# MASSACHUSETTS INSTITUTE of TECHNOLOGY <br> Department of Electrical Engineering and Computer Science 

6.161/6637 Practice Quiz 2

Spring Term, 2004

Issued X:XXpm 4/XX/2004
Due X:XX+1:30pm 4/XX/2004

Please utilize the space provided below each problem for your work. You may use extra paper if necessary. If you need additional clarification, be sure to ask. This quiz is open-book and open-notes. GOOD LUCK and DON'T STRESS!!! You may start as soon as you receive the quiz.

Please note that many of the problems on this practice quiz are NOT shortanswer questions like you will see on the upcoming quiz.

Please print your name here: $\qquad$

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Check Box:
$6.161 \square$
$6.637 \square$

## Problem 1 - Fiber Optics (15pts.)

In the laboratory you have found a cylindrical step-index 100-mode fiber that tapers down gradually to a 10 -mode step index fiber. The section of fiber in the neighborhood of the taper is shown below. The core and cladding refractive indices are unchanged throughout the entire length of the fiber.

(a) What is the ratio, $\left(d_{1} / d_{2}\right)$ of the core diameters of the two ends of the fiber.
(b) Assuming that light is injected into the larger end of the fiber, put a check mark in the appropriate box in the Table below to describe what happens as the light passes from the larger fiber into the smaller fiber.

| Parameter | Increases | Decreases | Unchanged | Does not Exist |
| :---: | :--- | :--- | :--- | :--- |
| Modal Dispersion |  |  |  |  |
| Material Dispersion |  |  |  |  |
| Waveguide Dispersion |  |  |  |  |
| Susceptibility to polarization dispersion |  |  |  |  |
| Output Power |  |  |  |  |
| Output Power density |  |  |  |  |

## Problem 2-Spatial Frequency(10 pts.)

A diver sets up a coordinate system with the surface of the water $\left(n_{w}=1.33\right)$ in in the $y-z$ plane. She then measures the direction of propagation of three beams (labelled A, B and C) that pass through the $z=1$ plane at the points $(1,1,1),(0.5,0,1)$ and $(-2,1,1)$ respectively (all dimensions are in meters) as shown. Beams A and B are measured above the surface of the water, while C is measured within the water of refractive index $n_{w}$. Beam A has a free-space wavelength of $\lambda_{a}$, B has a free-space wavelength of $\lambda_{b}$, and C has a free-space wavelength of $\lambda_{c}$. She finds that all three beams are propagating parallel to the $x-z$ plane and make an angle of 30 degrees with the $z$-axis. Later, she discovers that there is a fourth beam D of free-space wavelength $\lambda_{d}$ propagating in the water along the $x$-axis toward the origin of the coordinate system.
(a) What are the spatial frequencies $\overline{\omega_{a}}, \overline{\omega_{b}}, \overline{\omega_{c}}$ and $\overline{\omega_{d}}$ of the four beams?
(b) What is the angle of transmittance of beam C in air?


## Problem 3-Acousto-optic modulator (20pts.)

Collimated, collinear monochromatic red, green and blue (RGB) light from a display enters a thin Raman-Nath acoustic grating at normal incidence. The transducer of the cell is driven with a signal that is given by $S(t)=A \cos \left(\omega_{a} t\right)[h(t)=1]$, and the resulting acoustic grating periodicity is $\Lambda$. The seven lowest-order diffracted beams of light ( $n=1-7$ ) that emerge from the cell are shown below.
(a) If the wavelengths of the RGB beams are $\lambda_{r}, \lambda_{g}$, and $\lambda_{b}$, fill in the table below for each of the beam numbers (in the figure) to show the color, the angle $\alpha_{n}$, spatial frequency $f_{n}$, and the temporal frequency $\omega_{o}$ of the diffracted beams.


| Beam No. | Color | $\alpha_{n}$ | $f_{n}$ | $\omega_{o}$ |
| :---: | :--- | :--- | :--- | :--- |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |
| 6 |  |  |  |  |
| 7 |  |  |  |  |

(b) Collimated, collinear monochromatic red, green and blue (RGB) light from a display enters a thick acoustic cell at an angle $\theta$ as shown below. The transducer of the cell is driven with a signal that is given by $S(t)=A \cos \left(\omega_{a} t\right)[h(t)=1]$, and the resulting acoustic grating periodicity is $\Lambda$. The acoustic cell makes one counter-clockwise revolution (starting at $\theta=0$ ) about an axis $O$ at its center that is perpendicular to the plane of incidence. A stationary large-area photo-detector is placed in the 4th quadrant of the co-ordinate system as shown. Assume that propagation through the cell is possible through faces A and B only.


Fill in the table below to show the color, the angles and the temporal frequencies of the R, G and B light that reaches the photodetector as the acoustic cell rotates.

| Color | $\theta$ | Frequency, $\omega$ |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

## Problem 4 - Electro-optic modulator (20pts.)

A collimated ordinary-polarized beam of light of wavelength $\lambda$ from a laser is split into two equal intensity beams and passed through the $x^{\prime}$ and $y^{\prime}$ faces of an Potassium Dihydrogen Phosphate (KDP) crystal as shown in the diagram below (the $x^{\prime}$ and $y^{\prime}$ axes are oriented at $45^{\circ}$ to the crystallographic axes, $x$ and $y$ (not shown)). The output faces of the crystal are labelled A and B respectively. The dimensions of the crystal are $L \times L \times d \mathrm{~cm}$. The exiting beams (with a 90 degree angle between them) are combined on a screen as shown. A variable DC voltage is applied across the c-axis of the crystal as shown.

(a) Draw in and describe below the states of polarization of the two beams just before they enter the crystal.
(b) Draw in and describe below the polarization state of the light that exits the crystal at face A when voltage is applied to the crystal.
(c) Draw in and describe below the polarization state of the light that exits the crystal at face B when voltage is applied to the crystal.
(d) What is the separation between the fringes on the screen?
(d) Write an expression for the phase change of the light exiting face A of the crystal as a function of voltage.
(e) Write an expression for the phase change of the light exiting face B of the crystal as a function of voltage.
(f) A pinhole/detector system is placed behind one of the fringes. Write an expression for the detector current as a function of modulator voltage.

## Problem 5

(2-a) A given cylindrical step-index optical glass fiber is being operated at a wavelength of $1.3 \mu \mathrm{~m}$. It is specified by the manufacturer to have a numerical aperture of 0.20 . In an independent experiment you have measured the refractive index of the cladding at a wavelength of $1.3 \mu \mathrm{~m}$ to be $n=1.45$. What is the refractive index of the core?
(2-b) What is the largest diameter this fiber can have and still be a single-mode fiber at the operating wavelength?
(2-c) Which form of dispersion do you expect to be dominant in this single-mode fiber? Write an expression for the associated dispersion coefficient.
(2-d) If the source has a bandwidth of $\Delta \lambda=0.2 \AA$ and the fiber is 10 km long, what will be the maximum bit rate possible with this fiber?
(2-e) If instead of the given step-index fiber, you had used a single-mode graded-index fiber with profile parameter $\gamma=2$, what would have been the largest possible diameter for single-mode operation?
(2-f) If the ends of the fiber are overfilled with light, roughly how much more of the power from the same source (percentage) could be coupled into the graded-index fiber than into the step-index fiber?

## Problem 6

An integrated-optics Mach-Zehnder-interferometer switch is built by embedding two waveguides in a linear electro-optic material as shown. The arms of the interferometer are of equal length, and the input light gets divided equally into the two branches. The electrode length is $L$ meters and the electrode separation is $d$. The electro-optic material is uniaxial with refractive indices $n_{O}$ and $n_{E}$. Its non-zero electro-optic coefficients are $r_{13}=r_{23}$, $r_{22}=-r_{12}=-r_{61}, r_{33}$ and $r_{42}=r_{51}$. The form of the electro-optic tensor is shown below.

$$
\left(\begin{array}{ccc}
0 & r_{12} & r_{13} \\
0 & r_{22} & r_{23} \\
0 & 0 & r_{33} \\
0 & r_{42} & 0 \\
r_{51} & 0 & 0 \\
r_{61} & 0 & 0
\end{array}\right)
$$



The switch is first operated with horizontally polarized light $(\hat{e}=\hat{z})$ of amplitude $E_{\text {in }}$.
(3-a) Derive an expression for $V_{\text {off }}$ (the voltage that extinguishes the output light).
(3-b) Sketch the shape of the output intensity as a function of voltage, and label your sketch to show where $V_{\pi}$ occurs.


## Problem 7

Three identical acousto-optic modulators, each operated in the Bragg regime, are arranged to form the system shown below. Each modulator has a $64 \%$ diffraction efficiency. Assume diffraction into orders other than the +1 order are negligible. The system is exited with a plane monochromatic wave of wavelength $\lambda$ and amplitude $\bar{E}_{\text {in }}(\bar{r}, t)$ at the Bragg angle $\theta_{B}$.

(4-a) Write an expression for $\theta_{B}$ in terms of the acoustic velocity $v_{a}$ and the carrier frequency $\omega_{a}$.
(4-b) What are the optical frequencies of the optical waves $\bar{E}_{0}(\bar{r}, t), \bar{E}_{1}(\bar{r}, t), \bar{E}_{2}(\bar{r}, t)$ and $\bar{E}_{3}(\bar{r}, t)$ ?

$$
\begin{array}{ll}
\text { Wave } & \text { Amplitude } \\
\bar{E}_{0}(\bar{r}, t) \\
\bar{E}_{1}(\bar{r}, t) \\
\bar{E}_{2}(\bar{r}, t) \\
\bar{E}_{3}(\bar{r}, t) & \\
\hline
\end{array}
$$

(4-d) Ignoring all constant phase factors, write expressions for:
(1) $\bar{E}_{0}(\bar{r}, t)$ at the output face of AO 1 in terms of $\bar{E}_{i n}(\bar{r}, t)$.
(2) $\bar{E}_{1}(\bar{r}, t)$ at the output face of AO 1 in terms of $\bar{E}_{\text {in }}(\bar{r}, t)$.
(3) $\bar{E}_{2}(\bar{r}, t)$ at the output face of AO 2 in terms of $\bar{E}_{\text {in }}(\bar{r}, t)$.
(4) $\bar{E}_{3}(\bar{r}, t)$ at the output face of AO 3 in terms of $\bar{E}_{\text {in }}(\bar{r}, t)$.
(4-e) The detector array ignores d.c. signals and integrates only the a.c. component of the optical signal falling on it. The integration time is $T$, where $T$ is the transit time of the acoustic signal across the modulator. Write an expression for the charge $q(x, t)$ that accumulates in the detector over the period $T$.

Problem 8 (20 pts.)


An acousto-optic modulator is driven with a signal $S(t)=A g(t) \cos \omega_{a} t$
(a) Assuming that the refractive index change in the acousto-optic cell is proportional to $S(t)$, derive an expression for the acoustically induced phase change $\Delta \phi(x, t)$ in the acoustic cell.
(b) For the special case where $g(t)=1$, assume the cell is operated in the Raman Nath regime, write an expression for the intensity distribution in the back focal plane of the lens.
(c) Draw this distribution onto the diagram on the previous page, and label it with its key features.
(d) Again, with $g(t)=1$, if the cell were operated in the Bragg regime, how would the intensity distribution in the back focal plane change?
(e) Redraw the system to show the set-up for Bragg operation.

## Problem 9 (15 pts.)

A 3-level laser system is characterized by the electronic transitions shown in the diagram below. The system is optically pumped as shown in the diagram at a rate $R_{p}$. Laser action takes place between levels 2 and 3 .

(a) Write down the rate equations for the system when the laser is in operation.
(b) Write down a set of conditions that would be ideal to optimize the population inversion of this system.
(c) Use (a) and (b) to arrive at an expression for the population inversion in steady state laser operation.

## Problem 10 (20 pts.)

A cylindrical step-index multimode fiber has a core diameter of $100 \mu \mathrm{~m}$, a core index of 1.48 and a cladding index of 1.45 . The fiber is operated at a wavelength of $1.5 \mu \mathrm{~m}$.
(a) What is the normalized waveguide radius of this fiber?
(b) Estimate the number of modes in the fiber.
(c) What is the numerical aperture of the fiber?
(d) If single mode operation at $\lambda=1.5 \mu \mathrm{~m}$ was desired, what should have been the core diameter?

## Problem 11 (10 pts.)

In the following parts, you will need to perform troubleshooting as one might do in lab.
Your lab partner has set up the electro-optic crystal light modulator experiment exactly as shown below. He turns on the signal generator for a 40 MHz sine wave and upon turning on the high-voltage, high-frequency amplifier you both observe a nice 40 MHz sinusoidal signal on the oscilloscope. Explain the signal that you see on the oscilloscope.


## Problem 12 (15 pts.)

Three fibers of identical length were found in the laboratory. The refractive index at the center of their cores is 1.50 . In an effort to measure the length and determine the type of each fiber you launched a set of 1 ns rectangular pulses of light of equal power from a laser at time $t=0$, and you observed the outputs shown below.
(a) What is the length of these fibers?
(b) Fill in the column on the right with the correct fiber type.

| Fiber | 1nsec. on-axis monochromatic collimated pulse | 1nsec. divergent monochromatic pulse | Fiber type |
| :---: | :---: | :---: | :---: |
| A |  |  |  |
| B |  |  |  |
| C |  |  |  |

THE END

