MASSACHUSETTS INSTITUTE of TECHNOLOGY
Department of Electrical Engineering and Computer Science

6.161 Modern Optics Project laboratory 6.637 Optical Signals, Devices & Systems

Problem Set No. 8	**Fourier Optics**	Issued Thrs. 11/12/2009
Fall Term, 2009	**-**	Due Thrs. 11/19/2009

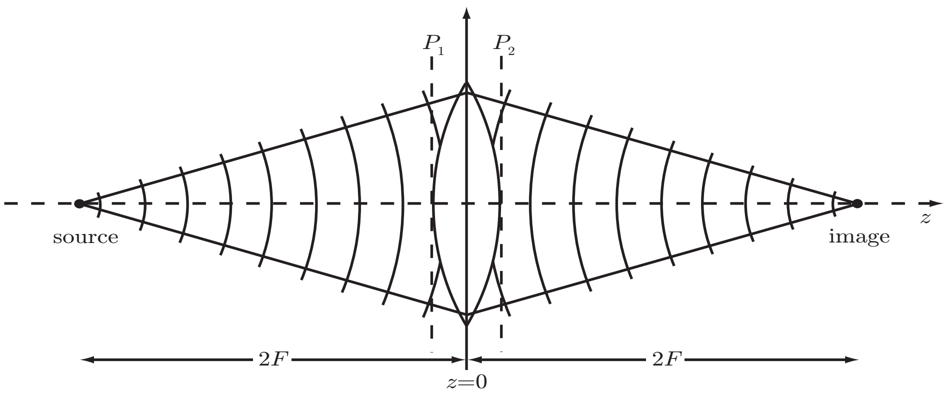
Reading recommendation: Class Notes, Chapter 8. Be neat in your work!

Problem 8.1

We know that if an object is placed in the front foal plane of a lens, the field in the back focal plane of the lens is related to the Fourier transform of the object. We also know that the far-field diffraction pattern of an object is related to the Fourier transform of that object. Make a list of the similarities and differences between the Fourier transforms formed by these two systems. Explain each of the differences with the aid of diagrams and an example such as a circular aperture, or a slit, or rectangular grid, etc.

Problem 8.2

It is well known that a lens will image a point-source at a distance $2F$ in front of the lens to a point at a distance $2F$ in back of the lens as shown.



(a) Write an expression for the wavefront incident on the lens $\underline{U}_1(\overline{\rho}_1)$ at the plane, P_1, located at $z = 0_-$ (lens is thin).

(b) Using the paraxial approximation, simplify the expression obtained in part (a).

(c) Similarly, write an expression for the wavefront exiting the lens $\underline{U}_2(\overline{\rho}_2)$ at the plane, P_2, located at $z = 0_+$ (lens is thin).

(d) Using the paraxial approximation, simplify the expression obtained in part (c).

(e) What then is the expression for the thin lens transformation $\underline{t}_l(\overline{\rho})$?

Problem 8.3

In a classical two-lens coherent optical processor with lenses of focal length F, two signal transparencies of transmittance $g_a(x_1, y_1)$ and $g_b(x_1, y_1)$ are inserted in the $\bar{\rho}_1$ (input) plane with centers at $(a, 0, 0)$ and $(-a, 0, 0)$ respectively. The Fourier-plane filter $\underline{t}_h(\bar{\rho})$ is a sinusoidal grating with amplitude transmittance

$$\underline{t}_h(\bar{\rho}) = \underline{t}_h(F\lambda\,\bar{f}) = \frac{1}{2} + \frac{1}{2}\sin(2\pi\, f_g x + \phi)$$

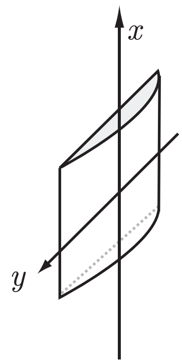
where f_g and ϕ represent the grating frequency and position respectively.

(a) Calculate the complex amplitude distribution at the output plane of the processor.

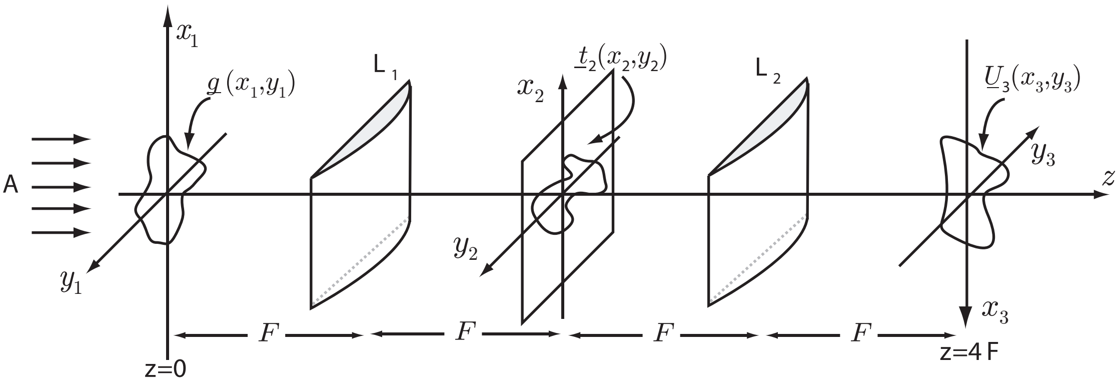
(b) For the special case where f_g= a/λF and $\phi = 0$, comment on the significance of the output. Repeat the process for $\phi = 90^\circ$.

Problem 8.4. - Cylindrical-lens Fourier Optics

(a) Consider the thin cylindrical lens shown below. Write an equation for the thin cylindrical lens transformation $\underline{t}_{cl}(x, y)$.



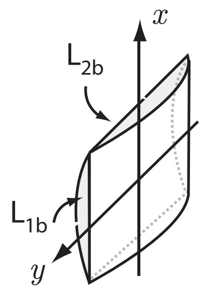
(b) A classical two-cylindrical lens coherent processor is built as shown below. The lenses L_1 and L_2 each have focal length F. Such a system can used to process 1-D objects $\underline{g}(x, y)$ using 1-D Fourier plane filters, $\underline{t}(x, y)$. Indicate on the diagram how the 1-D object, $\underline{g}(x, y)$ and the filter $\underline{t}(x, y)$ must be oriented to properly function in this system.



(c) Write an equation for the input-output characteristics of the system. That is, express $\underline{U}_3(x_3, y_3)$ in terms of $\underline{U}_1(x_1, y_1)$.

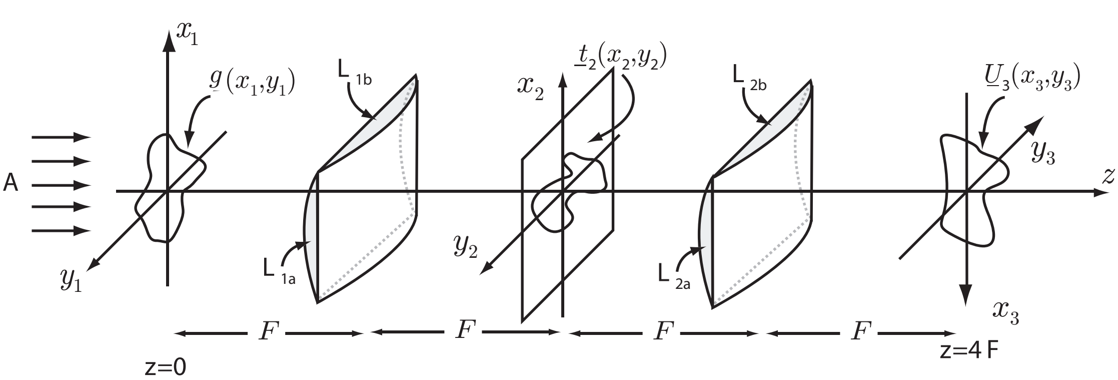
(d) Two plano-convex cylindrical lenses are crossed and fused together to form a compound lens as shown below.

d(1) Write the lens transformation for this compound lens assuming the two original lenses had the same focal length.



d(2) What is the focal length of the compound lens?

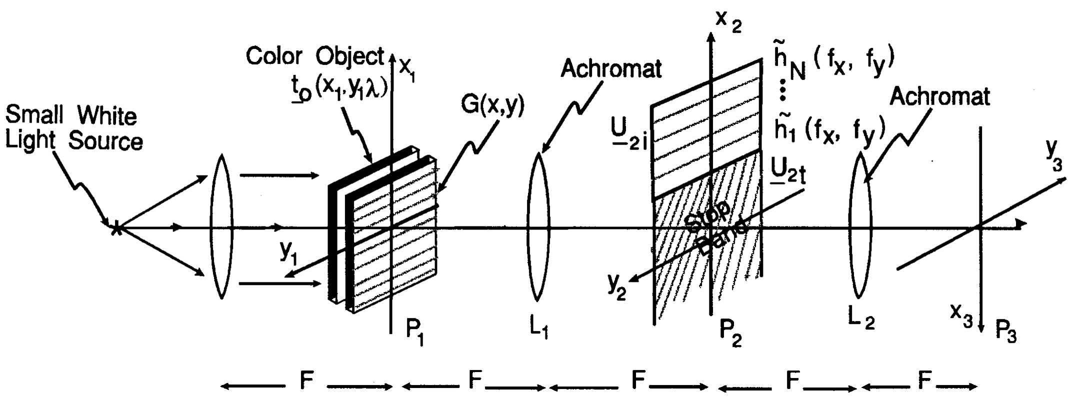
(e) A classical two-compound lens coherent processor is built as shown below. Write an equation for the input-output characteristic of the system. That is, express $\underline{U}_3(x_3, y_3)$ in terms of $\underline{U}_1(x_1, y_1)$



Problem 8.5 - 6.637 only

In the polychromatic optical processor shown below, S is a white-light point source, and L_0, L_1 and L_2 are achromatic lenses. Two three-color (each monochromatic) signal transparencies $\underline{g}_a(x, y, \lambda)$ and $\underline{g}_b(x, y, \lambda)$ are placed in the input plane P_1 at the points (0, d) and (0, -d) respectively and in contact with a high-efficiency diffraction grating $G(x, y)$. Assume the two input signal color transparencies, $\underline{g}_a(x, y, \lambda)$ and $\underline{g}_b(x, y, \lambda)$, each have a spatial frequency bandwidth limit of B_x x B_y. The transmission function of the in-plane grating is given by

$$t_G(x, y) = \frac{1}{2}[1 + cos(2\pi f_g x)]$$



(a) Write an expression for $U_{2i}(x, y, \lambda_n)$ where n is the color subscript. Here [n =1(red), 2(green), 3 (blue)].

(b) Explain/interpret each term in the equation you have derived in (a). For n = 1, 2, 3 make a sketch of the pattern of the light falling on the P_2 plane, and label the position of the centroid of each of the components.

(c) Suppose the Fourier-plane composite filter in the P_2 plane is composed of a set of non-overlapping subfilters given by

$$\underline{t}_h(x_2, y_2) = \tilde{h}(f_x, f_y) = \Sigma_n(1/2)[1 + sin(2\pi d f_{yn})] \tag{1}$$

for the first-order spectra, but is opaque for the zero and -1 orders. Sketch the three subfilters, corresponding to the three values of n, in physical space.

(d) Write an expression for the output $\underline{U}_3(f_x, f_y, \lambda)$ in the P_3 plane for one of the colors using its corresponding Fourier-plane subfilter.

(e) To perform full color subtraction, we must synthesize the composite filter $\tilde{h}(f_x, f_y)$. Make a sketch of the desired composite filter. On your diagram clearly specify and give expressions for: (1) its periodicity, Λ_{gy}, (2) the centroids and (3) widths of the filter segments (if any), and (4) the location of the desired subtraction image, $\underline{g}_a(x, y, \lambda)$ - $\underline{g}_b(x, y, \lambda)$, in the output plane.

(f) What are the conditions on λ_n, f_g, B_x, B_y and F so that the desired output image is clearly separated from other images in the output plane?

(g) Comment on the practical limitations of this system.