Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.242, Fall 2004: MODEL REDUCTION *

Take-home test 2[1]

Problem Q2.1

Construct a strictly proper rational transfer function $G = G(s)$ with no poles in the closed right half plane for which there exists a unique first order transfer function $\hat{G} = \hat{G}(s)$ such that $G(1) = \hat{G}(1)$, and $G(j) = \hat{G}(j)$ ($j = \sqrt{-1}$), and this $\hat{G}$ has a pole with positive real part. (As usually, G and $\hat{G}$ have *real* coefficients.)

Problem Q2.2

Proper transfer function $\Delta = \Delta(s)$ with no poles in the closed right half plane is such that $\Delta(1) = a$, where $a \in \mathbf{R}$ is a parameter. What is the minimal possible value of the integral

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{|\Delta(j\omega)|^2 d\omega}{1+\omega^2} \quad ?$$

Problem Q2.3

Among all proper rational transfer functions $G = G(s)$ with not more than one pole in the right half plane find the one for which $\|G(s) - 1/(s^2+s+1)\|_\infty$ is minimal.

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[1]To be returned by 2.30pm on December 8, 2004.

Problem Q2.4

G is a 100-th order rational transfer function with no poles in the right half plane, such that $|G(j\omega)| = 1$ for all $\omega \in \mathbf{R}$. Find all possible values of the 10-th Hankel singular number of $\Delta(s) = G(s) - 1/(s+1)$. Explain your answer.

Problem Q2.5

Find a positive integer n and an affine symmetric matrix-valued function $\alpha = \alpha(z)$ of a real vector parameter $z \in \mathbf{R}^{n+3}$, such that the conditions $a_0 > 0$ and

$$\sup_{\omega \in \mathbf{R}} \left| \frac{b_0 + b_1\omega^2}{a_0 + \omega^2} - \frac{1}{(\omega^2+1)^2} \right| < \frac{1}{2}$$

are jointly satisfied if and only if there exist real numbers $y_1, \dots, y_n$ such that $\alpha(z) > 0$ for

$$z = \begin{bmatrix} a_0 \\ b_0 \\ b_1 \\ y_1 \\ \vdots \\ y_n \end{bmatrix}.$$