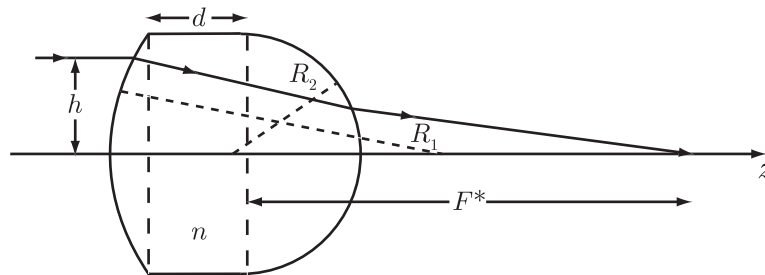


**Reading recommendation:** Class Notes, Chapter 2. Be neat in your work!

**Problem 2.1**

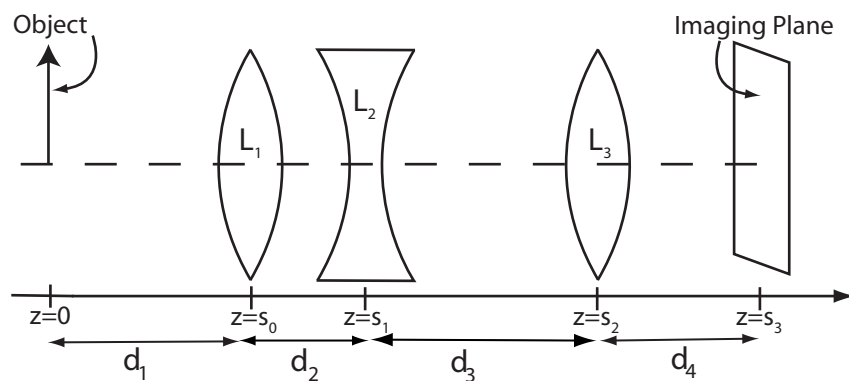


- (a) A ray of light enters a cylindrically symmetric glass bead (thick lens) of refractive index  $n$  at a height  $h$  above its principal axis. The lens has entrance and exit faces with radii of curvature  $R_1$  and  $R_2$  as shown. Assuming the usual small-angle and thin-lens approximations hold, use the ray-matrix approach to determine the approximate distance  $F^*$  at which the ray crosses the  $z$ -axis
- (b) Now turn the lens around so the ray, still at a height  $h$  above the principal axis, is incident of the  $R_2$  facet first. What is the new value  $F^{*'}$  of  $F^*$ ?
- (c) Comment on the use of such a bead as a lens. For example, does it have a well-defined focal length? What are its imaging properties?
- (d) When  $d = 0$ , do you get the expected result for two lenses in series?
- (e) When  $d = 0$ , show that your result for  $F^*$  is in agreement with the lens maker's formula.

**Problem 2.2**

An object is located in the  $z = 0$  plane of the 3-lens imaging system shown below.

- (a) Derive the ABCD ray-optics matrix (in terms of the focal lengths of the lenses and  $d_1$ ,  $d_2$ ,  $d_3$  and  $d_4$ ) for the system bounded by the given object and image planes. To help eliminate algebraic errors, you may want to use *Mathematica*, *Maple* or *Matlab* for this exercise.
- (b) Write an expression for the location of the image plane,  $s_3$ .
- (c) Write an expression for the image magnification.



Given that the focal lengths of the lenses are  $F_1 = 50\text{mm}$ ,  $F_2 = -50\text{mm}$ ,  $F_3 = 100\text{mm}$ , and the positions of the lenses, respectively, along the  $z$ -axis are  $s_0 = 100\text{mm}$ ,  $s_1 = 150\text{mm}$ ,  $s_2 = 300\text{mm}$ , answer the following questions.

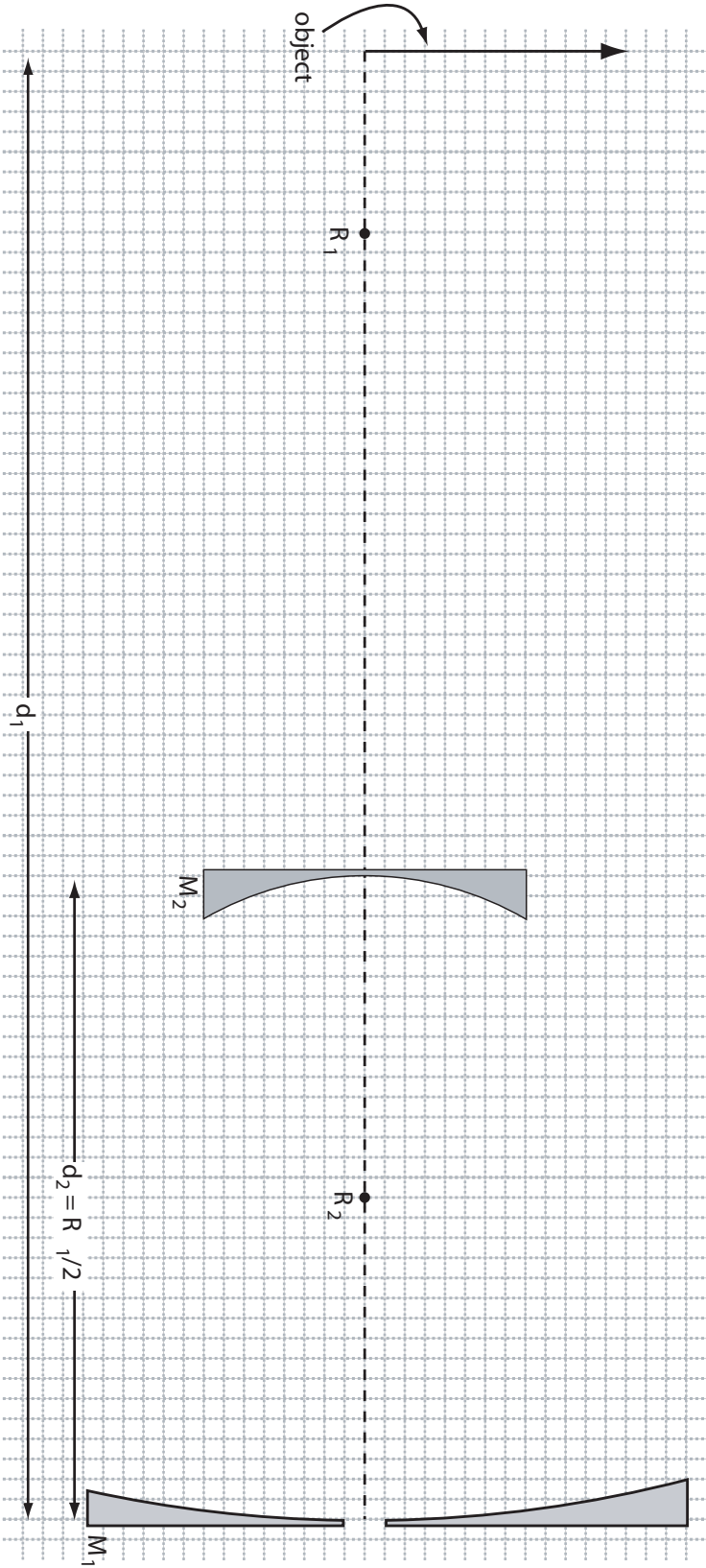
- (d) Is the image real or virtual?
- (e) What is the total magnification of the system? Given this magnification, is the image upside-down, or right-side up?
- (f) Let us now change the system so that  $L_2$  is a positive lens with focal length  $F_2$ . In the special case where  $d_1 = F_1$ , and  $d_3 = F_2 + F_3$ , what is the condition on  $d_2$  such that the image is at infinity?

### Problem 2.3

The two-mirror imaging system shown on the next page consists of a large primary mirror,  $M_1$ , with radius of curvature,  $R_1$ , and a small secondary mirror,  $M_2$ , with a radius of curvature,  $R_2$ . Both mirrors are concave. In the system,  $d_1$  is the distance of the object from the primary mirror,  $d_2$  is the separation between the mirrors, and  $d_3$  (not shown) is the distance of the final image from  $M_2$ .

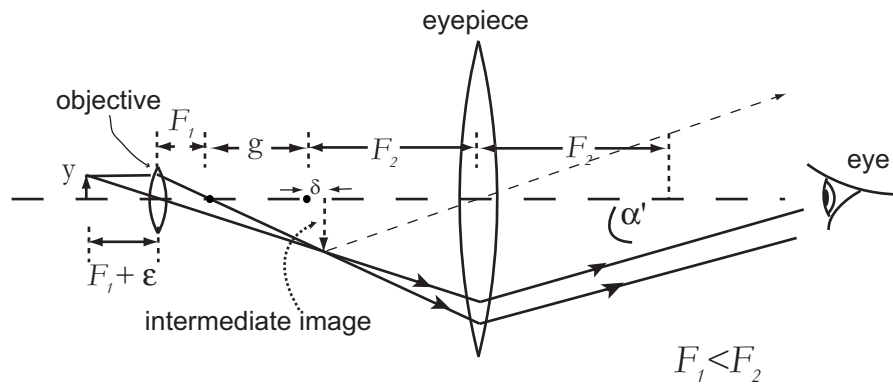
- (a) In the figure, you are given the special case where  $d_2 = \frac{R_1}{2}$ . Perform a geometric (ray-optics) construction (i.e., draw in the rays on the diagram) to show where the final image is formed.
- (b) Is the final image real or virtual?
- (c) Show the position and orientation of intermediate images, if any, and label them as real or virtual on the diagram.
- (d) For the case where both the mirror separation,  $d_2$ , is arbitrary and  $d_1 \gg \{d_2, R_1, R_2\}$ , and with the help of the class notes, write down and simplify an expression for the final image distance,  $d_3$ , in terms of  $d_1$ ,  $d_2$ ,  $R_1$ , and  $R_2$ .

Problem 2.3, Continued...



**Problem 2.4 - 6.637 only**

Consider a microscope with the geometry shown below.



- (a) Use the ABCD matrix method to show that the effective focal length of the two-lens combination is  $-F_1 F_2 / g$ .
- (b) Use the  $M_{system}$  equation in the notes for the two-lens system to calculate the **exact** (no approximations) angular magnification of the microscope. That is, assume  $d_1 = F_1 + \epsilon$ , the intermediate image is placed at a distance a little less than  $F_2$  from the eyepiece, and  $d_3 = -d_{min}$ .