Problem 5.1

You are given a red He-Ne laser (\( \lambda = 0.63 \mu m \)), a white source (not necessarily collimated, but you can collimate it if you wish) and a thick holographic recording material. Describe with the aid of diagrams how you would make a color RGB (red, green, blue) hologram that could be read out with the white light source with minimal misalignment of the output images.

Problem 5.2

(a) Our goal is to make a thick white light reflection hologram using the recording setup shown below. Assume the reference beam is a spherical wave. The hologram is to be read out with a center reconstruction wavelength \( \lambda_r = 532 nm \). However, owing to the recording material development process, we know that the recording emulsion will shrink to 80% of its original size. What must be the recording (write) wavelength \( \lambda_w \) to get a faithful reproduction of the object with the 532 nm spherical readout wave whose origin is at the same location as that of the reference beam?

(b) In the case where the same wavelength is used for writing and readout, assuming again that we have 20% emulsion shrinkage, how will the readout geometry have to change to view the hologram? Draw one or two diagrams to help clarify your explanation, and in these, show clearly the location of the readout source and location of the image thus formed.
Problem 5.3

A hologram is made in a recording material of refractive index \( n \) by interfering a nominally on-axis object beam \( U_0(x, y) \) with a tilted plane-wave reference beam. The writing light has wavelength \( \lambda_w \). The plane-wave reference beam has amplitude \( A_r \) and is incident from below in the \( x - z \) plane at an angle \( \theta \) as shown in the diagram below.

A. THIN HOLOGRAM CASE
First assume the recording medium is thin.

(A1) Does the writing geometry shown lead to a transmission or reflection hologram?

(A2) Assume that the hologram in Figure 1 is a thin hologram and that it is read out with a beam of amplitude \( B \) that has the conjugate phase of the reference beam. Derive the field expressions (amplitude and phase) of each beam exiting the hologram.

(A3) Draw a diagram of the readout geometry to show each entering and exiting beam with its appropriate field term [from (b)]. Also label the real and virtual images, if they exist.

B. THICK HOLOGRAM CASE
Now assume that the hologram is thick and that the write geometry is the same as that used in the Figure above. Thick reflection holograms are used for decorative purposes on credit cards, and are also worn (usually around the neck) as decorative jewelry. Large reflection holograms can also be found as art media in museums (e.g., MIT Museum).
(B1) Draw and describe the location of the images when the hologram is read out with the conjugate of the reference beam.

(B2) Next, the thick hologram is read out in an optimal way (Bragg matched) with a plane wave of wavelength \( \lambda' = 3\lambda_w/2 \) also incident from above on the emulsion side.

(a) What is the optimal angle, \( \psi \), between the \( z \)-axis and this readout beam?

(b) The angle \( \theta \) has a maximum value, \( \theta_{\text{max}} \), that should be respected if the intent is read out the hologram with light of wavelength \( 3\lambda_w/2 \). What is this angle?

(c) Describe the characteristics of the images produced, and draw a diagram of this optical readout configuration showing the location of the images.

(B3) Using the results from part (B2), describe the output images that are obtained when this hologram is read out with collimated white light incident from above on the front side.

(B4) Describe the output images that are obtained when this hologram is read out with non-collimated white light incident from above on the front side.
Problem 5.4

A hologram of an object is recorded through a lens $L_1$ into a very thick slab of material of refractive index $n$ using laser light of wavelength $\lambda_w$ and the geometry shown. Note that the recording plane-wave reference beam is incident from the back side of the material at an angle $\theta$ as shown.

For the 4 readout geometries below, draw in the dominant output images that emerge from the hologram. In each case state whether the image is real or virtual and show the angular direction of the output beams. Also Comment on what Observer A and B see.

Write Geometry

Read Geometry 1

Read Geometry 2
Problem 5.5 - 6.637 only

A hologram is made with waves derived from an on-axis object $U_o(\rho)$ located in the $z = 0$ plane and a point source of amplitude $A$ located on the $x$-axis at a distance $a$ above the principal axis as shown.

(a) Write an expression for the object wave, $U_o(\rho, z_0)$, at the recording medium.

(b) Write an expression for the spherical reference wave, $U_r(\rho, z_0)$, at the recording medium.

(c) Assuming the recording medium has a transmitted amplitude response that is proportional to exposure, write an expression for the amplitude transmittance of the resulting hologram.
(d) The hologram is to be read out with the conjugate of the reference wave. Write an expression for the conjugate reference wave at the $z = z_0$ plane and draw a diagram to show the wavefronts of the conjugate reference wave reading out the hologram.

(e) Compute the output terms corresponding to the waves generated by the hologram when read out with the conjugate reference beam, and illustrate these on the diagram you made in part (d).