MASSACHUSETTS INSTITUTE of TECHNOLOGY
Department of Electrical Engineering and Computer Science

*6.161 Modern Optics Project Laboratory*
*6.637 Optical Signals, Devices & Systems*

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| Problem Set No. 2 | **Geometric optics** | Issued Tues. 9/20/2011 |
| Fall Term, 2011 | **-** | Due Tues. 9/27/2011 |

**Reading recommendation:** Class Notes, Chapter 2. Be neat in your work!

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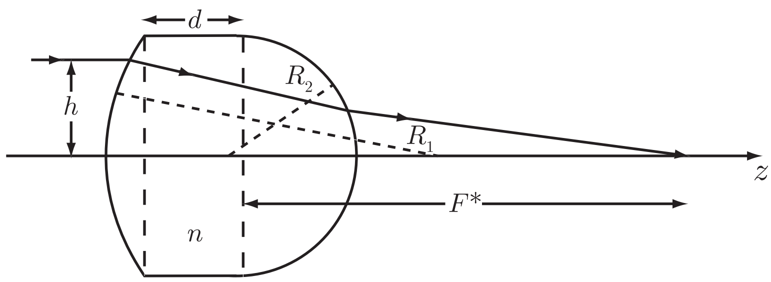
**Problem 2.1**




(a) A ray of light enters a cylindrically symmetric glass bead (thick lens) of refractive index $n$ at a height $h$ above its principal axis. The lens has entrance and exit faces with radii of curvature $R_1$ and $R_2$ as shown. Assuming the usual small-angle and thin-lens approximations hold, use the ray-matrix approach to determine the approximate distance $F^*$ at which the ray crosses the $z$-axis

(b) Now turn the lens around so the ray, still at a height $h$ above the principal axis, is incident of the $R_2$ facet first. What is the new value $F^{*\prime}$ of $F^*$?

(c) Comment on the use of such a bead as a lens. For example, does it have a well-defined focal length? What are its imaging properties?

(d) When $d = 0$, do you get the expected result for two lenses in series?

(e) When $d = 0$, show that your result for $F^*$ is in agreement with the lens maker's formula.

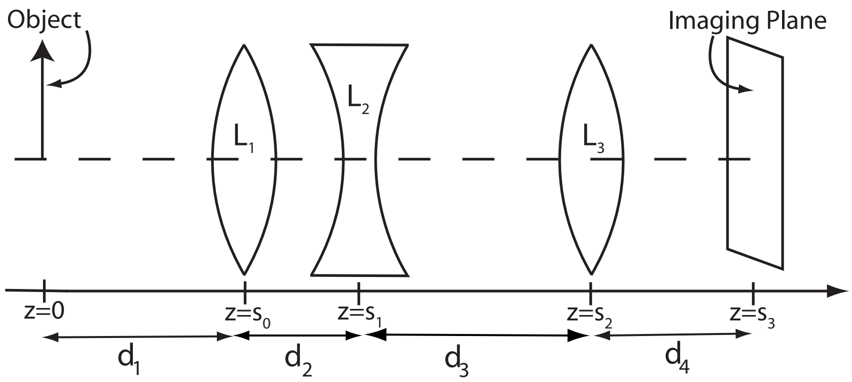
**Problem 2.2**

An object is located in the $z = 0$ plane of the 3-lens imaging system shown below.

(a) Derive the ABCD ray-optics matrix (in terms of the focal lengths of the lenses and $d_1$, $d_2$, $d_3$ and $d_4$) for the system bounded by the given object and image planes. To help eliminate algebraic errors, you may want to use *Mathematica*, *Maple* or *Matlab* for this exercise.

(b) Write an expression for the location of the image plane, $s_3$ .

(c) Write an expression for the image magnification.




Given that the focal lengths of the lenses are $F_1 = 50\text{mm}$, $F_2 = -50\text{mm}$, $F_3 = 100\text{mm}$, and the positions of the lenses, respectively, along the $z$-axis are $s_0 = 100\text{mm}$, $s_1 = 150\text{mm}$, $s_2 = 300\text{mm}$, answer the following questions.

(d) Is the image real or virtual?

(e) What is the total magnification of the system? Given this magnification, is the image upside-down, or right-side up?

(f) Let us now change the system so that $L_2$ is a positive lens with focal length $F_2$. In the special case where $d_1 = F_1$, and $d_3 = F_2 + F_3$, what is the condition on $d_2$ such that the image is at infinity?

**Problem 2.3**

The two-mirror imaging system shown on the next page consists of a large primary mirror, $M_1$, with radius of curvature, $R_1$, and a small secondary mirror, $M_2$, with a radius of curvature, $R_2$. Both mirrors are concave. In the system, $d_1$ is the distance of the object from the primary mirror, $d_2$ is the separation between the mirrors, and $d_3$ (not shown) is the distance of the final image from $M_2$.

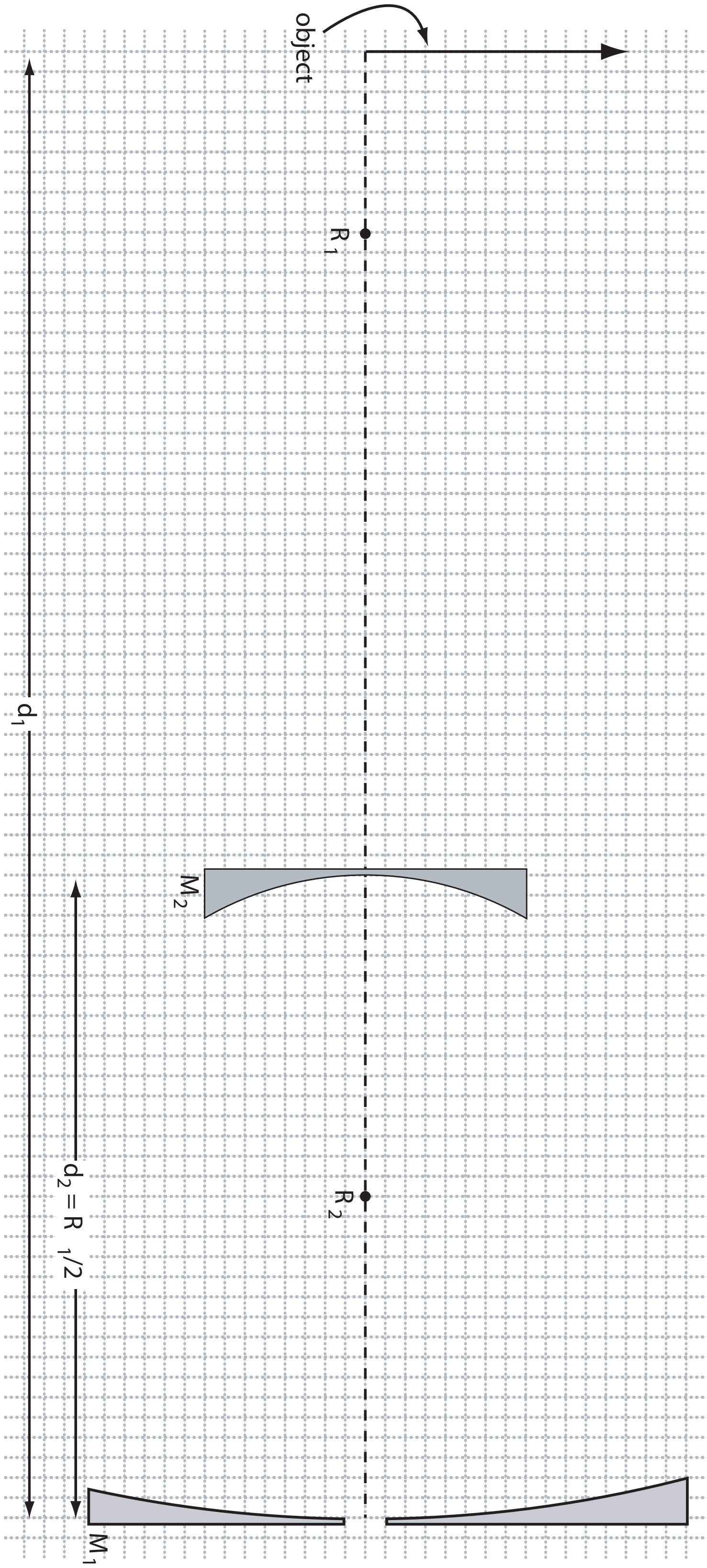
(a) In the figure, you are given the special case where $d_2 = \frac{R_1}{2}$. Perform a geometric (ray-optics) construction (i.e., draw in the rays on the diagram) to show where the final image is formed.

(b) Is the final image real or virtual?

(c) Show the position and orientation of intermediate images, if any, and label them as real or virtual on the diagram.

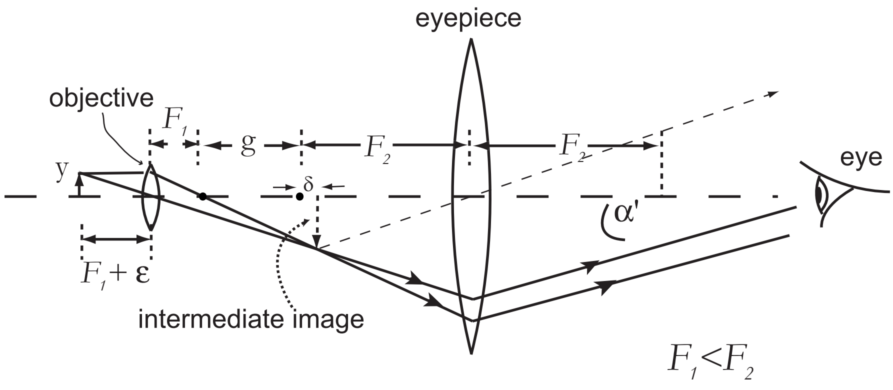
(d) For the case where both the mirror separation, $d_2$, is arbitrary and $d_1 \gg \{d_2, R_1, R_2\}$, and with the help of the class notes, write down and simplify an expression for the final image distance, $d_3$, in terms of $d_1$, $d_2$, $R_1$, and $R_2$.

**Problem 2.3, Continued...**



**Problem 2.4 - 6.637 only**

Consider a microscope with the geometry shown below.

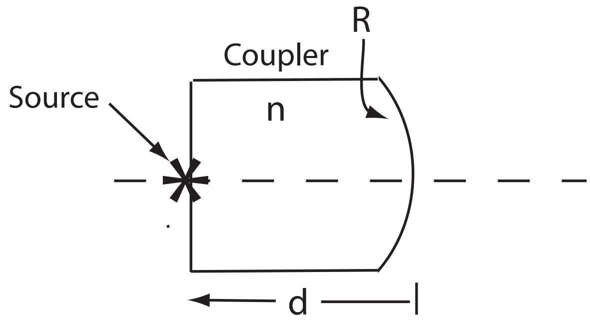



(a) Use the ABCD matrix method to show that the effective focal length of the two-lens combination (the distance behind $L_2$ that collimated input light comes to a focus)is $-d_2F_2/g$. For positive values of $g$, what is your interpretation of this result?

(b) Use the $M_{system}$ equation in the notes for the two-lens system to calculate the **exact** (no approximations) angular magnification of the microscope. That is, assume $d_1 = F_1 + \epsilon$, the intermediate image is placed at a distance a little less than $F_2$ from the eyepiece, and $d_3 =$ -$d_{min}$.

**Problem 2.5 - 6.637 only**

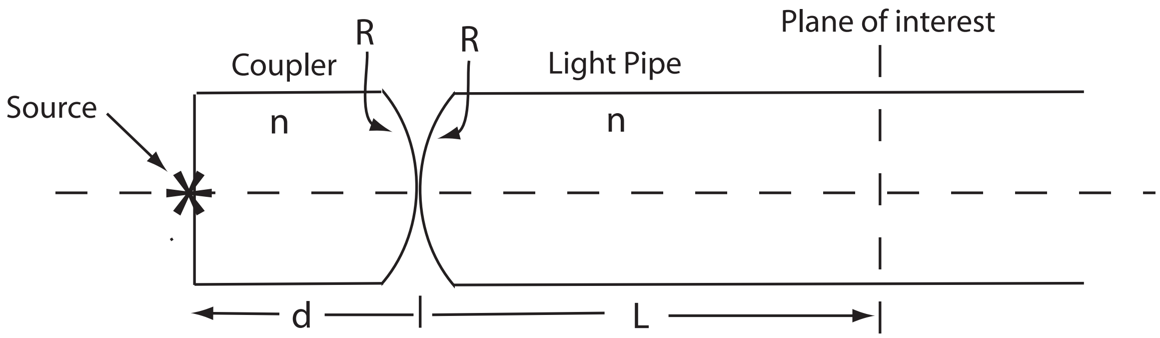
An illumination point source is located at the back of a short cylindrical glass slab (coupler) which has a radius of curvature $R$ at its other end as shown. The coupler has a length $d$ and a refractive index $n$.




(a) What is the length, $d_1$, of the coupler which produces a collimated exiting beam (in air)?

(b) What is the length, $d_2$, of the coupler so that the point source is imaged at an equal distance $d_2$ in air away from the the convex end of the coupler?

The coupler is now butted axially against a long glass rod (light pipe)of the same material and of the same diameter. The goal is to efficiently transfer light from the source into the light pipe, but it turns out that the contiguous end of the light pipe also has a convex surface of radius of curvature R as shown.




(c) Ignoring the outer boundaries of the coupler and the light pipe, what should be the length, $d_3$,of the coupler so that the light is collimated within the light pipe?

(d) Again ignoring the outer boundaries of the coupler and the light pipe, what should be the length, $d_4$,of the coupler so that the point source is is imaged at an equal distance $L = 2d_4$ in the light pipe away from the convex side of the interface between the coupler and the light pipe?