

MASSACHUSETTS INSTITUTE of TECHNOLOGY  
Department of Electrical Engineering and Computer Science

*6.161 Modern Optics Project Laboratory*  
*6.637 Optical Signals, Devices & Systems*

Problem Set No. 7  
Fall Term, 2009

**Lasers**  
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Issued Tues. 11/3/2009  
Due Tues. 11/10/2009

**Reading recommendation:** Class Notes, Chapter 7; Yariv, Chapter 5 - 7. Be neat in your work!

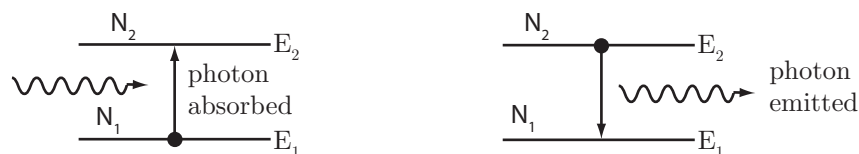
**Problem 7.1**

A certain He-Ne laser has a Doppler broadened linewidth of 1.5 GHz and its central operating wavelength of 632.8 nm. The radii of curvature of both mirrors is 1 m and the length of the cavity is 25 cm [assume  $n = 1$ ].

- (a) Is this cavity stable? (show your calculation).
- (b) What is the frequency difference between the longitudinal modes of the cavity?
- (c) How many longitudinal modes of the laser are active?
- (d) When this laser is mode-locked, what is the temporal separation between the output pulses as would be seen by a detector placed in the output beam?.
- (e) What is the maximum resonator length you would have chosen if single-longitudinal-mode operation was desired?

**Problem 7.2**

A certain two-level laser system has a total of  $N$  atoms per unit volume, and the ratio of  $N_2/N_1 = 1/e$  at room temperature (300 K). Here  $N_1$  and  $N_2$  are the number of atoms per unit volume in state 1 and state 2 respectively. ( $h = 6.624 \times 10^{-34} J \cdot s$      $k_B = 1.38 \times 10^{-23} J/K$ ).

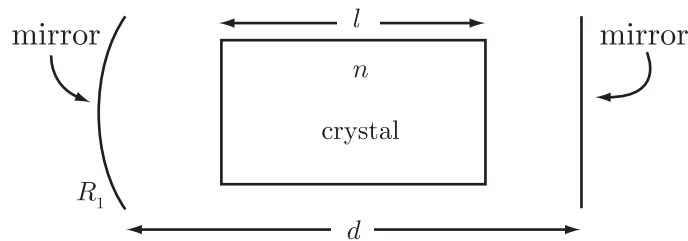


- (a) When broadband light is incident on this system, what is the frequency (in Hz) of the photons absorbed or emitted by the system?
- (b) What is the wavelength (in  $\mu\text{m}$ ) of the emitted light?
- (c) If the output has a spectral bandwidth of  $0.2 \text{ \AA}$ , what is the bandwidth in Hz?

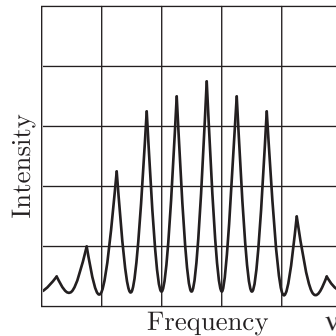
- (d) Write the rate equations for the system when the system is pumped at a general rate  $R_p$  (assume no stimulated transitions occur; i.e., only spontaneous emission).
- (e) If specifically,  $t_{21} = 0.5 \times 10^{-7}$  s, and the system is being pumped to steady state with  $N_2/N_1 = 3$ , what is the power density,  $P_{dr}$ , (in  $\text{W}/\text{m}^3$ ) radiated by the spontaneous emission process in this steady state condition?
- (f) What is the pump power density,  $P_{da}$ , (in  $\text{W}/\text{m}^3$ ) that is being absorbed to maintain this steady-state condition?

### Problem 7.3

A crystal laser has the design shown below. The refractive index of the crystal is  $n$  and its length is  $l$ . The mirror separation is  $d$ .



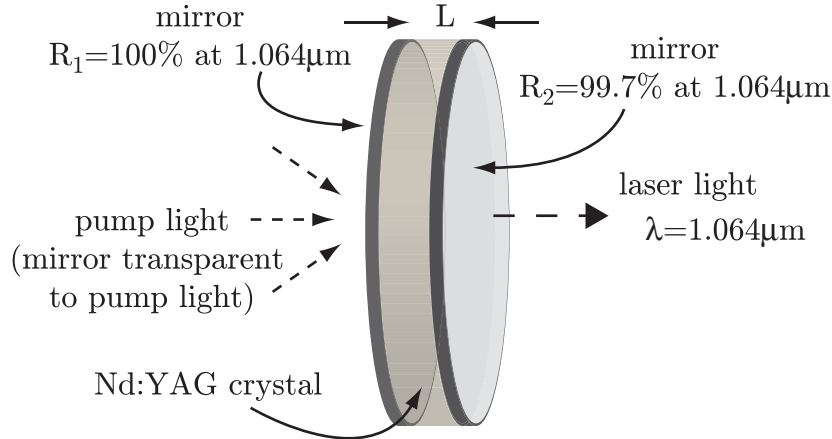
The output spectrum of the laser is shown in the figure below where each frequency division is 200MHz wide.



- (a) What is the length,  $d$ , of the laser resonator?
- (b) If one of the laser mirrors has a radius of curvature  $R_1$  and the other is flat, as shown:
- (1) What is the shortest possible length,  $d_s$ , between the mirrors for stable operation?
  - (2) What is the largest possible length,  $d_l$ , between the mirrors for stable operation?

### Problem 7.4

A Nd:YAG crystal micro-laser is in the form of a disc that is  $500 \mu\text{m}$  thick. The surfaces of the disc are plane-parallel and have mirror coatings with  $R_1 = 100\%$  and  $R_2 = 99.7\%$  at the Nd:YAG laser wavelength of  $1.064 \mu\text{m}$ . Assume the refractive index of Nd:YAG is 1.82 and its absorption coefficient (absorption loss per unit length)  $\alpha = 0.5\text{m}^{-1}$ . Such a disc would normally be optically pumped by a semiconductor laser operating at a wavelength at which the mirror coatings are transparent.

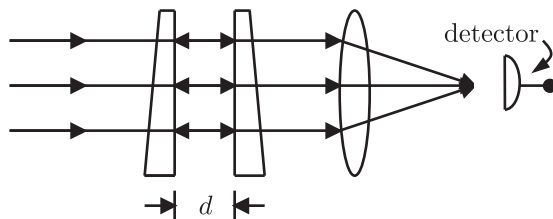


Assume the lower and upper laser levels ( $E_1$  and  $E_2$  respectively) have an energy separation corresponding to  $\lambda=1.064 \mu\text{m}$ .

- What is the energy difference, in eV, between these two levels? ( $h = 6.26 \times 10^{-34} \text{ J} \cdot \text{sec} = 4.14 \times 10^{-15} \text{ eV} \cdot \text{sec}$ )
- Considering these two levels only, what fraction of the population distribution is in level 2 at 300K?
- What is the round-trip optical path length?
- What is the round-trip time?
- What is the longitudinal mode spacing of this cavity? Do we have single mode operation?
- By treating the cavity absorption loss in the same way the mirror reflection losses are treated, using the material in Chapter 3 of the notes, write an expression for the finesse,  $F$ , of this cavity. Do not put the numbers into the equation.
- What is the relation between the finesse and the longitudinal mode width? Assuming  $F = 1650$ , what is the width of the longitudinal modes (in Hz)?

**Problem 7.5 - 6.637 only**

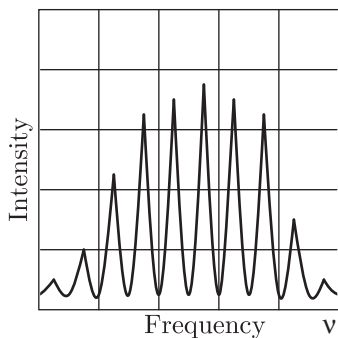
In the scanning Fabry-Perot spectrometer, the gap  $d$  between the mirrors is varied, and the intensity of the central spot transmitted through the spectrometer is monitored (see Figure below).



Let  $\lambda_m$  and  $\lambda_{m+1}$  be two adjacent wavelengths for which bright central maxima of order  $m$  and  $m + 1$  respectively occur. The free spectral range of the instrument is defined as the wavelength separation between adjacent maxima; i.e.,

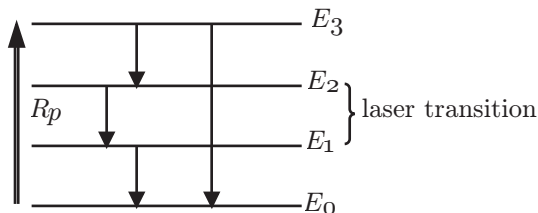
$$\Delta\lambda_{fsr} = \lambda_m - \lambda_{m+1}$$

- (a) Derive an expression for the free spectral range. The free spectral range is sometimes more useful when expressed in the frequency domain. Write down the expressions for  $\Delta\nu_{fsr}$ .
- (b) As  $d$  is varied, the wavelength corresponding to a maximum of fixed order  $m$  varies. How much must  $d$  be varied to scan through a wavelength range equal to the free spectral range?
- (c) These instruments have very high resolving power and are used to measure the fine structure of spectral lines. If you were given the task of designing an instrument to analyze the structure of the spectrum shown in the Figure below, how large would you make  $d$ , and what reflectivity would you choose for the mirrors? Show clearly your reasoning. Assume that for the spectrum shown in the figure below each frequency division is 200MHz wide.



**Problem 7.6 - 6.637-only**

For the ideal four-level homogeneously broadened laser system with allowed transitions as shown below,  $E_1 \gg kT$ .



- (a) Write down the rate equations for this system in the presence of stimulated coherent radiation for the laser transition shown.
- (b) List a set of 4 conditions on the relaxation times that would be desirable to optimize the population inversion?
- (c) Using the rate equations along with the following specific simplifying assumptions:
  - (1)  $t_{32} \ll t_{30}$
  - (2)  $t_{10} \ll t_{21}$
  - (3)  $t_{32} \ll t_{21}$
  - (4)  $t_{10} \ll t_{30}$

Derive an expression for the steady-state population inversion  $\Delta n$  in the presence of stimulated coherent radiation for the laser transition shown [hint: use only a subset of the rate equations that are independent (no redundancy) and give your answer in terms of  $N$  (total number of atoms per unit volume) - Try not to get too buried in the algebra]

- (d) What is the relation between the gain  $\gamma(\nu)$  and the population inversion  $\Delta n$  that you just calculated?

— END OF PROBLEM SET —