Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.242, Fall 2004: MODEL REDUCTION *

Problem set 5[1]

This problem set, to serve as preparation for Quiz 1, contains theoretical questions only.

Problem 5.1

For each of the stetements below, state if it is true or false. For false statements, give a counterexample. For correct statements, give a *brief* sketch of a proof.

(a) H-Infinity norm of system with transfer function

$$H(s) = D + C(sI - A)^{-1}B$$

is not smaller than $|D|$.

(b) If A, B, C, D are matrices of dimensions n-by-n, n-by-1, 1-by-n, and 1-by-1 respectively, and matrices

$$M_c = \left[\begin{array}{ccccc} B & AB & A^2B & \dots & A^{n-1}B \end{array} \right], \quad M_o = \left[\begin{array}{c} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{array} \right]$$

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have rank $n-1$ then order of system with transfer function

$$H(s) = D + C(sI - A)^{-1}B$$

equals $n-1$.

(c) If $Aq = 0$ for some vector $q \neq 0$ then

$$\lim_{s \to 0} sC(sI - A)^{-1}B = Cq.$$

(d) If A is a Hurwitz matrix, and matrix V is such that $V'V = I$ then $V'AV$ is a Hurwitz matrix as well.

(e) If A is a Hurwitz matrix, the pair (A, B) is controllable, the pair (C, A) is observable, and $W > 0$ is a diagonal matrix such that

$$AW + WA' = -BB', \quad WA + A'W = -C'C,$$

then $A_{11} < 0$, where A_{11} is the upper left corner element of A.

(f) If a proper rational transfer function $G = G(s)$ without poles in the closed right half plane satisfies $|G(j\omega)| \leq 1$ for all $\omega \in \mathbf{R}$, all Hankel singular numbers of G are not larger than 1.

Problem 5.2

What is the order of the LTI system with transfer matrix

$$H(s) = \begin{bmatrix} 1/(s+1) & 1/(s+2) \\ 1/(s+3) & 1/(s+3) \end{bmatrix} ?$$

Problem 5.3

Knowing that $G(j\omega) = 1 + j$ for $\omega = 1$, $G(0) = 1$, $G(-1) = 5$, and $G(\infty) = 4$, what is the best lower bound of the H-Infinity norm of a rational function G?

Problem 5.4

Find L2 gain of the system which maps scalar inputs $f(t)$ into outputs

$$y(t) = f(t-1)/(1 + t^2 + |f(t-1)|^2).$$

Problem 5.5

A, B, C, D are matrices of dimensions n-by-n, n-by-1, 1-by-n, and 1-by-1 respectively, such that $CA^{-3}B = 1$ and matrix UV, where

$$U = \begin{bmatrix} C \\ CA^{-1} \end{bmatrix}, \quad V = \begin{bmatrix} B & A^{-1}B \end{bmatrix},$$

is not singular. Is this information sufficient to find $\hat{C}\hat{A}^{-3}\hat{B}$, where

$$\hat{C} = CV, \quad \hat{B} = UB, \quad \hat{A} = UAV \ ?$$

Problem 5.6

What is the value of the 7th largest Hankel singular value of

$$H(s) = (s^2 - 2s + 2)^5/(s^2 + 2s + 2)^5 \ ?$$

Problem 5.7

A fifth order transfer function $\hat{G}$ is obtained by applying the standard balanced truncation algorithm to a seventh order transfer function G which has Hankel singular numbers $7, 6, 5, 4, 3, 2, 1$. What are the Hankel singular numbers of $\hat{G}$? What is the range of possible values of $\|G - \hat{G}\|_\infty$?