) such that the axis of the polarizer is in the plane of the page, and if the observer is still wearing her polarizing sunglasses where should the observer position herself to see maximum brightness from the light bulb? Draw rays on the diagram below to support your answer. Note that she is not allowed to go in front of the fisherman.

