MASSACHUSETTS INSTITUTE of TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.161 Modern Optics Project Laboratory
6.637 Optical Signals, Devices & Systems

Problem Set No. 4	**Diffraction**	Issued Tues. 10/04/2011
Fall Term, 2011	**-**	Due Thrs. 10/13/2011

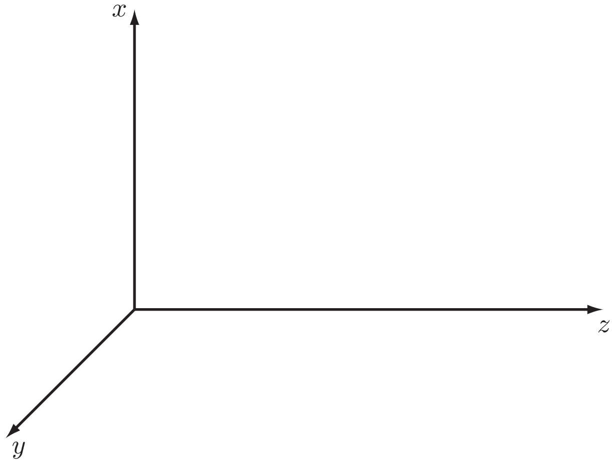
Good News: Quiz 1 moved to Thursday 10/20

Reading recommendation: Class Notes, Chapter 4. Be neat in your work!

Problem 4.1

In the coordinate system below, the following definitions hold:

$$\begin{aligned}\overline{\rho} &= x\hat{x} + y\hat{y}\\ \overline{r} &= x\hat{x} + y\hat{y} + z\hat{z}\\ \overline{k} &= k_x\hat{x} + k_y\hat{y} + k_z\hat{z}\end{aligned}$$



Show the direction of propagation, and algebraically derive, sketch and describe the shape of the wavefront associated with the following elementary unit amplitude waves:

(a) $\underline{U}(\overline{r}) = e^{j\overline{k}\cdot\overline{r}}$

(b) $\underline{U}(\overline{r}) = e^{jkz}e^{-j\frac{k\rho^2}{2F}}$

(c) $\underline{U}(\overline{r}) = e^{j\frac{k}{2z}(x^2+y^2)}$

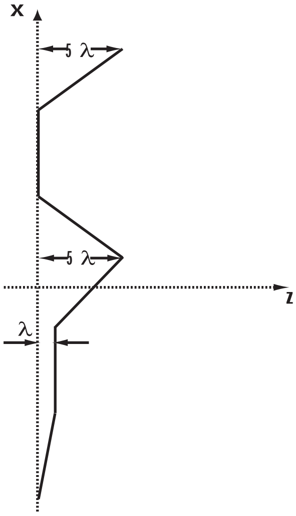
(d) $\underline{U}(\overline{r}) = e^{j\frac{k}{2z}[(x-x_0)^2+(y-y_0)^2]}$

(e) $\underline{U}(\overline{r}) = e^{jk(z^2+x^2+y^2)^{1/2}}$

(f) $\underline{U}(\overline{r}) = e^{jk[z^2+(x-x_0)^2+(y-y_0)^2]^{1/2}}$

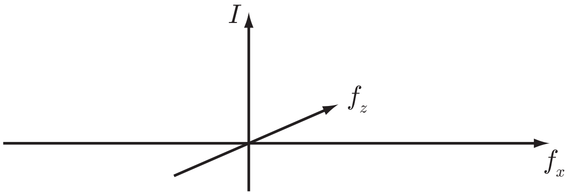
Problem 4.2

The figure below is a 1-D crosssectional plot of a 2-D piecewise-continuous approximation to an actual wavefront. The actual 2-D wavefront (not shown) is smooth (no sharp corners). For simplicity, the normal to the wavefront segments all lie in the $x - z$ plane, and the wavefront, of wavelength λ, is travelling nominally in the $+z$-direction. Assume all segments of the 2-D piecewise continuous wavefront are of the same area (10λ in the x-z plane, 10λ in y) and they all carry the same power density of I Watts/m^2.



(a) Write a frequency-domain expression, $\underline{U}(f_x, f_z)$, that describes the primary directions of power flow in this wavefront. (ignore any effective aperturing and, therefore, diffraction effects).

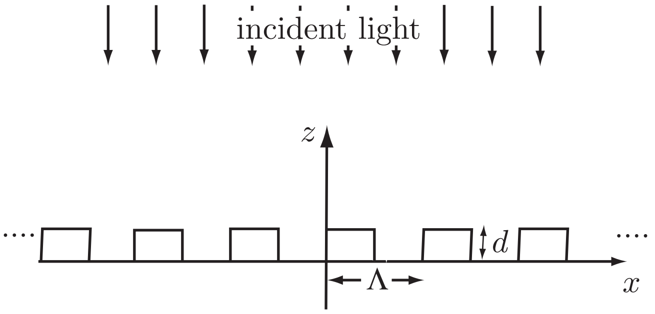
(b) Sketch the spatial-frequency content of this wavefront on a graph with the co-ordinate system shown below.



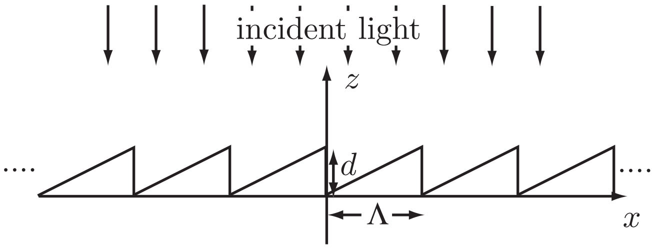
(c) Assume the above wavefront exists in the back focal plane of a lens of focal length F and is traveling nominally toward the lens. Sketch the intensity pattern that would be seen on a screen placed in the front focal plane of the lens, and label the positions and the sizes of any critical features that will be present on the screen.

Problem 4.3

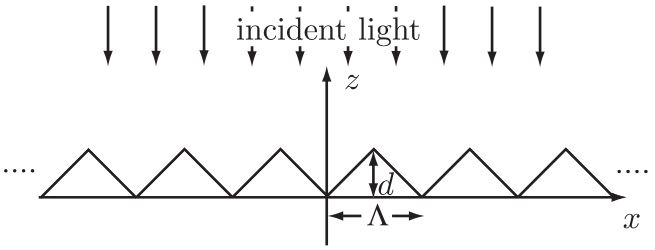
Consider the following three, infinitely-long one-dimensional, deformable grating mirrors (mirror surface with variable d) with the surface profiles shown below. The mirrors are illuminated at normal incidence (along the z-axis) with a plane wave of wavelength λ.



(i) square-wave grating profile



(ii) right-triangular grating profile



(iii) isosceles-triangular grating profile

For cases (ii) and (iii) above only:

(a) Write expressions $\phi(x)$ for the phase imparted by the mirrors on the wave.

(b) Using inspection techniques (no Fourier transforms), determine the minimum value of d that will extinguish the zero-order diffracted light (justify your answer with physical arguments).

Problem 4.4

A plane-wave of amplitude A and wavelength λ is incident at normal incidence in the $\hat{z}$-direction on the transmission object $\underline{U}_a(x, y)$ described below.

$$\underline{U}_a(x, y) = \begin{cases} e^{j\frac{\pi}{2}} & 0 < x < a \\ e^{-j\frac{\pi}{2}} & -a < x < 0 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Draw a sketch of this object in the x-y plane

(b) Give an example of how you would fabricate such an object.

(c) Compute analytically the intensity of the Fraunhofer diffraction field owing to this object.

(d) Plot the Fraunhofer diffraction intensity of part (c) using your favorite software package.

.

Problem 4.5

A Fresnel zone plate with a transparent central zone is illuminated with on-axis collimated light of wavelength λ. For an on-axis observation point P at a distance h behind the plate,

(a) How does the number zones, m, that fit within each transparent ring of the zone plate depend on h?

(b) If there are M zones in the first transparent ring, how many zones will there be in the 5th transparent ring?

(c) Large glass lenses are heavy. Design a large (about 30 cm diameter) thin (non-holographic) light-weight, diffractive-refractive plate that would be made out of transparent plastic material about 1 or 2 mm thick, and that uses Fresnel zone concepts to achieve lens-like behavior. Your resulting structure must be more optically efficient than a Fresnel zone plate (puts more light into a focused spot). Be sure to specify all dimensions in your design.

Problem 4.6 - 6.637 Only

A transmission grating is defined by

$$\underline{t}_g(x) = a\cos(2\pi x/\Lambda) \tag{1}$$

where the modulation depth a is such that $(0 \leq a \leq 1)$. The grating is placed in the $z = 0$ plane and illuminated with plane-wave collimated light of amplitude A and wavelength λ propagating along the z axis.

(a) Plot the magnitude and phase of this grating as a function of x on two separate graphs one above the other.

(b) Describe one method of fabricating such a transmission object

(c) Write the space-domain expression for the far-field intensity (i.e., $|\underline{U}_2(x_2)|^2$) on a screen placed on the z-axis at a distance L meters away from the grating.

(d) Sketch the brightness of the diffraction orders as the modulation depth a increases from zero.

Now assume that in fabricating this grating, you had inadvertently misaligned the phase section and the amplitude section by a relative shift of $\Lambda/4$ along the x axis.

(e) Sketch the new amplitude and phase graphs for the modified object one above the other.

(f) Write a mathematical expression for the transmission function $\underline{t}_{gm}(x)$ (over one period) of this modified object.

(g) Recompute the far-field intensity diffraction pattern $|\underline{U}_{m2}(x_2)|^2$) that you will observe on the screen for the modified objet.

(h) Comment on the similarities and the differences between the two diffraction patterns.

Problem 4.7 - 6.637 Only

Consider an infinitely long periodic grating that consists of slits of width a and periodicity Λ along the x-axis. The grating is illuminated with on-axis collimated light of wavelength λ traveling in the z-direction.

(a) Derive an analytical expression for the far-field diffaction pattern of this grating [Show, using sketches, the approach you used to arrive at your result].

(b) For $\Lambda = 10\ \mu m$, a $= 2\ \mu m$ and $\lambda = 0.5\ \mu m$, use your favorite software package to plot the intensity pattern of part (a) as a function of spatial frequency over a range that includes the central 9 maxima.

(c) A window of width W is placed over the grating. We can write the transmission function of the window as:

$$\underline{t}_W(x) = \left\{ \begin{array}{cc} 1 & -W/2 \leq x \leq W/2 \\ 0 & \text{elsewhere} \end{array} \right\}$$

(d) Derive a new analytical expression for the far-field diffaction pattern of this windowed grating [show, using sketches, the approach you used to arrive at your result].

(e) For $\Lambda = 10\ \mu m$, a $= 2\ \mu m$, $\lambda = 0.5\ \mu m$, and W $= 102\ \mu m$, use your favorite software package to plot the new far-field intensity pattern as a function of spatial frequency over a range that includes the central 9 maxima.

(f) For an N-slit grating of pitch Λ and slit-width a, derive an expression for the angular width of the zero-order fringe (between the adjacent nulls) when the grating is read out with light of wavelength λ.

(g) For $\Lambda = 10\ \mu m$, a $= 2\ \mu m$, $\lambda = 0.5\ \mu m$, Graphically plot the width of the zero-order fringe in part (f) from $N = 1$ to $N = \infty$.