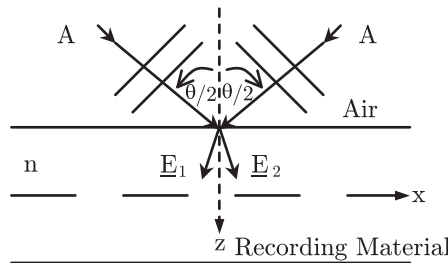


Reading recommendation: Class Notes, Chapter 3. Be neat in your work!

Problem 3.1

Two mutually coherent intersecting plane waves of equal amplitude A , wavelength λ_1 and separation angle θ , are incident symmetrically on a planar slab of holographic recording material of refractive index n as shown.

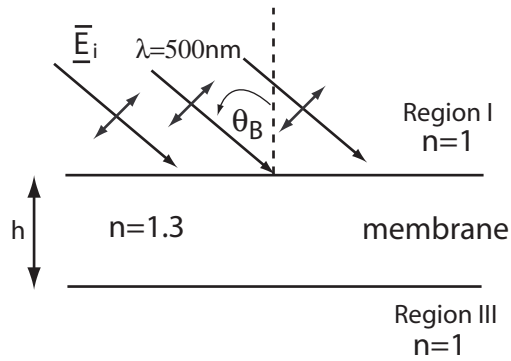


- Write equations for the two waves inside the slab.
- Assuming the phase difference between the waves is zero at $(x = 0, z = 0)$, use your equations to derive expressions for the separation Λ of the interference fringes in the material.
- Show via geometric construction of the fringes that your results are correct.

Problem 3.2(a)- for 6.161 students only

A plane wave of TM-polarized light ($\lambda = 500\text{nm}$) is incident upon a thin planar polymeric membrane of refractive index $n = 1.3$ and thickness $h \mu\text{m}$ at the Brewster angle. The wavelength of the light is $0.5 \mu\text{m}$

- What is this angle of incidence?
- Are there fringes in Region I arising from the reflected beam? Show your reasoning. If so, what is the interference condition for a bright maximum?
- Are there fringes in Region III arising from the transmitted beam? Show your reasoning. If so, what is the interference condition for a bright maximum?

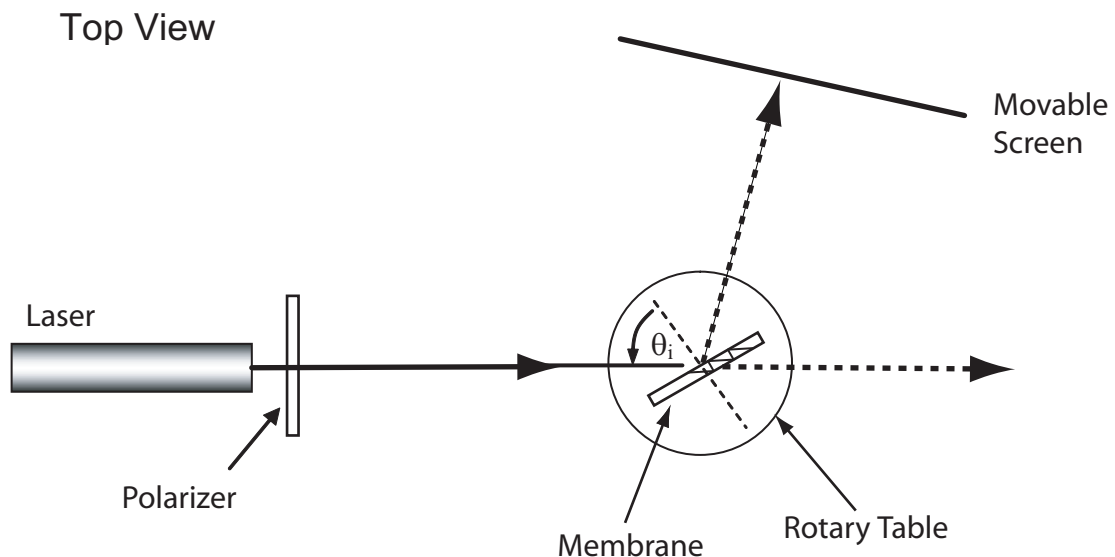


Suppose the polarization of the beam, still incident at the Brewster angle, is now changed to TE polarization:

- (d) Give an expression for the reflectivity R_{TE} of the air-membrane interface.
- (e) Ignoring diffraction, what is the minimum thickness h of the membrane (in microns) that puts maximum power into reflected wave at this angle of incidence? (Show your reasoning)

Problem 3.2 (b)- for 6.637 students only

A thin, plane-parallel transparent polymer membrane is stretched across a flat hoop as shown in the figure below. The index of refraction, n , of the membrane and its thickness h are unknown. In order to measure these two quantities, a student places the membrane on a rotary stage and illuminates the membrane with a collimated laser beam of wave length λ . She varies the angle of incidence, θ , and she observes that adjacent reflection minima occur for angles of incidence $\theta_1, \theta_2, \theta_3$. [need to change the diagram]



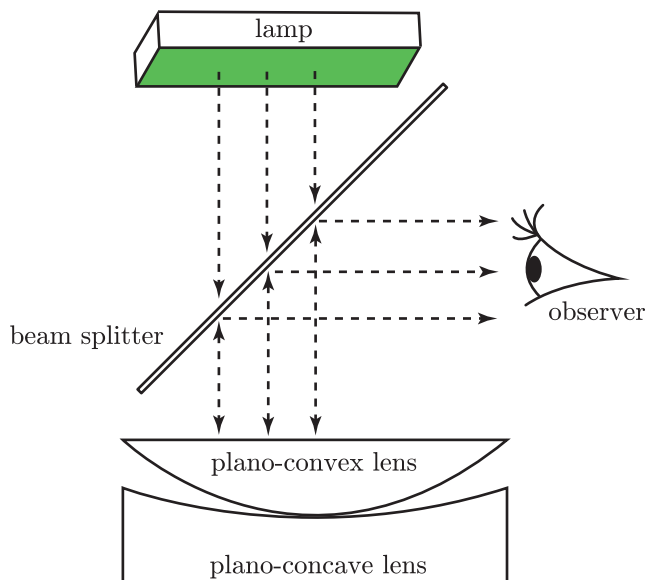
- (a) Derive an expression for the membrane thickness h .
- (b) Assuming the illumination wavelength is fixed, and the refractive index is constant but unknown, as the membrane thickness decreases, what are the experimental conditions that

limit the usefulness of this method for measuring membrane thickness and refractive index simultaneously?

- (c) What is the minimum optical membrane thickness that can be measured by this method?
- (d) If the index of refraction, n , of the membrane was known, what is the minimum physical membrane thickness that can be measured by this method?

Problem 3.3

Two lenses were found in the laboratory. One is plano-convex with a 50cm radius of curvature, and the other is plano-concave with an unknown radius of curvature. The refractive index of the lens material is $n = 1.52$ for both lenses. So you decided to set up the Newton's rings interference experiment shown below in order to find the radius of curvature of the plano-concave lens. In the setup, the curved surfaces of both lenses are placed in contact and the system illuminated with collimated monochromatic light of wavelength 550nm. With the aid of a beamsplitter, you observe a set of circular fringes called "Newton's rings". In particular you witness that the 20th bright ring has a diameter of 12mm.



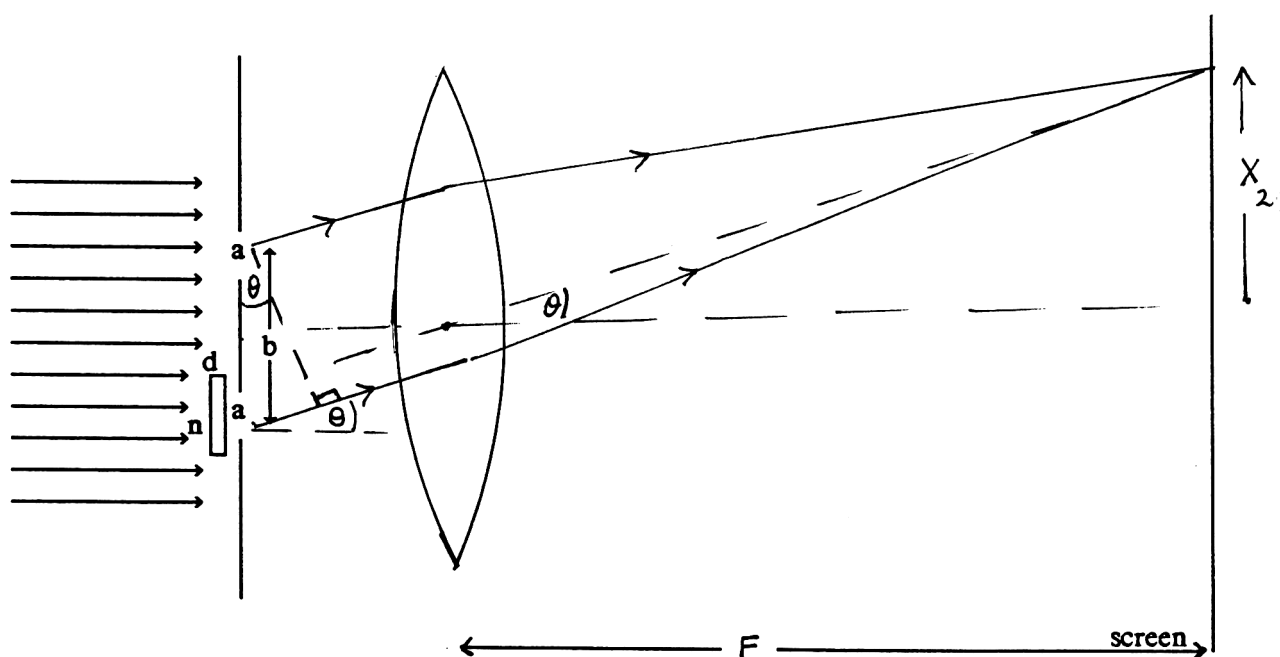
- (a) What is the radius of curvature, R_{concave} , of the concave lens surface? [Hint: Replace curved lens surfaces with piece-wise continuous surfaces so you can ignore refraction].
- (b) What are the focal lengths, F_{concave} and F_{convex} , of both lenses?

Problem 3.4 - 6.637 only

The Young's double slit experiment is modified, as shown below, by placing a thin parallel glass plate of thickness d and refractive index n over one of the slits. Here a is the slit width in the x direction and b is the slit separation also in the x direction. The system is excited with on-axis collimated laser light of wavelength λ . A lens of focal length F is placed immediately behind the

slits and a screen is placed a distance F away from the lens, so that a set of well-defined interference fringes are visible on the screen. Ignore all phase changes from reflection and transmission in this problem.

- (a) For the case where the glass plate is absent, derive an expression for the position, X_{2m} , of the m th maximum on the screen.
- (b) Derive an expression for the spatial frequency shift $\Delta f = [(\sin \theta')/\lambda - (\sin \theta)/\lambda]$ that occurs when the glass slide is inserted in the position shown in the diagram. What important conclusion do you draw from your expression?
- (c) Derive an expression for the lateral spatial shift $\Delta X_{2m} = X'_{2m} - X_{2m}$ on the screen as a result of inserting the glass slide.
- (d) In which direction do the fringes shift?



Problem 3.5 - 6.637 only

The goal here is to compare the quality of the interference fringes written with TM waves with those written with TE waves. In the two systems shown below, two mutually coherent intersecting plane waves of equal amplitude A , wavelength λ_1 and separation angle θ , are incident symmetrically on a planar slab of recording material of refractive index n as shown. One system uses TE light, the other TM light.

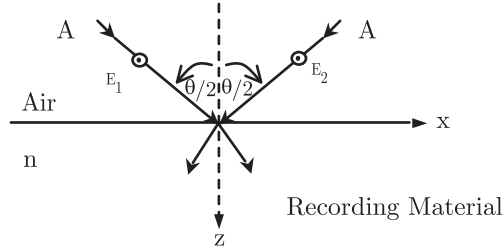


Figure 1: Two mutually-coherent intersecting TE plane waves of equal frequency, $\omega_1 = \omega_2$, incident on a slab of index of refraction, n .

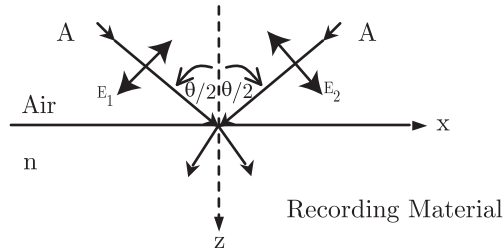


Figure 2: Two mutually-coherent intersecting TM plane waves of equal frequency, $\omega_1 = \omega_2$, incident on a slab of index of refraction, n .

- (a) Write equations for the the intensity fringe patterns in the material in both cases.
- (b) What are the similarities and differences in these patterns? For example, which pattern offers the higher contrast fringes? [contrast ratio = I_{max}/I_{min}]
- (c) Suppose the material is uniaxial with n_E aligned parallel to the surface (along x). What are the new expressions for the intensity fringe patterns.
- (d) Suppose beams E_1 and E_2 are unpolarized and the medium is still uniaxial. Describe the intensity fringe pattern inside the material.
- (e) In part (d) what is the Moire fringe spacing?