MASSACHUSETTS INSTITUTE of TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.161 Modern Optics Project Laboratory
6.637 Optical Signals, Devices & Systems

Problem Set No. 6	**Light Modulation**	Issued Tues. 10/25/2011
Fall Term, 2011	**-**	Due Tues. 11/01/2011

Reading recommendation: Class Notes, Chapter 6; Yariv, Chapter 9. Be neat in your work!

Problem 6.1 - Transverse ADP Modulator

Consider the transverse electro-optic modulator shown below. The crystal material is Ammonium Dihydrogen Phosphate (ADP), a tetragonal $\bar{4}$2m crystal. The input light is polarized at 45° to the z and y' axes for intensity modulation. The z axis is the optical axis of the crystal, and the x' and y' axes are oriented at 45° to the crystallographic axes, x and y (not shown).

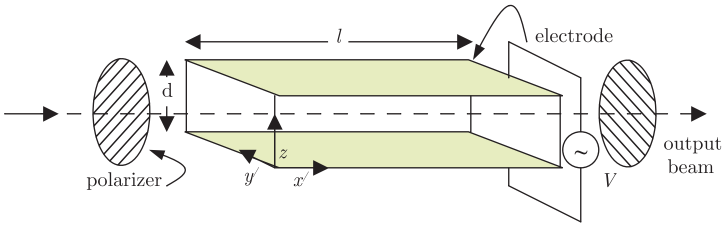


Figure 1: ADP crystal under transverse modulation

(a) For ADP with an electric field applied along is z axis, show that, in general, the indices of refraction for light polarized along x', y' and z are

$$\begin{aligned} n_{x'}(V) &\approx n_o - \frac{1}{2}n_o^3 r_{63}\frac{V}{d} \\ n_{y'}(V) &\approx n_o + \frac{1}{2}n_o^3 r_{63}\frac{V}{d} \\ n_z(V) &= n_E \end{aligned}$$

(b) For the specific modulator configuration shown above, show that the total phase retardation $\Gamma = \phi_{y'} - \phi_z$ is given by

$$\Gamma = \frac{2\pi l}{\lambda}\left\{(n_o - n_E) + \frac{n_o^3}{2}r_{63}\frac{V}{d}\right\}$$

(c) If the crystal thickness d is 2 mm, and its half-wave voltage (corresponding to $\Delta\Gamma(V_\pi) = \Gamma(V_\pi) - \Gamma(0) = \pi$) as measured in transverse operation (see figure above) is 1200 volts, what is the length of the crystal? (electro-optic coefficients and indices of refraction for ADP can be found in one of the tables of Yariv, Chap 9. Assume λ=6328Å).

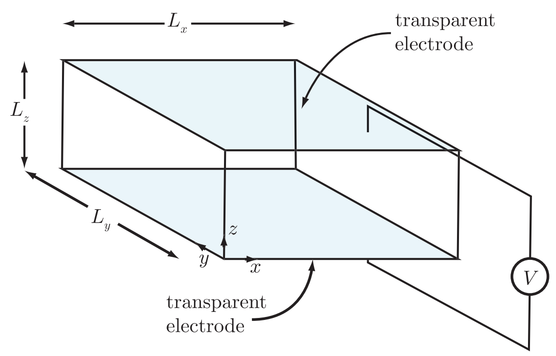
(d) Calculate the corresponding quarter-wave voltage, $\Delta\Gamma(V_{\frac{\pi}{2}})$, for transverse operation. Discuss the advantages and disadvantages of transverse vs longitudinal modulators.

Problem 6.2 - Lithium Niobate Modulator Design

The goal here is to design intensity and phase modulators for a collimated, **unpolarized** light beam of intensity, I_{in} , using the electro-optic effect in a given lithium niobate crystal. Lithium niobate is an uniaxial material ($n_x = n_y$) and its electro-optic tensor contains the following non-zero elements:

$$r_{22} = -r_{12} = -r_{61} \qquad r_{13} = r_{23} \qquad r_{42} = r_{51} \text{ and } r_{33}$$

The given crystal has transparent electrodes on the faces perpendicular to the c-axis as shown, and its dimensions are L_x, L_y and L_z. Thus, you can apply electric fields along the z-axis only. However, you can propagate the light beam to be modulated along either the x, y or z axes.



(a) First, I would like you to make a phase-only modulator but without the use of polarizers. Draw a diagram to show how you would operate the crystal to achieve this goal. For your chosen operation, what is your expression for $\Delta\Phi(V)$?

(b) Now I would like you to design an intensity modulator. Place the crystal between polarizers (not necessarily crossed), and for each of the three propagation directions, derive an expression for $I_{\text{out}}(V)/I_{\text{in}}$. Be sure to clearly specify your chosen orientation of the polarizers relative to the axes of the crystal (a diagram is required for each case).

(c) Suppose that the wavelength of the light to be modulated is $\lambda = 0.6\mu$m, and that $L_x = L_y = L_z$. Given that $n_O = 2.286, n_E = 2.200$ and that

$r_{22} = -r_{12} = -r_{61} = 6.8$ pm$/V$ (pm=pico meter= 10^{-12}m)

$r_{13} = r_{23} = 9.6$pm$/V$

$r_{42} = r_{51} = 32.6$ pm$/V$

$r_{33} = 30.9$ pm$/V$

Which of the three cases makes the most efficient intensity light modulation system in terms of intensity change per volt? What is the half wave voltage for this system? How could you significantly lower the halfwave voltage?

Problem 6.3

A plane parallel slab of $LiNbO_3$ 5 mm thick is cut such that the z-axis of the crystal is perpendicular to the faces of the slab. The faces are coated with transparent electrodes so as to convert it to light modulator (see Fig. 1). A voltage ramp (see top of Fig. 2)is applied to the electrodes and the crystal is illuminated with an axial plane wave of wavelength λ as shown in Fig. 1. The electro-optic tensor for $LiNbO_3$ contains the following non-zero elements:

$$r_{22} = -r_{12} = -r_{61} \qquad r_{13} = r_{23} \qquad r_{42} = r_{51} \text{ and } r_{33}$$

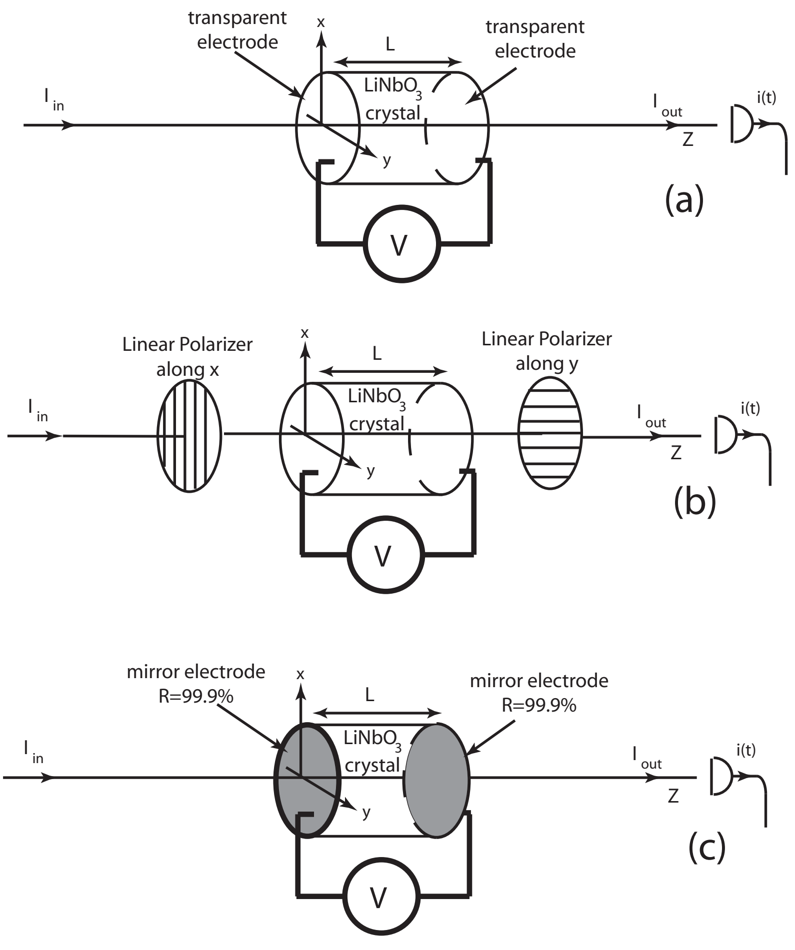


Fig. 1

(a) Using the above information for the electro-optic coefficients write down the simplified electro-optic tensor for $LiNbO_3$.

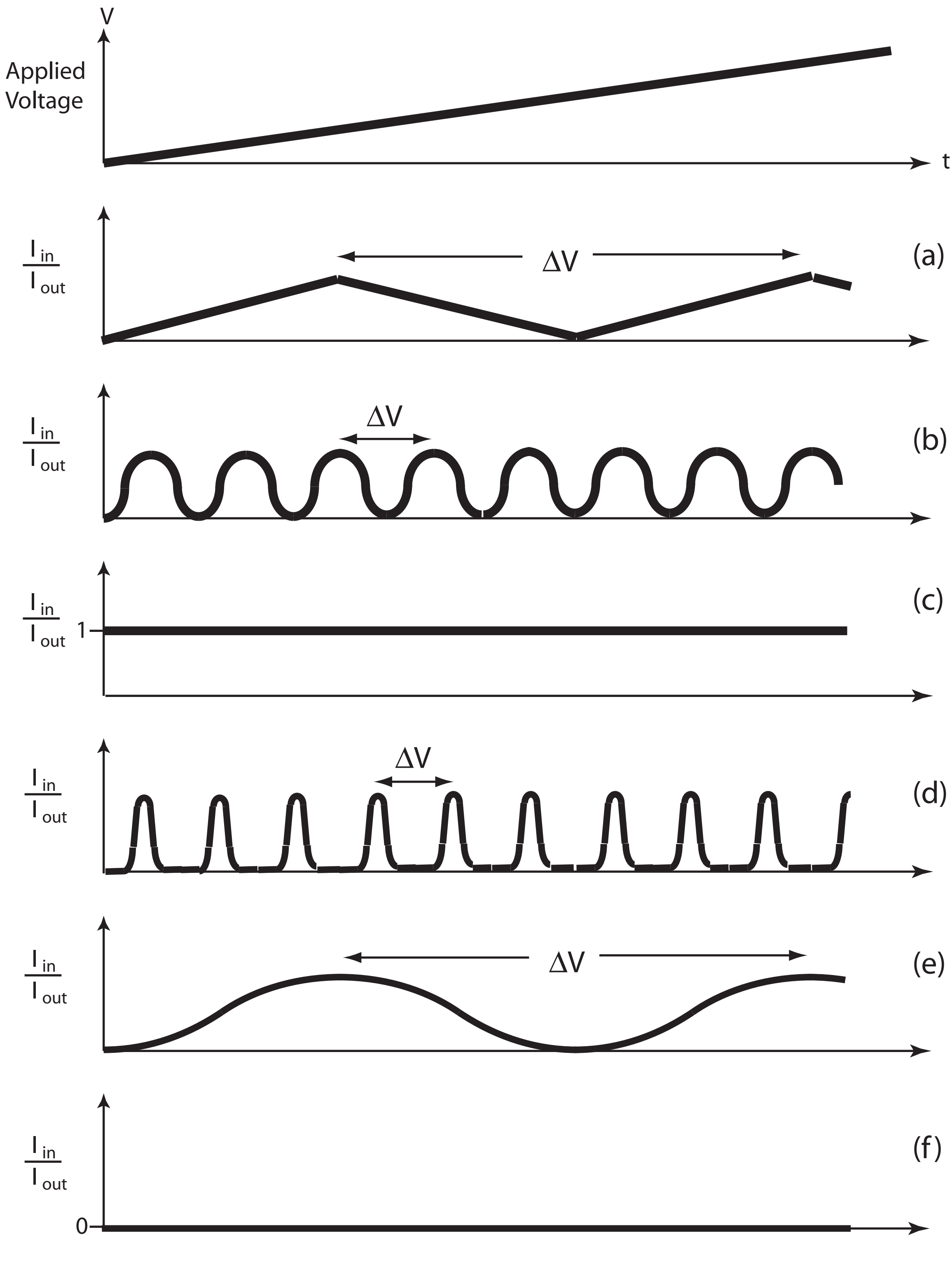


Fig. 2

(b.1) Figure (1a) above shows the above-described basic modulator with its transparent electrodes. Choose the waveform in Figs 2 (a) - (f) that best describes its output intensity variation with voltage, and write your choice in the space below. Show your reasoning. No credit will be given for correct answers without an explanation as to how you arrived at your result.

b.2 For your chosen I-V curve in part (b.1) derive an expression for the period ΔV, if any (see Fig. 2 for definition of ΔV). Show your reasoning.

(c.1) In Fig (1b) the crystal with its transparent electrodes is placed between crossed polarizers oriented as shown. Choose the waveform in Figs 2 (a) - (f) that best describes the output intensity variation with voltage. Show your reasoning. No credit will be given for correct answers without an explanation as to how you arrived at your result.

(c.2) For your chosen I-V curve in part c.1 derive an expression for the period ΔV (if any). Show your reasoning.

(d.1) In Fig 1(c) the transparent electrodes are replaced with conducting mirror electrodes. Choose the waveform in Figs 2 (a) - (f) that best describes the output intensity variation with voltage. Show your reasoning. No credit will be given for correct answers without an explanation as to how you arrived at your result.

(d.2) For your chosen I-V curve in part d.1 derive an expression for the period ΔV (if any). Show your reasoning.

Problem 6.4 - Acousto-optic Fringe Projector - 6.637 only

Shown below is a fused quartz acousto-optic modulator followed by some spatial-filtering/imaging optics. This spatial-filtering/imaging optics consists of a lens placed a distance a away from the modulator, a double-slit spatial filter placed in the back focal plane of the lens, and a screen in the image plane at a distance b from the lens. The focal length of the lens is F. A beam of coherent, collimated light ($\lambda = 633$nm) is incident normal to the surface of the bulk acousto-optic cell.

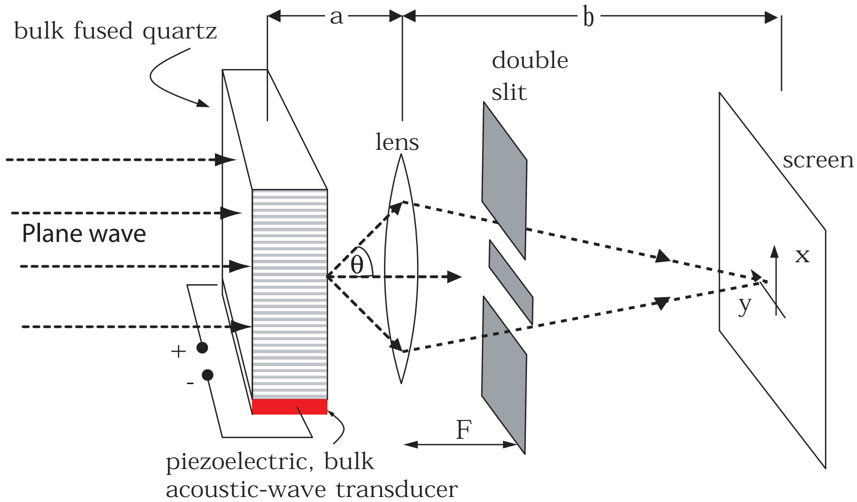


Figure 2: Diagrammatic view of a fringe-projection system consisting of an AOM, lens, and a double-slit

A sinusoidal acoustic wave of frequency $f_a = 50$ MHz is launched from the piezoelectric transducer at the bottom of the modulator such that the refractive index change in the material is given by

$$\Delta n(x,t) = -\frac{1}{2}An^3\rho\cos(k_a x - \omega_a t)$$

The crystal properties for $f_a = 50$MHz are $n = 1.46$, $v_s = 6$km/s, $\rho = 0.17$ and $A = 1 \times 10^{-4}$. Assume the acousto-optic material is thin ($<< 1mm$), so the interaction length is small.

(a) What is the spatial period Λ_a of the acoustic wave inside the medium?

(b) What is the ratio of the change in refractive index, Δn to the average index of refraction, n, of the crystal?

(c) If the focal length F of the lens is $\frac{9}{10}a$ and the lens has a radius of 4cm, what is the value of θ and the minimum value of a such that only the $m = 0, -1, 1$ diffracted orders are captured by the lens?

(d) What is the screen distance b in terms of a.

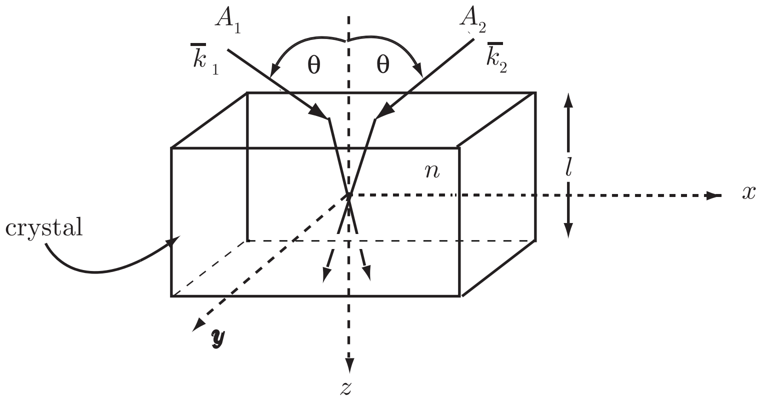
The double slit shown in the diagram is placed in the back focal plane of the lens and has been designed to let the $m = 1$ and $m = -1$ diffracted orders pass, while blocking the $m = 0$ order (assume the slits are large enough that diffraction may be neglected).

(e) Calculate the intensity pattern seen on the screen as a function of x (assume that the $m = -1$ and $m = 1$ orders are wide enough to interfere at the screen). Plot the pattern on the screen as a function of x. In which direction does the fringe pattern move on the screen?

(f) Given that the source of the light beam is a diode laser, which can be switched on and off at very high speed, devise a timing scheme for the diode laser that would produce a **stable** interference pattern on the screen. Remember, since the repetition rate (pulse rate) is high, the pattern on the screen will only seem stable to a slow detector, such as the human eye or a video camera.

Problem 6.5 - Photorefractive Effect - 6.637 only

Two mutually coherent TE-polarized plane-wave light beams of unequal amplitudes (A_1 and A_2) and polarizations $\hat{e}_1$ and $\hat{e}_2$, respectively, traveling in the x-z plane intersect in a photorefractive crystal at an angle 2θ as shown. The wavelength of the two writing beams is λ_w, the unperturbed refractive index of the crystal is n_0, and the thickness of the crystal is l in the z-direction. Assume the writing beams overfill the crystal.



(a) Write an expression for the intensity pattern I(x) in the crystal and give an explicit equation for the fringe spacing, Λ.

(b) Define the modulation index as

$$m = < \hat{e}_1 . \hat{e}_2 > \frac{(I_1 I_2)^{1/2}}{(I_1 + I_2)}$$

rewrite the expression for I(x) in terms of m.

(c) Let us assume that before saturation occurs, the local number of mobile charges generated per unit volume, $N(x)$, is proportional to the local exposure, $\mathcal{E}(x)$. Write an expression for $N(x)$ and the corresponding space charge density $\rho_{sc}(x)$.

(d) Assuming that the more mobile negative charges move by diffusion (motion owing to concentration gradients) towards the least bright regions within the crystal, derive an expression for the resulting space-charge electric field, $E_{sc}(x)$ at steady-state equilibrium.

(e) Assuming the modulation index is small, rewrite the approximate expression for $E_{sc}(x)$.

(f) Further assume that the electric-field induced refractive index change in the crystal is given by

$$\Delta n(x) = \frac{1}{2} n_o^3 r E_{sc}(x)$$

where r is the electro-optic coefficient. Make a plot of $I(x)$, $\rho_{sc}(x)$, $E_{sc}(x)$ and $\Delta n(x)$ all on the same graph, one below the other for the case of small modulation depth.

(g) Consider the case where the crystal can be considered a "thin" grating, and is read out with a plane wave of wavelength λ_r and amplitude A_r propagating along the +z axis:

(1) Write and expression for the transmitted wave at the output face of the crystal.

(2) What is the photorefractively-induced phase change, $\Delta\phi(x)$, that the crystal imparts to the readout beam?

(3) What is the intensity of the light that is diffracted into the mth order?

(g) Now consider the case where the crystal is thick, $\lambda_r > \lambda_w$, and the crystal is readout at the Bragg angle. What is this angle? What is the expected first-order diffraction efficiency?