Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science
6.182
Laboratory 2
Adaptive Measurement of Intensity Discrimination
Issued: March 9, 2009 Due: (First Draft) March 20, 2009

1 Background

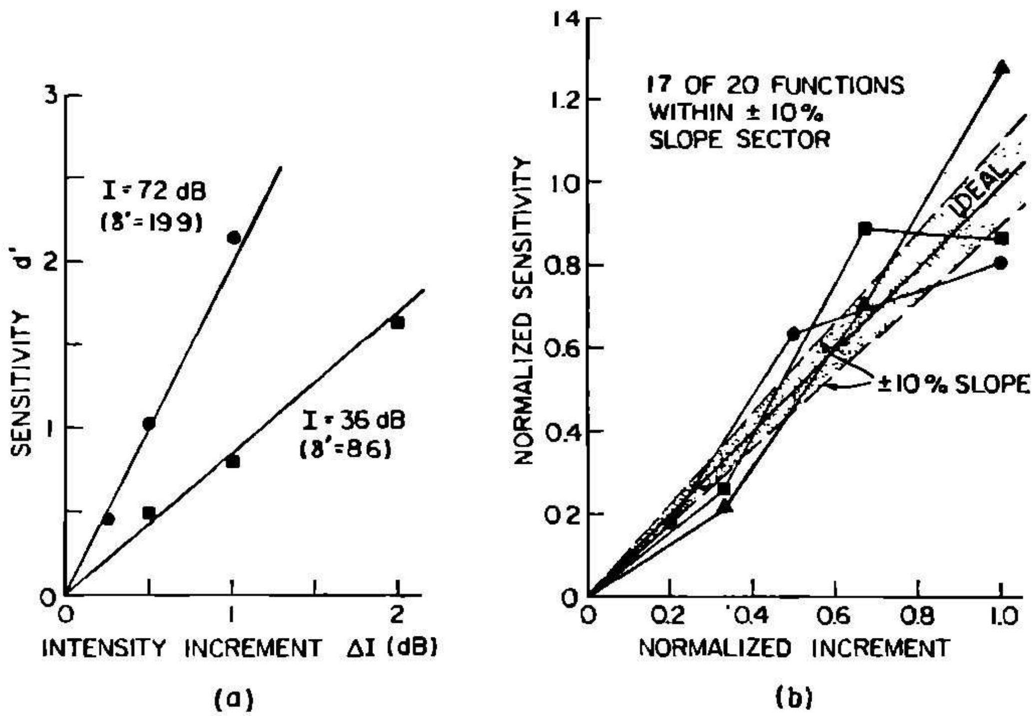

FIG. 1. Psychometric functions (from Rabinowitz, 1970): (a) shows sample functions for $I=36$ and 72 dB SL [ΔI(dB) means $10\log(1+\Delta I/I)$]; (b) summarizes the normalized psychometric functions. See text for details.

Figure 1: Dependence of d' on $10\log_{10}(I+\Delta I)/I$. Left panel: The relation for $I = 36$ and $I = 72$ dB SL. Right panel: The relation normalized to the largest increment ΔI tested. Note that 17 of 20 measurements of the relation fall within the hatched area.

In this laboratory you are to design and conduct experiments that measure how well humans can discriminate between stimuli that differ only in intensity. The ability to discriminate between intensities I and $I + \Delta I$ (see Appendix I for a discussion of sound intensity) is conveniently expressed in terms of

$$d'(I, I+\Delta I)$$

(see the recitation notes for February 26, 2008 for a discussion of d')

Some measurements of this ability made by Rabinowitz et al.[1] showed that whereas the ability depends *locally* on

$$\frac{I+\Delta I}{I}$$

[1] Rabinowitz, W.M., Lim, J.S., Braida, L.D., and Durlach, N.I. (l976). "Intensity Perception. VI. Summary of Recent Data on Deviations from Weber's Law for 1000-Hz Tone Pulses," J. Acoust. Soc. Am., 59, 1506-1509.

so that

$$d'(I, I + \Delta I) = K 10 \log_{10} \frac{I + \Delta I}{I}$$

(Fig. 1, panel 1B) the ability depends *globally* on I (Fig. 1, panel 1A), i.e. K is not a true constant but is a function of I, $K(I)$.

Rabinowitz et al. determined the relation between d' and intensity by holding ΔI constant and estimating d' from measurements of the percentage of correct reponses (similar to Protocol 3 of Laboratory 1). You will examine the relation using an adaptive technique that estimates the value of ΔI which produces specified target percentages of correct response (similar to Protocol 2 of Experiment 1).

The experiments are to answer these questions:

1. Is d' a linear function of the log of the ratio of the sound intensities (i.e., a linear function of the difference in intensity levels)?

2. Does Weber's Law apply to intensity discrimination?

2 Experiments

The experiments are to use an adaptive procedure with the symmetrical, two-interval, two-alternative forced choice format described in the class notes on the Decision Model and used in Protocol 2 of Laboratory Exercise 1. Each trial of the experiment consists of two observation intervals that contain clearly audible tone bursts, and so need not be marked by lights. The stimuli are identical except for pressure amplitude, and consequently intensity (which is proportional to the square of amplitude, see the Appendix). The tones are to have a frequency of 1000 Hz.

The bursts are to have envelopes that have 250 ms on-times with rise/fall times of 25 ms. The inter-burst interval is to be 400 ms. On a given trial the intensities of the stimuli[2] are denoted as I_1 and $I_2 = I_1 + \Delta I$. There are two types of trials that differ with respect to the order of presentation of the intensities (I_1, I_2) and (I_2, I_1) When an adaptive procedure is used, ΔI is varied from trial to trial to try to achieve a specified level of performance.

Choice of the Adaptation Variable

There are many variables that can be specified as the adaptation variable in these experiments. Perhaps the easiest one to use is the intensity increment ΔI.

A convenient iniital value and step size for an adaptive experiment may be expressed as follows. At the beginning of a run, set $\Delta I = I$, so that the more intense stimulus is 3 dB greater than the less intense $(I + \Delta I = 2I)$. A reasonable value for the initial step size is the initial value of $\Delta I/3.16$, or 5 dB less than I.

There are reaons that this may not be an optimal adaptaion variable. Based on the assumption that the probability of a correct response, Q, is given by

$$Q = \Phi\left(\frac{d'}{2}\right), \tag{1}$$

where

$$d' = k\,(I_1)\,10\log_{10}\left(I_2/I_1\right) = k\,(I_1)\left(10\mathcal{I}_2 - 10\mathcal{I}_1\right) = k\,(I_1)\,10\Delta\mathcal{I} \tag{2}$$

the dependence of Q on the SPL difference (in dB, $10\Delta\mathcal{I}$) is plotted in the left panel of Fig. 2 for $k = 5, 10,$ and, 20.

Fig. 2 illustrates a difficulty that would arise if one attempted to measure adaptively the value of $\Delta\mathcal{I}$ that produced any specified value of Q. Recall that in a Levitt adaptive procedure, the next test level L_{i+1} is related to the current test level L_i by

$$L_{i+1} = L_i \pm S_i$$

where S_i is the current step size. According to this relation, L_{i+1} is not guaranteed to be positive, e.g. if $L_i < S_i$ and the listener responds correctly. If $\Delta\mathcal{I}$ becomes negative, the

[2] These can be measured with the HP Type 35660A Dynamic Signal Analyzer using long duration stimuli with the same amplitude.

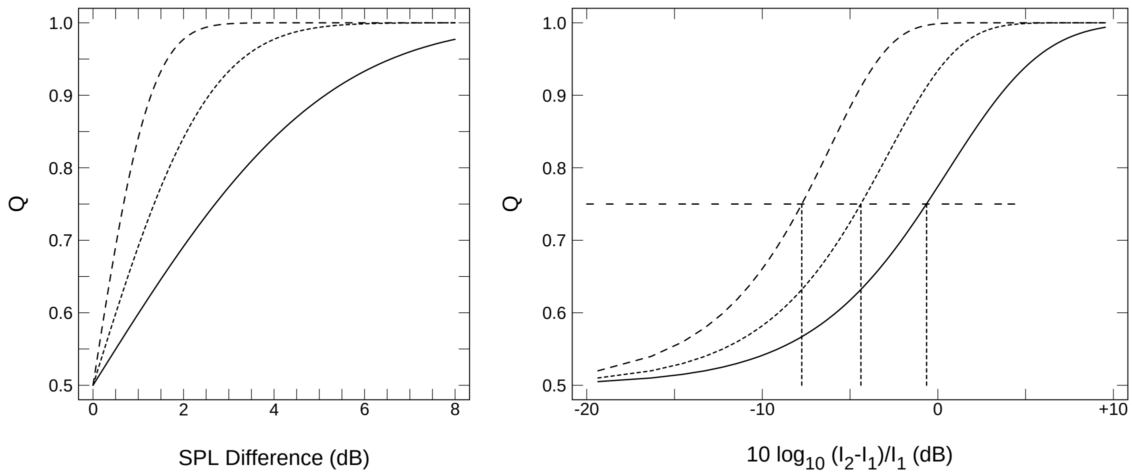

Figure 2: Psychometric functions for Intensity Discrimination. The solid curve corresponds to $k = 5$, the dotted to $k = 10$, and the dashed to k=20. In the left panel the functions are plotted as a function of the SPL difference between the two intensities. In the right panel, the functions are plotted as a function of the logarithm of the Weber fraction (in dB).

mapping of stimuli to responses becomes distorted. The nominally more intense stimulus is actually less intense. The probability of a correct response drops below 50%.

This problem does not arise if the adaptation variable is $10 \log_{10} \Delta I/I$ rather than $\Delta \mathcal{I}$. Note that:

$$10 \log_{10} \frac{\Delta I}{I} = 10 \log_{10} \frac{I_2 - I_1}{I_1} \tag{3}$$

Graphs of the probability of a correct response as a function of $10 \log_{10} \Delta I/I$ are plotted in the right panel of Fig. 2. They indicate that both positive and negative values of $10 \log_{10} \Delta I/I$ lead to probability of a correct response greater than 50%. They also show an additional advantage of using $10 \log_{10} \Delta I/I$ rather than $\Delta I/I$ as the adaptation variable: differences in the value of $k\,(I)$ become differences in the location of the curve rather than differences in the slope and location of the curve.

When plotted as a function of the logarithm of the fractional increase in intensity, the psychometric functions are ogival in shape. Since $Q = \Phi(0.5k(I) \log_{10} \frac{1+\Delta I}{I}) \approx \Phi(0.217k(I)\frac{\Delta I}{I})$, one expects that when $k(I)$ doubles, $\frac{\Delta I}{I}$ is halved to achieve the same value of Q. Thus when Q is plotted as a function of $\log_{10} \frac{\Delta I}{I}$, different values of k yield functions that are translations of one another, the translation being roughly 3 dB for a doubling of k.

This translates into the following. If the adaptive procedure requires an adjustment of intensity I_1 by D (to determine I_2)

$$10 \log_{10} \frac{I_2 - I_1}{I_1} = D$$

then

$$\frac{I_2}{I_1} = 10^{D/10} + 1$$

or

$$10\log_{10}\frac{I_2}{I_0} = 10\log_{10}\frac{I_1}{I_0} + 10\log_{10}\left(10^{D/10}+1\right)$$

that is, to the SPL of a stimulus whose intensity is I_1 add the quantity

$$10\log_{10}\left(10^{D/10}+1\right)$$

to determine the SPL of the stimulus whose intensity is I_2.

As in the case discussed above, setting an initial $\Delta I = I$ so that I_2 is 3 dB higher than I_1 (an initial $D = 0$). The initial stepsize corresponding to $\Delta I/3.16 = I/3.16$ is $D = -5$.

Code

The code for this experiment is a modified version of the code used in Lab2A available on the web:

`http://web.mit.edu/6.182/www/`

as `Lab2b.html`. An outline of the code in presented in Appendix 2.

This code implements an adaptive two-alternative forced-choice tone-detection experiment. You should familiarize yourself with the operation of the experiment and with the code provided before changing the code to control intensity discrimination experiments,

The user specifies:

- Whether the level of the tone or that of the noise is adjusted.
- Whether correct-response feedback is provided on a trial-by-trial basis.
- The target probability for the adaptation.
- The results that are printed at the end of the experiment.

The user may enter a brief description of the experiment that will be printed before the results.

The user is prompted to enter the following parameters of the experiment:

- The calibration level - the dB SPL of the maximum level tone the system can produce.
- The frequency of the tone to be detected.
- The initial rms level of the tone.
- The minimum rms level of the tone.
- The rms level of the noise.
- The initial step size.
- The minimum step size.

There are many parameters that are used to control the experiment. Parameters unlikely to change, such as the sample rate and durations, are not displayed by including `display:none` in the list of `style` parameters (in the `<body>`). You may access these parameters by removing the `display:none` from the list of `style` parameters.

The code is configured for a masked detection experiment. It can be used to perform a crude discrimination experiment by setting the noise level to 0 dB. the minimum tone level to the base SPL (e.g., 70 dB SPL), the inital tone level a few dB higher (e.g. 80 dB SPL), and the iniial step size to -5 dB. Provided the tone level does not decrease below the minimum tone level, the discrimination experiment will proceed correctly. It is strongly suggested that you modify the code to perform a more general discrimination experiment by changing the adaptation process (e.g., Section 2).

You should not have to modify the code for `Element.extend` or for the `Class`es `LevittChanger` and `ExpResults`.

Your changes will largely be resricted to the `Class LevittExp`. Function `initialize` obtains the parameters set by the user. These parameters are accessed via the

`document.getElementById('<parameter>')` or `$(<parameter>)`

construct.[3] The value of each parameter is saved in the `params` string, which will be printed at the beginning of the output if you select `Full` for `Results`.

Most of your modifications will be confined to function `randtone`. Depending on your choice of adaptation variable, you may need to use some of the following `javaScript` mathematical functions, described in

`http://www.w3schools.com/jsref/jsref_obj_math.asp`

- Math: `Math.sqrt(x)`, `Math.ranmdom()`
- Exponential: `Math.exp(x)`, `Math.log(x)`, `Math.pow(x,y)`
- Magnitude: `Math.abs(x)`, `Math.ceil(x)`, `Math.floor(x)`, `Math.round(x)`
- Comparison: `Math.max(x,y)`, `Math.min(x,y)`
- Trigonometric: `Math.sin(x)`, `Math.cos(x)`, `Math.tan(x)`
- Inverse Trigonometric: `Math.asin(x)`, `Math.acos(x)`, `Math.atan(x)`, `Math.atan2(x,y)`

[3] The text of `<parameter>` is obtained by `$(<parameter>).value`. To interpret this text as a floating point number use `$(<parameter>).value.toFloat()`, as an integer use `$(<parameter>).value.toInt()`.

3 Measurements

The durations (rise, fall, and on times; interval between bursts) the bandwidth of the noise, the order parameters for the noise filtering, and sample rate, although read by the `initialize` function are not displayed and thus ae effectively fixed.

Using the Levitt adaptive procedure with a target probability of 0.707, determine the value of ΔI (or equivalently $\log_{10}[1 + \Delta I/I]$) that yields 70.7% correct discrimination responses. Note that 70.7% correct responses corresponds to $d' \approx 1.0885$.

One listener should perform this experiment with I is 40 dB SL $\approx$ 50 dB SPL, and then again at 80 dB SL. The other listener should perform this experiment with I is 80 dB SL $\approx$ 90 dB SPL, and then again at 50 dB SL. When you retest in a second lab session, reverse the order of testing: the first listener tests at $I = 80$ dB SL then at $I = 40$ db SL, the second tests first at $I = 40$ dB SL then at $I = 80$ db SL.

Repeat this measurement with a target percentage of 84.1%, corresponding to $d' \approx 1.9971$. When you are retest in a second lab session, reverse the order of testing. Test first at a target percentage of 84.1%, then at 70.7%.

You should perform each of these four measurements at least three times at each of two separate laboratory sessions.

Make sure you have had an adquate amount of training and practice on the experiment before taking data. As in Laboratory 1, precede each formal measurement with a practice run wholse results are discarded. As in Protocols 2 and 3 of Laboratory 1, you should make all measurements without trial-by-trial correct-response feedback. Feel free to use feedback during practice experiments.

For each level (value of I) estimate the increments corresponding to 70.7% and 84.1% correct responses. Record the estimated size of the increment and the number of stimulus presentations.

4 Theory

Sensitivity – d'

Sensitivity, d', is often assumed to be logarithmically related to the ratio of the sound intensities:

$$d'(I_1, I_2) = k(I_1) \log_{10} \frac{I_2}{I_1}. \tag{4}$$

This is readily seen to be

$$d'(I_1, I_2) = k(I_1)(\mathcal{I}_2 - \mathcal{I}_1) = \frac{k(I_1)}{10}(10\mathcal{I}_2 - 10\mathcal{I}_1) \tag{5}$$

or

$$d' = k(I_1)\Delta\mathcal{I} = \frac{k(I_1)}{10} 10\Delta\mathcal{I}. \tag{6}$$

where $\mathcal{I} = 10\log_{10} I$.

Weber's Law

When applied to intensity discrimination, Weber's Law states that a fixed level of performance (i.e. a fixed percentage of correct responses or equivalently a fixed value of d'), is associated with a fixed ratio of intensities (I_2/I_1), independent of the base intensity. This implies that $k(I_1)$ is independent of I, i.e. $k(I_1) = k$. Let the level of performance be D, then (by Eq. 6

$$D = k\Delta\mathcal{I}, \tag{7}$$

or

$$\Delta\mathcal{I} = \frac{D}{k}. \tag{8}$$

Thus, assuming that d' is logarithmically related to the ratio of the sound intensities is equivalent to assuming that it is linearly related to the difference in intensity levels.

Analysis

The results of four intensity discrimination experiments are shown in Table 1. The experiments develop estimates of intensity discrimination at 40 and 80 dB SL (50 and 90 dB SPL), each at two performance levels, corresponding to target probabilities of 0.707 and 0.841.

The hypothesis that $d' = k(I_1)\log(I_2/I_1)$ can be tested, independent of the value of $k(I_1)$, by examining the results of sets of experiments in which I1 is held constant. For experiments A and B with equal I_1,

$$d'_A = k(I_1)\Delta\mathcal{I}_{\mathcal{A}} \tag{9}$$

and

$$d'_B = k(I_1)\Delta\mathcal{I}_{\mathcal{B}} \tag{10}$$

	Intensity	
Pr (Correct)	50 dB SPL	90 dB SPL
0.841	$\Delta\mathcal{I}_{21}$	$\Delta\mathcal{I}_{22}$
0.707	$\Delta\mathcal{I}_{11}$	$\Delta\mathcal{I}_{12}$

Table 1: The results of an adaptive intensity discrimination experiment conducted at two intensities (50 and 90 dB SPL) and with two target probabilities of a correct response (0.707 and 0.841), corresponding to $d' = 1.1$ and $d' = 2.0$.

solving eqs. 9 and 10 for $k(I_1)$ yields

$$\frac{d'_A}{\Delta\mathcal{I}_{\mathcal{A}}} = \frac{d'_B}{\Delta\mathcal{I}_{\mathcal{B}}} \tag{11}$$

or

$$\frac{d'_A}{d'_B} = \frac{\Delta\mathcal{I}_{\mathcal{A}}}{\Delta\mathcal{I}_{\mathcal{B}}}. \tag{12}$$

In the context of the experimental results given in Table 1, if experiment A has $\Pr(\text{Correct}) = 0.841$ and experiment B has $\Pr(\text{Correct}) = 0.707$, then

$$\frac{d'_A}{d'_B} = \frac{2z(0.841)}{2z(0.707)} = \frac{2.0}{1.1} = 1.8. \tag{13}$$

So, if the hypothesis that $d' = k(I_1)\log(I_2/I_1)$ is true, then Eq. 12 shows that for 50 dB SPL,

$$1.8 = \frac{\Delta\mathcal{I}_{21}}{\Delta\mathcal{I}_{11}} \tag{14}$$

and for 90 dB SPL,

$$1.8 = \frac{\Delta\mathcal{I}_{22}}{\Delta\mathcal{I}_{12}} \tag{15}$$

The hypothesis that Weber's law applies to intensity discrimination is tested by determining whether $k(I)$ is independent of I. It is assumed that

$$d'_A = k(I_A)\,\Delta\mathcal{I}_{\mathcal{A}} \tag{16}$$

and

$$d'_B = k(I_B)\,\Delta\mathcal{I}_{\mathcal{B}} \tag{17}$$

but if $d'_A = d'_B = d'$, then

$$k(I_A) = k(I_B) = k \tag{18}$$

if and only if

$$\frac{d'}{k} = \Delta\mathcal{I}_A = \Delta\mathcal{I}_B. \tag{19}$$

Thus, in the context of the experimental results, there are two tests for determining whether Weber's Law applies:

$$\Delta\mathcal{I}_{11} = \Delta\mathcal{I}_{12} \text{ and } \Delta\mathcal{I}_{21} = \Delta\mathcal{I}_{22}. \tag{20}$$

5 Report

The report on this laboratory exercise will be a major contributor to satisfying the CI-M requirement associated with 6.182 as well as satisfying the technical aspects of the subject. CI-M subjects (Communication Intensive in the Major) teach the specific forms of written, oral, and/or visual communication appropriate to the field's professional and academic culture.

The draft version of your laboratory report is due on March 20, 2008. You will receive feedback on this submission by April 10, 2008. If you choose, you may revise your report and submit it by April 17, 2008 to receive a final grade.

In writing up your Laboratory Report

1. Feel free to discuss both your and your laboratory partner's data.
2. Present your raw data and the code that you have modified in an appendix.
3. Make sure to discuss the details of your experiments:
   - Who were the listener(s).
   - What was the order of the experiments.
   - Mention the details of the adaptive procedure, e.g. the inital level, step size, the adaptation variable, and for the Levitt up-down procedure, the total number of reversals and number of reversals you included in your estimates of the data points.
4. Consider presenting your processed data in both graphical and tabular form.
   - If you present the data as graphs, consider the axes carefully (e.g., Appendix III). Plotting the dB increment vs. percentage of correct responses may not be so good an idea as plotting the dB increment vs. d', because when the dB increment is zero, d' is zero.
   - Be sure to present both the average and standard deviation of your measurements.
5. Consider performing statistical tests on your measurements. Either the t-test or the Mann-Whitney (Wilcoxon) test would appear to be appropriate.
6. You should be familiar with other data with which your results can be compared. Are your results in qualitative agreement with that data or in qualitative disagreement. Are your results in quantitative agreement with that data or in quantitative disagreement.

6 Appendix I.

Sound Pressure

Pressure is a measure of the force (Newtons) per unit area (square meters). It consists of an average value, or barometric pressure, plus a time varying component.

$$p_T(t) = P_Q + p(t) \tag{21}$$

Sound consists of the propagating time varying component, $p(t)$. Necessarily this has zero average value and a root-mean-square or *rms* value P. The rms value is measured relative to a reference, $P_{\text{ref}} = 2 \times 10^{-5}$ Newtons/square-meter, or $P_{\text{ref}} = 2 \times 10^{-5}$ Pascals.

Sound Intensity and dB SPL

Sound intensity I is a measure of the sound power (watts) passing through a given area (square meter). It is typically measured relative to a reference, $I_{\text{ref}} = 1 \times 10^{-12}$ watts/square-meter. The intensity level of a sound, $\mathcal{I}$ is a logarithmic measure of the sound intensity,

$$\mathcal{I} = \log_{10} \frac{I}{I_{\text{ref}}} \tag{22}$$

It is measured in Bels. More commonly, we use decibel measure

$$10\mathcal{I} = 10 \log_{10} \frac{I}{I_{\text{ref}}} \tag{23}$$

Sound intensity is proportional to the square of sound pressure. The references for sound pressure P_{ref} and I_{ref} are consistent with each other, so that

$$\begin{aligned} \frac{I}{I_{\text{ref}}} &= \left(\frac{P}{P_{\text{ref}}}\right)^2 \\ 10 \log_{10} \frac{I}{I_{\text{ref}}} &= 20 \log_{10} \frac{P}{P_{\text{ref}}} \end{aligned}$$

provides a measure that is often referred to as *dB SPL*.

The *sound pressure level* of a sound, $\mathcal{P}$, is a logarithmic measure of the rms sound pressure. It is commonly in decibels, where 1 decibel (1 dB) is 0.1 Bels. Thus we use

$$20\mathcal{P} = 20 \log_{10} \frac{P}{P_{\text{ref}}} \tag{24}$$

to measure the sound pressure in dB SPL.

7 Appendix II.

Code Outline

```
<html>
<head>
<script>
//      Global variables

function begin()
function soundgen_callback(obj)

var LevittExp = new Class({
initialize: function(),
randtone: function(),
soundgen_callback: function(obj),
reportresp: function(response)
})

var LevittChanger = new Class({
initialize: function(nc, ni, initial_stepsize, min_stepsize, torn),
receive_response: function(accuracyArray),
turnAround: function()
})

var ExpResults = new Class({
initialize: function(),
savePresenteddB: function(db, presentationNumber),
saveAccuracy: function(response, correctResponse, presentationNumber),
saveTurnAround: function(turnAroundNumber, presentationNumber),
formatResultsAsHTML: function(avgTurns)
})

Element.extend()

function square(x)

function format(total_length,postdecplaces,n)
</script>
</head>
<body>
</body>
</html>
```

8 Appendix III. Graphs of Results.

Graphs can be used both to display data and also to test theories.

Consider the theoretical expression for sensitivity (d') in an intensity discrimination experiment:

$$d'(\Delta\mathcal{I}) = k(\mathcal{I})\,\Delta\mathcal{I}, \tag{25}$$

where $\mathcal{I}$ is the base intensity (in dB SPL or dB SL) and $\Delta\mathcal{I}$ is the difference (in dB) between the two intensities being discriminated. This theoretical construct is made *real* by the predictive relation

$$Q = \Phi(d'/2) \tag{26}$$

which specifies how the probability of a correct response, Q, is related to d'.

The goals of this laboratory are to determine 1) whether Eq. 25 applies *locally* to discrimination experiments and 2) whether Weber's Law applies, i.e. whether $k(\mathcal{I})$ is a constant independent of $\mathcal{I}$. The experiment estimates adaptively the values of $\Delta\mathcal{I}$ that produced 70.7% and 84.1% correct responses (corresponding to $Q = 0.707$ and 0.841) for two different values of $\mathcal{I}$ = 50 and 90 dB.

Now consider a graphical representation of the results of this laboratory. Although it is tempting to graph $\Delta\mathcal{I}$ vs Q such a representation of the data is essentially the same as a tabulation of the data because the relation between Q and $\Delta\mathcal{I}$ is nonlinear. Representing such graphs with straight lines, although visually useful for grouping results, is highly misleading. Such two-point graphs are trivially fit by straight lines.

Representing the relation between $\Delta\mathcal{I}$ vs Q by a straight line misses an important fact: the graph of $\Delta\mathcal{I}$ vs d' must be a straight line that passes through the origin (Eq. 25). It is highly unlikely that three points will be perfectly fit by a straight line.

% Correct	d'	Level - dB SL	Mean - dB	Standard Error - dB
70.7%	1.1	50	1.00	0.1
70.7%	1.1	90	0.55	0.1
84.1%	2.0	50	2.20	0.1
84.1%	2.0	90	0.85	0.1

Table 2: Results four experiments (two target percentages and two sound levels) in terms of the mean and standard error of $\Delta\mathcal{I}$.

An illustration of this difference that uses the data in Table 1 is shown in the two graphs below.

Note that the goodness of fit of the straight lines to the data in the right panel provides a visual measure of the degree to which Eq.25 is satisfied, whereas the

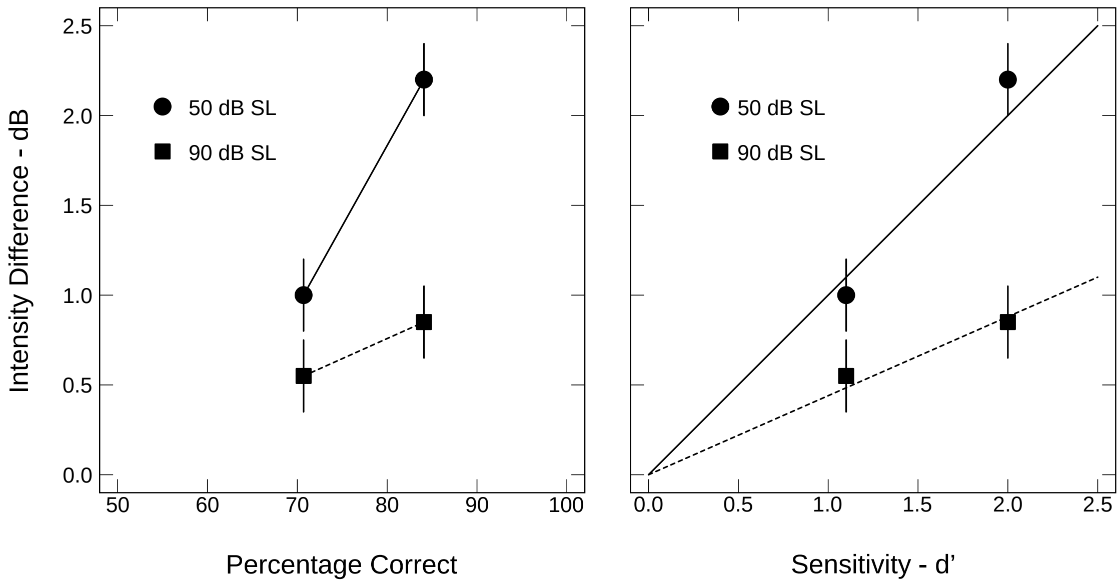

Figure 3: Left panel: Percentage Plot; right panel: Sensitivity Plot.

agreement between the slopes of the two lines provides a visual measure of the degree to which Weber's Law is satisfied (i.e., that $k(\mathcal{I})$ is independent of $\mathcal{I}$). Neither of these two benefits are available in the plots in the left panel.