Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.242, Fall 2004: MODEL REDUCTION *

Problem set 3[1]

Problem 3.1

Hankel singular numbers of a stable causal LTI system G of very large order are given by

$$\sigma_k(G) = 2^{-m} \quad \text{for} \quad k = 2^m + 1, \dots, 2^{m+1}, \quad (m = 0, 1, \dots), \quad \sigma_1(G) = 2.$$

(a) Suggest a good a-priori lower bound for the quality $\|G - \hat{G}\|_\infty$ of approximating G by a system $\hat{G}$ of order 8.

(b) Suggest a good a-priori upper bound for the quality $\|G - \hat{G}_{btr}\|_\infty$ of approximating G by a system $\hat{G}_{btr}$ of order 8 using the method of balanced truncation.

(c) What are the Hankel singular numbers of $\hat{G}_{btr}$ from (b)?

Problem 3.2

Give an explicit description of the set of all possible n-vectors

$$[\sigma_1(G), \sigma_2(G), \dots, \sigma_n(G)],$$

formed by the Hankel singular values $\sigma_k(G)$ of all stable "all pass" transfer functions G of order n (i.e. such that $|G(j\omega)| = 1$ for all ω).

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[1] Version of September 29, 2004. Due October 6, 2004.

(a) Design a numerical experiment, utilizing MATLAB functions `lyap`, `chol`, and `eig`, to collect data on the topic. (Do not use `sysbal` and similar "full service" model reduction functions.)

(b) Formulate a hypotheses on what the answer is.

(c) Prove the hypotheses (at least for the case $n = 2$). **Hint:** one relatively easy way to do this in general is by using the KYP Lemma.

Problem 3.3

Use the method of balanced truncation to find a 10th order reduced model for the system described in Problem 2.3, with $M = 1$, $B = 0.2$, $K = 4$, and $n = 50$. (Do not use `sysbal` and similar "full service" model reduction functions.)