MASSACHUSETTS INSTITUTE of TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.161 Modern Optics Project Laboratory
6.637 Optical Signals, Devices & Systems

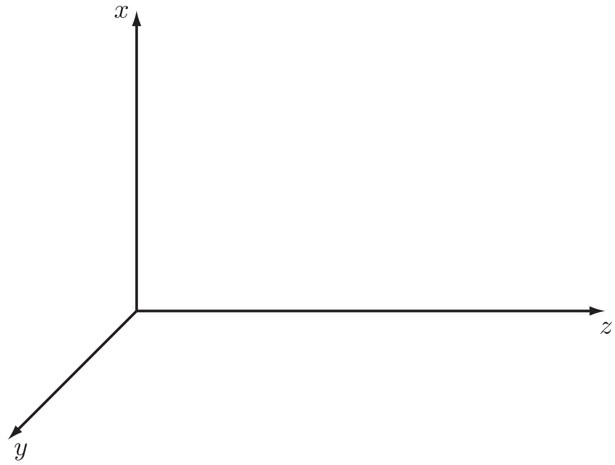
Problem Set No. 4	**Diffraction**	Issued Tues. 10/6/2009
Fall Term, 2009	**-**	Due Tues. 10/13/2009

Please hand in this Pset at 13-3102 before 5 pm on 10/13. Pick up solutions outside my door after 5pm on 10/13 - Reminder: In-class Quiz on Thursday 10/15

Reading recommendation: Class Notes, Chapter 4. Be neat in your work!

Problem 4.1
In the coordinate system below, the following definitions hold:

$$\begin{aligned}\overline{\rho} &= x\hat{x} + y\hat{y}\\ \overline{r} &= x\hat{x} + y\hat{y} + z\hat{z}\\ \overline{k} &= k_x\hat{x} + k_y\hat{y} + k_z\hat{z}\end{aligned}$$



Show the direction of propagation, and algebraically derive, sketch and describe the shape of the wavefront associated with the following elementary unit amplitude waves:

(a) $\underline{U}(\overline{r}) = e^{j\overline{k}\cdot\overline{r}}$

(b) $\underline{U}(\overline{r}) = e^{jkz}e^{-j\frac{k\rho^2}{2F}}$

(c) $\underline{U}(\overline{r}) = e^{j\frac{k}{2z}(x^2+y^2)}$

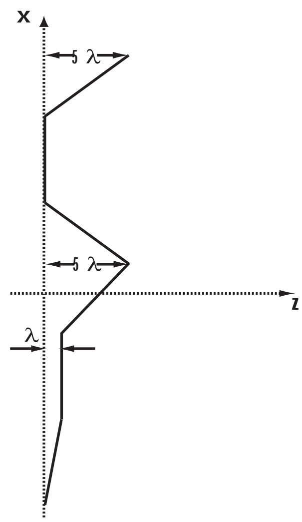
(d) $\underline{U}(\overline{r}) = e^{j\frac{k}{2z}[(x-x_0)^2+(y-y_0)^2]}$

(e) $\underline{U}(\overline{r}) = e^{jk(z^2+x^2+y^2)^{1/2}}$

(f) $\underline{U}(\overline{r}) = e^{jk[z^2+(x-x_0)^2+(y-y_0)^2]^{1/2}}$

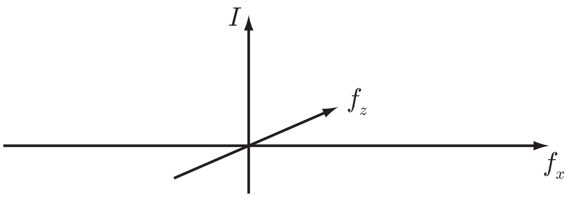
Problem 4.2

The figure below is a 1-D crosssectional plot of a 2-D piecewise-continuous approximation to an actual wavefront. The actual 2-D wavefront (not shown) is smooth (no sharp corners). For simplicity, the normal to the wavefront segments all lie in the $x-z$ plane, and the wavefront, of wavelength λ, is travelling nominally in the $+z$-direction. Assume all segments of the 2-D piecewise continuous wavefront are of the same area (10λ in the x-z plane, 10λ in y) and they all carry the same power density of I Watts/m^2.



(a) Write a frequency-domain expression, $\underline{U}(f_x, f_z)$, that describes the primary directions of power flow in this wavefront. (ignore any effective aperturing and, therefore, diffraction effects).

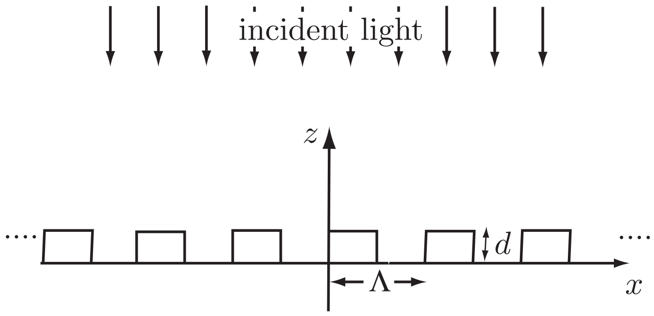
(b) Sketch the spatial-frequency content of this wavefront on a graph with the co-ordinate system shown below.



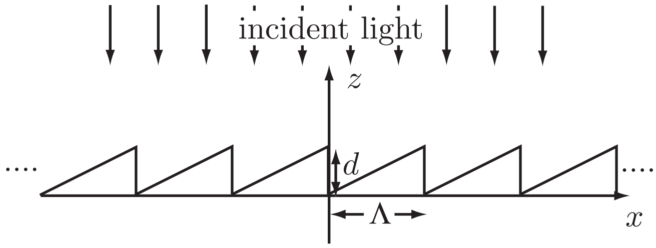
(c) Assume the above wavefront exists in the back focal plane of a lens of focal length F and is traveling nominally toward the lens. Sketch the intensity pattern that would be seen on a screen placed in the front focal plane of the lens, and label the positions and the sizes of any critical features that will be present on the screen.

Problem 4.3

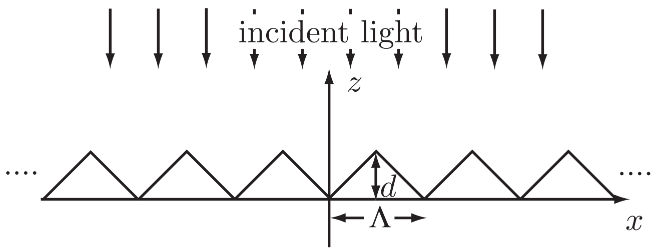
Consider the following three, infinitely-long one-dimensional, deformable grating mirrors (mirror surface with variable d) with the surface profiles shown below. The mirrors are illuminated at normal incidence (along the z-axis) with a plane wave of wavelength λ.



(i) square-wave grating profile



(ii) right-triangular grating profile



(iii) isosceles-triangular grating profile

<u>For cases (ii) and (iii) above only:</u>

(a) Write expressions $\phi(x)$ for the phase imparted by the mirrors on the wave.

(b) Using inspection techniques (no Fourier transforms), determine the minimum value of d that will extinguish the zero-order diffracted light (justify your answer with physical arguments).

Problem 4.4

A plane-wave of amplitude A and wavelength λ is incident at normal incidence in the $\hat{z}$-direction on the transmission object $\underline{U}_a(x, y)$ described below.

$$\underline{U}_a(x,y) = \begin{cases} e^{j\frac{\pi}{2}} & 0 < x < a \\ e^{-j\frac{\pi}{2}} & -a < x < 0 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Draw a sketch of this object in the x-y plane

(b) Give an example of how you would fabricate such an object.

(c) Compute analytically the intensity of the Fraunhofer diffraction field owing to this object.

(d) Plot the Fraunhofer diffraction intensity of part (c) using your favorite software package.

.

Problem 4.5 - 6.637 Only

Consider an infinitely long periodic grating that consists of slits of width a and periodicity Λ along the x-axis. The grating is illuminated with on-axis collimated light of wavelength λ traveling in the z-direction.

(a) Derive an analytical expression for the far-field diffraction pattern of this grating [Show, using sketches, the approach you used to arrive at your result].

(b) For $\Lambda = 10\ \mu m$, a $= 2\ \mu m$ and $\lambda = 0.5\ \mu m$, use your favorite software package to plot the intensity pattern of part (a) as a function of spatial frequency over a range that includes the central 9 maxima.

(c) A window of width W is placed over the grating. We can write the transmission function of the window as:

$$\underline{t}_W(x) = \left\{ \begin{array}{ll} 1 & -W/2 \leq x \leq W/2 \\ 0 & \text{elsewhere} \end{array} \right\}$$

(d) Derive a new analytical expression for the far-field diffraction pattern of this windowed grating [show, using sketches, the approach you used to arrive at your result].

(e) For $\Lambda = 10\ \mu m$, a $= 2\ \mu m$, $\lambda = 0.5\ \mu m$, and W $= 102\ \mu m$, use your favorite software package to plot the new far-field intensity pattern as a function of spatial frequency over a range that includes the central 9 maxima.

(f) For an N-slit grating of pitch Λ and slit-width a, derive an expression for the angular width of the zero-order fringe (between the adjacent nulls) when the grating is read out with light of wavelength λ.

(g) For $\Lambda = 10\ \mu m$, a $= 2\ \mu m$, $\lambda = 0.5\ \mu m$, Graphically plot the width of the zero-order fringe in part (f) from $N = 1$ to $N = \infty$.