Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.242, Fall 2004: MODEL REDUCTION *

Problem set 6[1]

Problem 6.1

(a) Find an analytical expression for the coefficients $c_1, \dots, c_n$ of the linear combination

$$\hat{G}_n(s) = \sum_{k=1}^{n} \frac{c_k}{s + 1/k},$$

which minimizes the integral

$$\|G - \hat{G}_n\|_{H2}^2 = \frac{1}{2\pi} \int_{-\infty}^{\infty} |G(j\omega) - \hat{G}_n(j\omega)|^2 d\omega,$$

where

$$G(s) = \frac{s^{1/3}}{s+1}.$$

(b) For $n = 1, 2, \dots, 50$, use MATLAB to compute and compare $\|G - \hat{G}_n\|_{H2}$ and $\|G - \hat{G}_n\|_\infty$.

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[1]Version of November 11, 2004. Due November 19, 2004.

Problem 6.2

(a) Is it true or false: if $f:\ [0,1]\mapsto(-\infty,0)$ is convex then $1/f$ is convex as well?

(b) Is it true or false: the sum of two quasi-convex functions $f_1,f_2:\ \Omega\mapsto\mathbf{R}$ is always quasi-convex?

(c) For $m,n\in\{1,2,3,\dots\}$ let $\Phi:\ \Omega_{n,m}\mapsto\mathbf{R}$ be the function

$$\Phi_{n,m}(x)=\sum_{k=1}^{n}\left|\frac{p(k/n)}{q(k/n)}-y_k\right|,$$

where

$$p(t)=p_0+p_1t+\dots+p_mt^m,\quad q(t)=1+q_1t+\dots+q_mt^m,$$

and $\Omega_{n,m}=\{x\}$ is the set of vectors

$$x=[p_0;p_1;\dots;p_m;q_1;q_2;\dots;q_m;y_1;y_2;\dots;y_n]$$

such that $q(t)\neq 0$ for all $t\in[0,1]$. For which values of $m,n\in\{1,2,3,\dots\}$ is $\Phi_{n,m}$ quasi-convex?

Problem 6.3

(a) For every $n\in\{1,2,\dots\}$ and $\epsilon>0$, define $N=N(n)$ and an affine symmetric matrix function $A=A(x)$ of vector

$$x=[b_0;b_1;\dots;b_n;a_0;a_1;\dots;a_{n-1};y_1;\dots;y_N]$$

such that, given $b_0,\dots,b_n$ and $a_0,\dots,a_{n-1}$, the inequality $A(x)>0$ has a solution with respect to $y_1,\dots,y_N$ if and only if

$$b(\omega^2)>0,\quad a(\omega^2)>0,\quad \frac{b(\omega^2)}{a(\omega^2)}<1+\epsilon\ \ \forall\ \omega\in\mathbf{R},$$

$$\frac{b(\omega^2)}{a(\omega^2)}>1-\epsilon\ \forall\ |\omega|\le 1,$$

and

$$\frac{b(\omega^2)}{a(\omega^2)}<\epsilon\ \forall\ |\omega|\ge 1+\epsilon,$$

where

$$a(\theta)=a_0+a_1\theta+\dots+a_{n-1}\theta^{n-1}+\theta^n,\quad b(\theta)=b_0\theta+b_1\theta+\dots+b_n\theta^n.$$

(b) Use the result from (a) and a semidefinite program solver (see section 4 of Lecture 9 for some options) to write a MATLAB code for designing high quality low pass filters, in the form of a stable n-th order transfer function G such that

$$\|G\|_\infty^2 < 1 + \epsilon, \quad 1 - |G(j\omega)|^2 > 1 - \epsilon \ \forall\, \omega \in [0, 1], \quad |G(j\omega)|^2 < \epsilon \ \forall\, \omega \in [1 + \epsilon, \infty),$$

where $\epsilon > 0$ is a given small parameter.