MASSACHUSETTS INSTITUTE of TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.237 Modern Optics Project Laboratory
6.637 Optical Signals, Devices & Systems

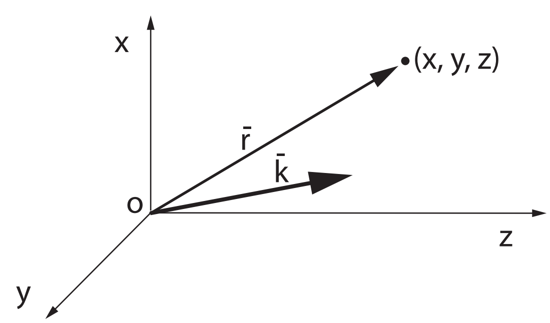
Problem Set No. 4	**Diffraction**	Issued Tues. 10/08/2024
Fall Term, 2024	-	Due Tues. 10/15/2024

Reading recommendation: Class Notes, Chapter 4. Be neat in your work!

6.237 STUDENTS: Do any four
6.637 STUDENTS Do all five

Problem 4.1
In the coordinate system below, the following definitions hold:

$$\begin{aligned} \overline{\rho} &= x\hat{x} + y\hat{y} \\ \overline{r} &= x\hat{x} + y\hat{y} + z\hat{z} \\ \overline{k} &= k_x\hat{x} + k_y\hat{y} + k_z\hat{z} \end{aligned}$$



In our diffraction theory analysis, the simplifying assumption we made (paraxial approximation) led us to approximate wavefront components some of which are similar to those associated with the unit amplitude "waves" described below:

(a) $\underline{U}(\overline{r}) = e^{j\overline{k}\cdot\overline{r}}$

(b) $\underline{U}(\overline{r}) = \frac{e^{jk(z^2+x^2+y^2)^{1/2}}}{(z^2+x^2+y^2)^{1/2}}$

(c) $\underline{U}(\overline{r}) = e^{jkz}e^{-j\frac{k\rho^2}{2F}}$

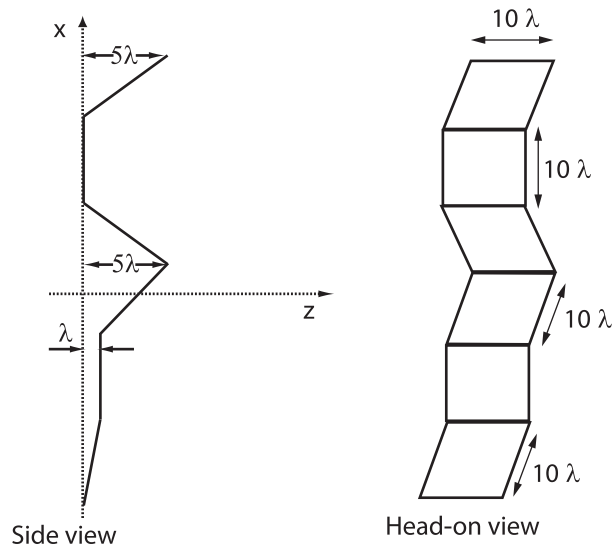
(d) $\underline{U}(\overline{r}) = e^{jk[z^2+(x-x_0)^2+(y-y_0)^2]^{1/2}}$

(4.1a) Which of these functions do not satisfy the wave equation and why? Show your work!

(4.1b) However, for all of them we can still describe the inferred approximate wavefront shape to give us a feel for what the mathematics means. In each case: (1) algebraically <u>derive</u> the equation describing the wavefront shape, and (2), <u>sketch</u> the shape of the wavefront components as well as the nominal direction of propagation.

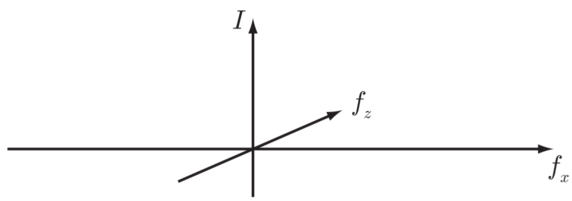
Problem 4.2

The figure below is a 1-D crosssectional plot of a 2-D piecewise-continuous approximation to an actual wavefront exiting a certain device in the $z = 0$ plane. The actual 2-D wavefront (not shown) is smooth (no sharp corners). For simplicity, the normal to the wavefront segments all lie in the $x-z$ plane, and the wavefront, of wavelength λ, is travelling nominally in the $+z$-direction. Assume all segments of the 2-D piecewise continuous wavefront are of the same area (10λ in the x-z plane, 10λ in y) and they all carry the same power density of I Watts/m^2.



(a) Write a frequency-domain expression, $\underline{U}(f_x, f_z)$, that describes the primary directions of power flow in this wavefront. (ignore any effective aperturing and, therefore, diffraction effects).

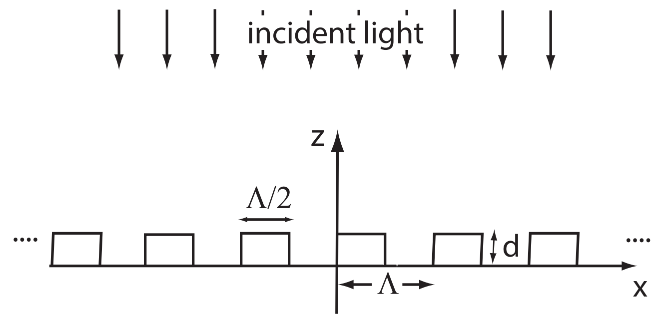
(b) Sketch the spatial-frequency content of this wavefront on a graph with the co-ordinate system shown below.



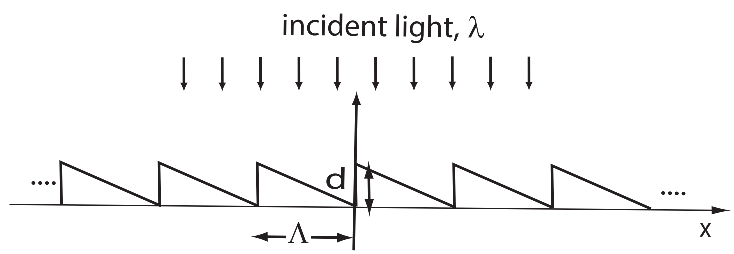
(c) Assume the above wavefront exists in the back focal plane of a lens of focal length F and is traveling nominally toward the lens. Sketch the intensity pattern that would be seen on a screen placed in the front focal plane of the lens, and label the positions and the sizes of any critical features that will be present on the screen.

Problem 4.3

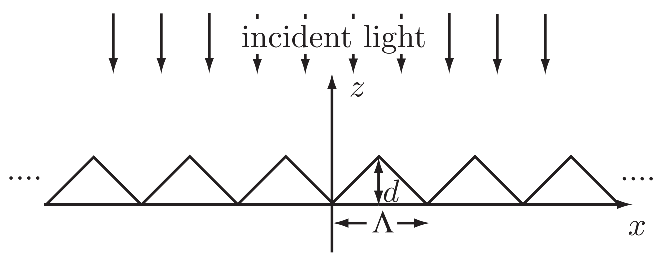
Consider the following three, infinitely-long one-dimensional, deformable grating mirrors (mirror surface with variable d) with the surface profiles shown below. The mirrors are illuminated at normal incidence (along the z-axis) with a plane wave of wavelength λ.



(i) square-wave grating profile



(ii) right-triangular grating profile



(iii) isosceles-triangular grating profile

For the three cases above:

(a) Write expressions $\phi(x)$ for the phase imparted by the mirrors on the wave.

(b) Using inspection techniques (do not use Fourier transform approach), determine the minimum value of d that will extinguish the zero-order diffracted light (justify your answers with physical arguments).

Problem 4.4

A plane-wave of amplitude A and wavelength λ is incident at normal incidence in the $\hat{z}$-direction on the transmission object $\underline{U}_a(x, y)$ described below.

$$\underline{U}_a(x, y) = \begin{cases} e^{j\frac{\pi}{2}} & 0 < x < a \\ e^{-j\frac{\pi}{2}} & -a < x < 0 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Draw a sketch of this object in the x-y plane

(b) Give an example of how you would fabricate such an object.

(c) Compute analytically the intensity of the Fraunhofer diffraction field owing to this object.

(d) Plot the Fraunhofer diffraction intensity of part (c) using your favorite software package.

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Problem 4.5

A Fresnel zone plate with a opaque central zone is illuminated with on-axis collimated light of wavelength λ. Several on-axis points of highly focused light are observed at distances h_j behind the plate.

(a) If there are M zones in the first transparent ring of the Fresnel Zone plate, how many zones will there be in the 5th transparent ring?

(b) How do the h_j depend on the number of fictitious zones, m, that fit within each transparent ring of the zone plate?

(c) Large glass lenses are heavy, and the Fresnel zone plates described in the notes absorbs 50% of the light. Design a large (about 30 cm diameter) thin light-weight, Fresnel Zone plate that would be made out of transparent plastic material about 2 mm thick, and achieve lens-like behavior using almost 100% of the light. Be sure to specify all dimensions in your design.